$\delta -$ CONVERGENCE OF FILTER

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Abstract : The notion of convergence is most important concepts in topology. Nets and Filters are two notions to fill this need for introducing the concept of convergence in more general setting of topological space.By introducing the concept of $\delta -$ convergence of filters we have proved various results.

1.Introduction : In 1922,E.H.Moore and H.L. Smith developed a generalized version of sequence to produce the notion of nets,whereas in 1936,Henri Cartan presented a formal theory is called filter.

Although they lead to essentially equivalent theories, the nets have the advantage that they are very natural and are direct generalization of sequences with which, we are all too familiar.On the other hand,they are some thing about filters that have no natural analogues for nets.

Moreover filters can be defined and studied in other contexts such as Boolean Algebras.As a result,recently filters are more vogue than nets.

Jong Suh Park in the paper [7] has got many interesting results related with H-closed spaces.By using the notion of

σ-continuous maps,**ω**-closure,**ω**-accumulation point etc.various results are proved concerned with these concepts

In this paper we have introduced the concept of $\delta -$ convergence and have got many results.While proving the theorems many concepts like $\delta -$ convergence of filters,

$\delta -$ cluster point of filters and $\delta -$ Hausdorff spaces.

Throughout the paper, spaces are topological spaces, symbols X,Y,Z are used for topological spaces and f,g,h are used for maps between topological spaces. For terms and notation not explained here we refer the reader to [3,8,9].

2.$\delta -$ Convergence of Filters :

The section begins with the following definition.

2.1 Definition : Let (X, $\Im$) be a topological space and let F be a filter on X. F is said to $\boldsymbol{\delta} -$ Converge to x if every neighbourhood U of x such that Int Cl U ∈ F whereU ∈ η_x.

2.2 Definition : Let (X, $\Im$) be a topological space and let F be a filter on X. F is said to $\delta -$ accumulate to a point x of X denoted by F $\overset{\delta}{\underset{\alpha}{}}$ x if A ∩ Int Cl U $\neq \phi$ for all neighbourhood U of X and A∈ F.

2.3 Proposition : Let S : D→ X be a net and F be the associated filter with it.Then S : D→ X $\boldsymbol{\delta} -$ Converge to x∈ X

As a net iff F $\boldsymbol{\delta} -$ Converge to x as a filter.

Proof : It is given that S : D→ X be a net and F is the associated filter with it.Assume that S : D→ X $\boldsymbol{\delta} -$ Converge to x∈ X as a net.To show that F $\boldsymbol{\delta} -$ Converge to x as a filter take a neighbourhood U of x in X.Since S : D→ X $\boldsymbol{\delta} -$ Converges to x ∈ X there exists p∈D such that S(n) ∈ Int Cl U

For all n≥p.

Recall from the construction of associated filter that

B_m={ S(n) }| n∈D, n≥p}.

It means that

B_p⊂ Int Cl U

Thus Int Cl U ∈ F

Since U is arbitrary, F $\boldsymbol{\delta} -$ Converge to x

Conversely assume that F $\boldsymbol{\delta}-$ Converge to x as a filter.

To show that S : D→ X $\boldsymbol{\delta}-$ Converges to x ∈ X as a net take a neighbourhood U of x in X.

Since F $\overset{\delta}{\rightarrow}$ x, Int Cl U ∈ F.

By construction of associated filter there exists m∈D such that B_m⊂ Int Cl U

B_m={ S(n) }| n∈D, n≥m}.

It means that

S(n) ∈ Int Cl U For all n≥m.Since U is arbitrary we conclude that S : D→ X $\boldsymbol{\delta}-$ Converges to x $in\ X$.

2.4 Proposition : Let F be a filter in a space X and S : D→ X be the associated net in X. Let x ∈ X, x is a $\boldsymbol{\delta}-$ cluster point of the filter F iff it is a $\boldsymbol{\delta}-$ cluster point of the netS : D→ X.

Proof : It is given that F be a filter in a space X and S : D→ X be the associated net in X.Assume that F has x ∈ X as a

$\boldsymbol{\delta}-$ cluster point of the associated net S : D→ X,taking

D={(y,G):G ∈ F, y∈ G} and putting S(y,G)=y.Take a neighbourhood U of x ∈ X and an element (y,G) of D be given.

Since F has x ∈ X as a $\boldsymbol{\delta}-$ cluster point G∩ Int Cl U ≠ ∅.

Let z∈ G ∩ Int Cl U

Then (z,G) ∈D, (z,G) ≥ (y,G)

And S(z,G)=z∈ Int Cl U,since U is arbitrary. We conclude that x is $\boldsymbol{\delta}-$ cluster point of S : D→ X.

Conversely, suppose that x is a $\boldsymbol{\delta}-$ cluster point of S : D→ X.

Take a neighbourhood U of x ∈ X and f∈ F to show that

F ∩ Int Cl U ≠ ∅.

Let z be any point of F.Then (z,F) ∈D.

Since x is a $\boldsymbol{\delta} -$ cluster point of S, there exists (y,G) ∈ D such that (y,G) ≥ (z,G) and S(y,G) ∈ Int Cl U.

But then y∈ G, G⊂F and y∈ Int Cl U showing that

F∩ Int Cl U ≠ ∅ as desired.We conclude that x is a

$\boldsymbol{\delta} -$ cluster point of the filter F.

2.5 Theorem : A topological space is $\boldsymbol{\delta} -$ Hausdorff iff no filter can $\boldsymbol{\delta}$-converge to more than one point in it.

Proof : Let X be a $\boldsymbol{\delta} -$ Hausdorff space.To show that no filter can $\boldsymbol{\delta}$-converge to more than one point in it,take a filter F which is $\boldsymbol{\delta}$-converges to x as well as y.It means that $\eta_x \subset$ F and $\eta_y \subset$ F.

Now if x≠ y,there exists U∈ η_x, V∈ η_y such that

Int Cl U∩ Int Cl V= ϕ,which will contradict that F has the finite intersection property so x=y. Thus limits of convergent filters in X are unique.

Conversely, assume that no filter in X has more than one limit in X. To show that X is $\boldsymbol{\delta} -$ Hausdorff space, suppose on the contrary that X is not $\boldsymbol{\delta} -$ Hausdorff space,there exsit x,y ∈ X, x≠ y such that every neighbourhood of x intersects every neighbourhood of y ,i.e. $\text{Int Cl U} \cap \text{Int Cl V} \neq \phi$.

From this it follows that the family $\text{Int Cl U} \cap \text{Int Cl V}$ where

U∈ η_x, V∈ η_y has the finite intersection property.Then there exists a filter F on X which $\boldsymbol{\delta}$-converges to both x and y contradicting our hypothesis.Thus X is $\boldsymbol{\delta} -$ Hausdorff space.

2.6 Proposition: Let F be a filter in a space X and

S : D→ Xbe the associated net in X.Then F $\boldsymbol{\delta}$-converges to x ∈ X,as a filter if and only if S : D→ X $\boldsymbol{\delta}$-converges to x ∈ X as a net.

Proof : Assume that F $\boldsymbol{\delta}$-converges to x. To show that

S : D→ X $\boldsymbol{\delta}$-converges to x. Since F $\boldsymbol{\delta}$-converges to

x∈ X,take a neighbourhood U of x∈ X, such that

Int Cl U ∈ F .It is given that S : D→ Xbe the associated net in X .Recall from the construction of the associated net.

S(x, Int Cl U)=x

And G ⊂ Int Cl U

It follows that

S(x, Int Cl U) ∈ Int Cl U

Since U is arbitrary.

We conclude that S : D→ X $\boldsymbol{\delta}$-converges to x as a net.

Conversely ,assume that S : D→ X $\boldsymbol{\delta}$-converges to

x∈X.To show that F is $\boldsymbol{\delta}$-converges to x in X take a neighbourhood U of x in X. By construction of associated net

D={(x,F) ∈X× F:x ∈ F}

For (x,F), (y,G) ∈ D which is defined by (x,F) ≥(y,G).

Since S : D→ X $\boldsymbol{\delta}$-converges to x in X,

S(y,G) ∈ Int Cl U.

It means that G ⊂ Int Cl U.

Then Int Cl U ∈ F.

Since U is arbitrary, F $\boldsymbol{\delta}$-converges to x in X.

2.7 Proposition : Let S : D→ X be a net in a space X and F be the associated filter in X. Then x∈X $\boldsymbol{\delta}$-cluster point of the net S : D→ X iff it is a $\boldsymbol{\delta}$-cluster point of the filter F .

Proof : It is given that S : D→ X be a net and F be the associated filter with it .Assume that S : D→ X has x∈X

As a $\boldsymbol{\delta}$-cluster point,for any neighbourhood Uof x and n there is an m such that S(m) ∈ Int Cl U:for all m ≥ n.

Recall from the construction of associated filter

B_m={ S(n) }| n∈D, n≥m}.

It is given that

$B_p \subset A$

Where B_p={ S(n) }| n∈D, n≥p} and A∈ F .

Therefore there exist m≥p such that S(m) ∈ Int Cl U:for all m ≥ p.

It is seen that S(m) is a common point between A and Int Cl U

i.e. A ∩ Int Cl U ≠ ∅.

Since A and U are arbitrary we conclude that x is a $\boldsymbol{\delta}$-cluster point of the filter F .

Conversely suppose that F has x∈X as a $\boldsymbol{\delta}$-cluster point.

To show that x is a $\boldsymbol{\delta}$-cluster point S : D→ X,take a neighbourhood U of x∈X and n∈D. Since F has x∈X as a

$\boldsymbol{\delta}$-cluster point

A ∩ Int Cl U ≠ ∅ for all A∈ F .

Now from the construction of associated filter

B_n={ S(p) }| p∈D, p≥n}

It means that

$B_n \cap$ Int Cl U ≠ ∅.

Which implies that

S(p) ∈ Int Cl U : p∈D, p≥n.

Hence S : D→ X has x∈X as a $\boldsymbol{\delta}$-cluster point.

2.8 Example : Let (X,$\Im$) be a topological space and be the filter in X where

X={a,b,c)

$\Im$ ={X, ∅,{a},{b},{a,b}}

$\zeta(X)$={ ∅,X,{b,c} ,{a,c},{c} }

F ={X, {a,b}}

We see that this filter neither convergence nor $\boldsymbol{\delta}$-converges to the point

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