PROBABILITY LABELING ON ROAD NETWORK

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Absract:- In this paper we use probability values as labels of streams in a traffic network of a city. Using these values we can easily manage the timing of traffic lights at the junction/city. We introduce a new labeling called Probability Labeling of Graph and it is illustrated with an example.

Key words: Labeling, Fuzzy Graph

1. Introduction

There are a number of famous and difficult graph-theoretical problems that arose over the past four decades from the design of networks such as computer networks, road networks and many others. Fuzzy Graphs were introduced by Rosenfeld, ten years after Zadeh's paper "Fuzzy Sets". Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like paths, cycles and connectedness and established some of their properties.

2. Preliminaries

2.1 A ***Graph***[2]consists of a finite nonempty set V=V(G) of p-vertices together with a prescribed set E of q unordered pairs of distinct vertices of V

2.2 A **Graph Labeling**[5] is an assignment of integers to the vertices or edges, or both, subject to certain conditions

2.3 A ***Fuzzy graph*** [3] $G=(V,\sigma,\mu)$ is a triple consisting of a nonempty set V together with a pair of functions $\sigma:V\rightarrow[0,1]$ and $\mu:VxV\rightarrow [0,1]$ such that for all $x,y\in V$, $\mu(x,y)\leq\sigma(x)\wedge\sigma(y)$ for all x,y .

2.4 Sample Space[6] The totality of all possible outcomes of a random experiment is called the sample space.

2.5 An **Event**[6] is any subset of a sample space

2.6 Probability Axioms[6]: Let S be a sample space of a random experiment and A be an event. The probability function P(.) must satisfy the following Kolmogrov's axioms;

(i) For any event A, $P(A)\geq 0$

(ii) P(S)=1

(iii) P(AUB)= P(A)+P(B) provided A and B are mutually exclusive events(i.e.,$A\cap B=\emptyset$)

2.7 Discrete Random Variable and probability mass function: Let X be a discrete random variable with possible values $x_1,x_2, \ldots x_n$ then probability mass function of X is a function P from X to [0,1] satisfying the condition $\sum_i P(x_i)=1$

3. Formulation of the Problem

If there are 4 roads meet at a traffic junction. Let these roads are represented by the numbers 1,2,3 and 4(clock wise order). Now the possible streams of vehicle movement are from 1 to 2, from 1 to 3, from 1 to 4, from 2 to 1, from 2 to 3, from 2 to 4, from 3 to 1, from 3 to 2, from 3 to 4, from 4 to 1, from 4 to 2 and from 4 to 3(Fig:1)[4]. The number of vehicles at a particular time 't' on these streams can be represented by f_{12}, f_{13}, f_{14}, f_{21}, f_{23}, f_{24}, f_{31}, f_{32}, f_{34}, f_{41}, f_{42}, f_{43}.

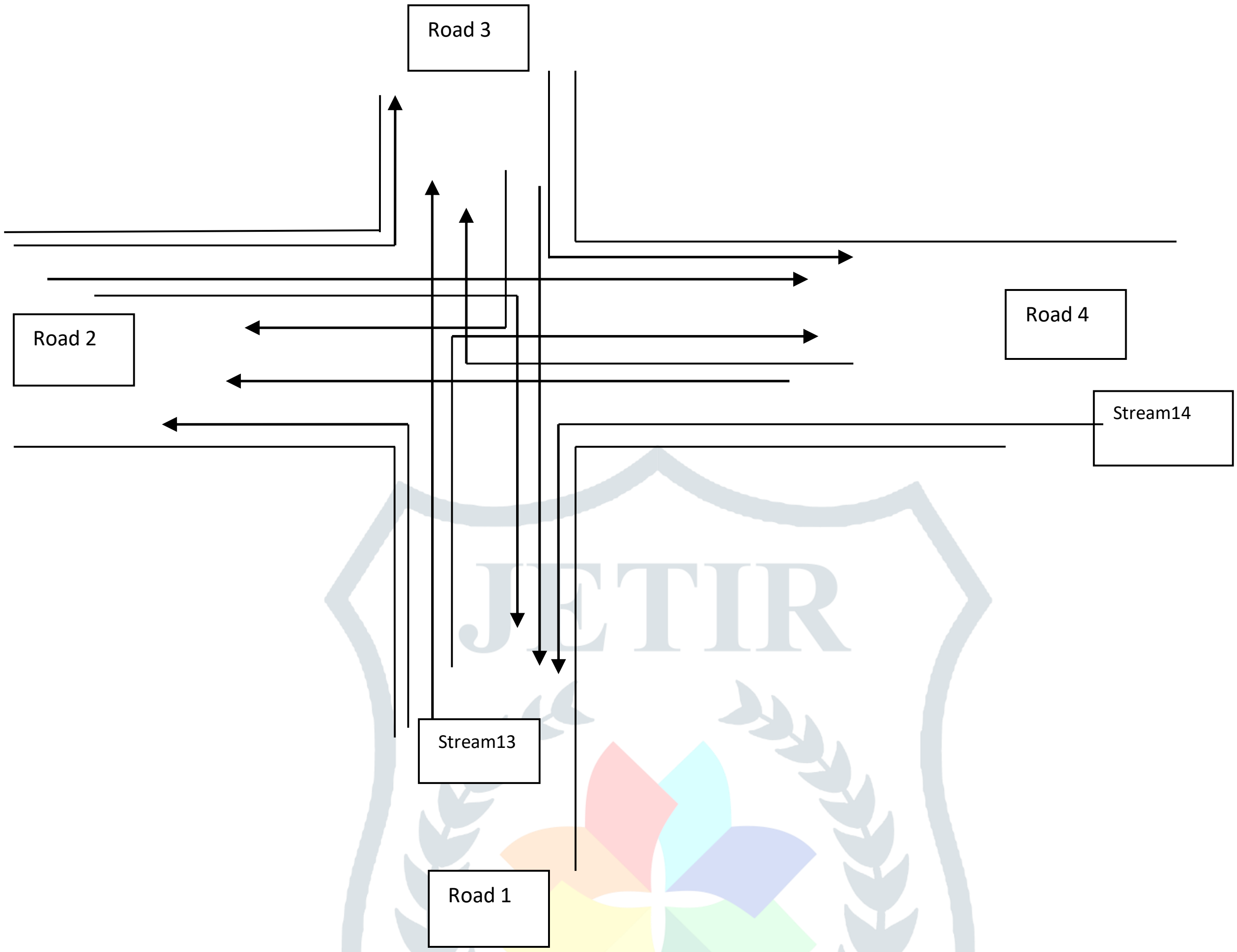
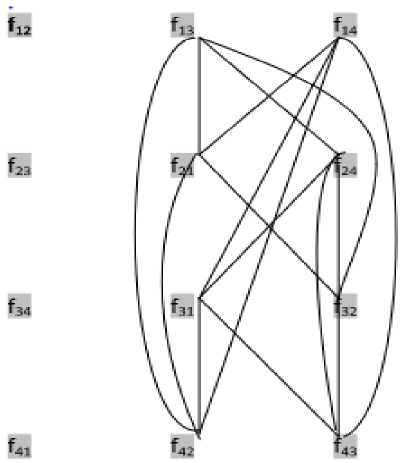


Fig:1 Traffic Streams at junction of 4 roads

Generally there are 'n' roads meet at a junction(n>2) . Then the possible streams are (1,2),(1,3),(1,4),... (1,n); (2,1),(2,3),(2,4),…(2,n); (3,1),(3,2),(3,4),…..(3,n); (4,1),(4,2),…(4,n); …..; (n,1),(n,2),…(n,(n-1)). Total of n(n-1) streams. Set of all these streams are denoted by V. In which the streams (1,2),(2,3),(3,4),….(n,1) are easily moving streams.(no intersection with other streams) and these streams do not need traffic control. The remaining n(n-2) streams may have intersection with other streams. These streams are represented by vertices v_{13},v_{15},…..v_{21},…..etc. and these are the elements of the vertex set V*(elements have at least one adjacency with other elements). The vertices of V are adjacent if there is crossing of vehicles between them. Let E be the set of edges thus obtained. Now G= (V, E) and G*= (V*, E*) are graphs where E*=E. An example of such graph G=(V,E) is shown below (when n=4)Fig:2.Since there are 16 crossings between the streams in Fig1 therefore 16 edges in the following graph. For convenience let us denote stream (i,j) as ij or $\boldsymbol{x}$ in the following sections.

Fig2: Graph of G=(V,E) corresponding to the junction of 4 roads



Consider the random experiment of selecting a vehicle from the junction (within a fixed radius of the junction) and looking its stream of movement. For example a selected vehicle is moving on stream 13 (coming from road 1 and going to road 3). Now the sample space is S={12,13,14,…,21,…..}. We can assign membership value to each vertex **x** in V by defining

$\sigma_x(t) = \frac{f_x(t)}{\sum_{x\in V} f_x(t)}$;. ……….(1) where $f_x(t)$ is the number of vehicles on the stream x near the junction(within a specified distance from the junction) at a time **t**. Assign a value to an edge $xy \in E$ by

$$\mu_{xy}(t) = \min\{\sigma_x(t), \sigma_y(t)\} \quad \text{……….(2)}$$ where x, y are two vertices.

3.1 Definition: Probability vertex Labeling of a graph

A function P :V → [0,1] P defined on the vertex set V of a graph G=(V,E) is called a **Probability vertex labeling** of graph if P satisfies the following conditions

(i) For any vertex $v \in V$, P(v)≥0 (ii) $\sum_{v\in V} P(v) = 1$

3.2 Definition: Probability edge Labeling of a graph

A function P:E→[0,1], defined on the edge set E of a graph G=(V,E) is called a **Probability edge labeling** of graph G if P satisfies the conditions

(i) For any edge $e \in E$, P(e)≥0 (ii) $\sum_{e\in E} P(e) = 1$

3.3 Definition *Probability labeling of a Graph:* Probability vertex labeling and Probability edge labeling are called Probability Labeling of a Graph

Theorem 3.1

The membership function defined by $\sigma_x(t) = \frac{f_x(t)}{\sum_{x\in V} f_x(t)}$; where $x \in V$ (defined in (1) is a **probability vertex labelling** of G=(V,E).

Proof : (i) Since $f_x \geq 0$ (number of vehicles), there fore $\sigma_x \geq 0$

(ii) $\sum_{x\in V} \sigma_x = \frac{\sum_x f_x}{\sum_x f_x} = 1.$

Therefore $\boldsymbol{\sigma_x}$ satisfies all the required conditions for a probability vertex labeling.

Theorem 3.2

(V,σ,μ) is a **fuzzy graph** where σ is any probability vertex labelling on V and $\mu_{xy}(t)=\min\{\sigma_x(t),\sigma_y(t)\}$

Proof: By the property of probability vertex labeling $0\le\boldsymbol{\sigma_x}\le 1$ for all x

Also $0\le\mu_{xy}\le 1$ (Since $0\le\boldsymbol{\sigma_x}\le 1$ and $\mu_{xy}=\min\{\sigma_x,\sigma_y\}$)

By the definition of μ it is clear that $\mu_{xy}\le\sigma_x\wedge\sigma_y$

Therefore (G,σ,μ) is a **fuzzy graph.**

Theorem 3.3: σ^* is a probability vertex labeling and (V^*,σ^*,μ^*) is a fuzzy graph where $\sigma^*_x=\dfrac{f_x}{\sum_{x\in V^*} f_x}$ ……(3) is a function on V* (set of non isolated vertices of V) $\mu^*_{xy}=\min\{\sigma^*_x,\sigma^*_y\}$; x,y vertices in V* and xy is an edge joining x&y.

Proof : Similar to proofs of theorem 3.1 and 3.2

4. Application

Consider vertex labels on V* $\sigma^*_x(t)=\dfrac{f_x(t)}{\sum_{x\in v^*} f_x(t)}$ on V* and edge labels as $\mu^*_{xy}=\min\{\sigma^*_x,\sigma^*_y\}$, where x,y are vertices of V* and xy is an edge joining x and y.

Let T be the time period at a junction to give signals to vehicles on all roads. We suggest to giving $T\sigma^*_x$ time for green light to a stream x. The total time for green light in that junction for a cycle of period is $\sum_{x\in V^*} T\sigma^*_x(t)$ and which equal to T (since $\sum_{x\in v^*}\sigma^*_x(t)=1$). Practically we can give $\sum_{x\in V_1} T\sigma^*_x(t)$ time for green signal to streams in V_1 at a time, where V_1 is the intersection of set of vertices of streams starting from road 1 and the set V*. Generally $\sum_{x\in V_i} T\sigma^*_x(t)$ is the total time for green signal to streams in V_i, where V_i is the intersection of set of vertices (streams) starting from road i and the set V*, where $i=1,2,3,..$ Let the time duration for green signal for vehicles on road i at an instant $'t'$ is denoted by $T_i(t)$. Now $T_i(t)=\sum_{x\in V_1} T\sigma^*_x(t)$

The above all calculations are based on an arbitrary time 't'. But the calculation of $T_i(t)$ using $f_{ij}(t)$ is quite difficult because both the calculation and data collection are at the same time 't'. So avoiding this complication we define a modified formula for $T_i(t)$

$$T_i(t)=\begin{cases}\sum_{x\in V_1} T\sigma^*_x(t-1); t=1,2,3,\ldots\ldots. \\ T/n; t=0\end{cases} \quad \_\_\_\_\_\_\_\_\_(4)$$

5. Illustration.

The above method can be illustrated with an example. The number of vehicles on a stream 'ij' is $\mathbf{f_{ij}}$ and its membership value is $\boldsymbol{\sigma_{ij}}$, time allotment for green light on road 'i' is **Ti . .** 'T' is the total time period (in seconds or minutes) for one complete cycle.

Table 5.1: Number of arriving vehicles and time for crossing junction

Stream(x)		T_0	t=1		T_1	t=2		T_2	t=3		T_3	t=4		T_4
			f_x	σ_x		f_x	σ_x		f_x	σ_x		f_x	σ_x	
12			5			3			3			5		
13	V_1	T/4	4	4/34	9T/34	5	5/38	8T/38	6	6/50	10T/50	3	3/28	5T/28
14			5	5/34		3	3/38		4	4/50		2	2/28	
21	V_2	T/4	6	6/34	8T/34	6	6/38	10T/38	7	7/50	12T/50	4	4/28	6T/28
24			2	2/34		4	4/38		5	5/50		2	2/28	
23			3			4			5			6		
31	V_3	T/4	4	4/34	9T/34	6	6/38	13T/38	8	8/50	17T/50	4	4/28	9T/28
32			5	5/34		7	7/38		9	9/50		5	5/28	
34			1			2			4			3		
41			3			1			2			5		
42	V_4	T/4	4	4/34	8T/34	3	3/38	7T/38	5	5/50	11T/50	3	3/28	8T/28
43			4	4/34		4	4/38		6	6/50		5	5/28	
Total		T	46	1	T	48	1	T	64	1	T	47	1	T

■ -shaded rows represent easily moving streams which don't need traffic controls

Remark. 5.1

If two vertices (streams) x and y are adjacent, then there is a possibility of collision between vehicles on x and y. This possibility depends on the membership value μ_{xy} of the edge connecting x and y. If $\mu_{xy} > 1/n$ then the crossing between x and y is dangerous. So traffic management team can give special attention to those crossings which have membership value greater than 1/n.

5. Conclusion

In this paper we have introduced Probability Labeling on road network Graph. Using this we can manage timing of traffic lights at a junction of roads. We hope, we can extend this idea for predicting the time at which max traffic will happen on a road/junction.

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