A STUDY ON COLOURING IN GRAPHS AND ITS SCOPE.
GRAPH THEORY

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Abstract:

Graph Theory is a delightful playground for the exploration of proof techniques in discrete mathematics and its results have applications in many areas of the computing, social and natural sciences. Development of graph theory started since 19th century. Gradually the growth of Graph Theory helped almost all areas of mathematics. But Graph Theory is still young and now discuss about the authors who contribute to development of Graph Theory.

INTRODUCTION:

1.1 HISTORY OF GRAPH COLOURING:

The first results about graph colouring deal almost exclusively with planar graphs in the form of the colouring of maps. While trying to colour a map of the counties of England, Francis Guthrie postulated the four colour conjecture, noting that four colours were sufficient to colour the map so that no regions sharing a common border received the same colour. Guthrie's brother passed on the question to his mathematics teacher Augustus de Morgan at University College, who mentioned it in a letter to William Hamilton in 1852. Arthur Cayley raised the problem at a meeting of the London Mathematical Society in 1879. The same year, Alfred Kempe published a paper that claimed to establish the result, and for a decade the four colour problem was considered solved. For his accomplishment Kempe was elected a Fellow of the Royal Society and later President of the London Mathematical Society.

In 1890, Heawood pointed out that Kempe's argument was wrong. However, in that paper he proved the five colour theorem, saying that every planar map can be coloured with no more than five colours, using ideas of Kempe. In the following century, a vast amount of work and theories were developed to reduce the number of colours to four, until the four colour theorem was finally proved in 1977 by Kenneth Appel and Wolfgang Haken. The proof went back to the ideas of

Heawood and Kempe and largely disregarded the intervening developments. The proof of the four colour theorem is also noteworthy for being the first major computer-aided proof.

1.2 OTHER TYPES OF COLOURINGS:

I. Edge colouring:

An edge colouring of a graph is a proper colouring of the edges, meaning an assignment of colours to edges so that no vertex is incident to two edges of the same colour. An edge colouring with k colours is called a k-edge-colouring and is equivalent to the problem of partitioning the edge set into k matchings. The smallest number of colours needed for an edge colouring of a graph G is the chromatic index, or edge chromatic number.

II. Total colouring:

Total colouring is a type of colouring on the vertices and edges of a graph. When used without any qualification, a total colouring is always assumed to be proper in the sense that no adjacent vertices, no adjacent edges, and no edge and its end-vertices are assigned the same colour. The total chromatic number chi square of (G) a graph G is the least number of colours needed in any total colouring of G.

III. Unlabeled colouring:

An unlabeled colouring of a graph is an orbit of a colouring under the action of the automorphism group of the graph. If we interpret a colouring of a graph on d vertices as a vector in Z^d the action of an automorphism is a permutation of the coefficients of the colouring. There are analogues of the chromatic polynomials which count the number of unlabeled colourings of a graph from a given finite colour set.

BASIC DEFINITIONS:

Definition 1.1:

A graph G = (V (G), E (G)) consists of two sets, V (G) = $\{v_1,v_2,v_3\ldots\}$ is called vertex set of G and E(G) = $\{e_1,e_2,\ldots\}$ called edge set of G. Sometimes we denote vertex set of G as V(G) and edge set of G as E(G). Elements of V (G) and E(G) are called vertices and edges respectively.

Definition 1.2:

An edge of a graph that joins a vertex to itself is called a loop. A loop is an edge e

= V_iV_i.

Definition 1.3:

If two vertices of a graph are joined by more than one edge then these edges are called multiple edges.

CONCLUSION:

OUTLINE OF THE THESIS:

Chapter-2, provides the basic definitions and presents The Greedy algorithm, some of the upper and lower bounds for a chromatic number of a graph, The DSATUR Algorithm, The Recursive Largest First (RLF) Algorithm, each algorithm is illustrated by an example.

Chapter-3, deals with advanced techniques for graph colouring. This chapter introduces exact algorithms, backtracking approaches, graph colouring using integer programming and application of inexact, heuristic and metaheuristic based approaches of graph colouring.

Chapter-4, presents The α-graph colouring problem, Connectivity property, Colouring property of a minimal α-graph and The searching process for minimal α-graph.

Chapter-5, elaborates some of the real life application for graph colouring like splitting groups of friends, construction of time tables, scheduling of call taxis and compiler register Allocation and short circuit testing are presented with examples.

ACKNOWLEDGEMENT:

This work started at the Graph and Study on Colouring in Graphs and Network Visualization (GNV) session of IISA Mumford. C.New order –based crossovers for the graph coloring problem.In parallel problem Solving from Nature (PPSN) IX, volume 4193 of LNCS. Pages 880-889.Springer ,2006. We thanks the anonymous reviwers of this paper for their valuable suggestions.

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