AN EOQ MODEL WITH MODIFIED WEIBULLY DISTRIBUTED DETERIORATION RATE, TIME DEPENDENT DEMAND WITH LINEAR HOLDING COST

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Abstract : In this paper, an inventory model is developed with linear as well as quadratic demand. Three-parameters modified Weibull distribution is used to represent the distribution of time for deterioration. In the model considered here, shortages are allowed to occur and they are partially backlogged. The model is solved analytically to obtain the optimal solution of the problem. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.

IndexTerms **-** Linear demand, Three parameters modified Weibully distributed deterioration rate, Quadratic demand, Shortages.

1. Introduction:

Inventory of deteriorating items first studied by Whitin [23]. He considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader [8] extended the classical EOQ formula to include exponential decay, wherein a constant fraction of on hand inventory is assumed to be lost due to deterioration. Covert and Philip [6] and Shah and Jaiswal [22] carried out an extension to the above model by considering deterioration of Weibull and general distributions respectively. Dave and Patel [7] first developed an inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no shortages allowed. Many researchers such as Park [14] and Hollier and Mak [11] also considered constant backlogging rates in their inventory models. Nahmias [13] gave a review on perishable inventory theory. Rafaat [18] described survey of literature on continuously deteriorating inventory model. He focused to present an up-to-date and complete review of the literature for the continuously deteriorating mathematical inventory models.

All researchers assume that during shortage period all demand either backlogged or lost. In reality, it is observed that some customers are willing to wait for the next replenishment. Abad [1] considered this phenomenon in his model, optimal pricing and lot sizing under conditions of perishable and partial backordering. He assume that the backlogging rate depends upon the waiting time for the next replenishment. But he does not include the stock out cost (back order cost and lost sale cost).

J. Jagadeeswari and P. K. Chenniappan [12] developed an order level inventory model for deteriorating items with time – quadratic demand and partial backlogging. Sarala Pareek and Garima Sharma [20] developed an inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging. R. Amutha and Dr. E. Chandrasekaran [17] developed an inventory model for deteriorating items with time – varying demand and partial backlogging.

According to Sarhan and Zaindin [21], it is interesting to observe that the modified Weibull distribution has a nice physical interpretation. It represents the lifetime of a series system. This system consists of two independent components. The lifetime of one component follows exponential distribution and the lifetime of the other follows Weibull distribution. Alin Rosca and Natalia Rosca [2] have developed an EOQ Model for an item with modified Weibull distribution deterioration rate with exponential demand and shortages with partial backlogging.

Bhojak and Gothi [3] developed an EOQ model with time – dependent demand and Weibully distributed deterioration. Further, Bhojak and Gothi [4] have developed an EPQ model with time dependent holding cost and Weibully distributed deterioration under shortages. Chatterji and Gothi [5] have developed an inventory model for two – parameter Weibully deteriorated items with exponential demand rate and completely backlogged shortages. Gothi, Shah, and Khatri [9] recently developed two warehouses inventory model for deteriorating items with power demand and time-varying holding cost where shortages are permissible and are a mixture of partial backlog and lost sales. Further, Parmar and Gothi [15] have developed an EPQ model for deteriorating items under three-parameter Weibull distribution and time-dependent IHC with shortages.

The Inventory system for deteriorating items with power demand pattern and time depending holding cost with two-parameter Weibull distribution and Pareto Type-I distribution was recently developed by Pooja D. Khatri and U.B. Gothi [16]. An EOQ model for Weibully distributed deteriorating items with the effect of permissible delay in payments and partially backlogged shortages was recently developed by Gothi, Shah and Rohida [10]. Also, Rohida and Gothi [19] have developed a deterministic inventory model with Weibull deterioration and quadratic demand rate under trade credit.

In this paper, we have developed an inventory model by considering three parameters Weibull distributed deterioration rate in the interval [0, μ) with linear demand. While in the interval [μ, t_1], the distribution of the time to deteriorate is a random variable following two parameter Pareto type-I distribution. The probability density function of two parameter Pareto type-I distribution, given by $f(t)=\frac{\theta}{\mu}\left(\frac{t}{\mu}\right)^{-\theta-1}; t \geq \mu,$ where θ and μ are parameters with positive real value. The instantaneous rate of deterioration θ(t) of the non-deteriorated inventory at time t, can be obtained from $\theta(t)=\frac{f(t)}{1-F(t)}$ where $F(t)=1-\left(\frac{t}{\mu}\right)^{-\theta}$ is the cumulative distribution function for the two parameter Pareto type-I distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $\theta(t)=\frac{\theta}{t}$ and demand rate is quadratic function of time. In this model shortages are allowed to occur and they are partially backlogged.

2. Notations:

We use the following notations for the developed mathematical model:

1. I(t) : The instantaneous state of the inventory level at any time t ($0 \leq t \leq T$).
2. R(t) : Demand rate varying over time.
3. θ(t) : Deterioration rate per unit per unit time.
4. A : Ordering cost per order.
5. C_d : Deterioration cost per unit per unit time.
6. C_h : Inventory holding cost per unit per unit time.
7. C_s : Shortage cost per unit.
8. C_p : The penalty cost of a lost sale including lost profit (unit time).
9. Pc : Purchase cost per unit.
10. T : The fixed length of each cycle.
11. TC : The average total cost for the time period [0, T].

3. Assumptions:

The model is developed under the following assumptions:

1. Inventory system deals with a single item.
2. The annual demand rate is a function of time which is R(t) = (a + bt) for [0, μ) and R(t) = ($a + bt + ct^2$) for [μ, t_1]. (a, b, c > 0)
3. Holding cost is a linear function of time expressed by $C_h = h + rt$ (h, r > 0)
4. $\theta(t) = \alpha+\beta\gamma t^{\gamma-1}$ Three parameters modified Weibull deterioration rate (unit/unit time). ($0 < \alpha < 1, 0 < \beta < 1, \gamma > 0$)
5. Lead-time is zero.
6. Time horizon is finite.
7. Shortages are allowed to occur and partially backlogged.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real, continuous function which is convex to the origin.
10. α and β are very small and so in the derivation of the model their higher powers are neglected.

4. Mathematical Model And Analysis:

At the beginning of the cycle period the on hand inventory level is S_1 units and it gradually reduces to zero at time t_1. This is due to demand and deterioration of the units during this time period. In the interval $[0, \mu)$ demand is linear and time to deteriorate follows modified Weibull distribution with three parameters while in the interval $[\mu, t_1]$, demand is quadratic and time to deteriorate follows Pareto type-I distribution. Thereafter, shortages are allowed to occur during the time interval $[t_1, T]$, and all of the demand during the period $[t_1, T]$ is partially backlogged.

The graphical presentation is shown in **Figure 1**.

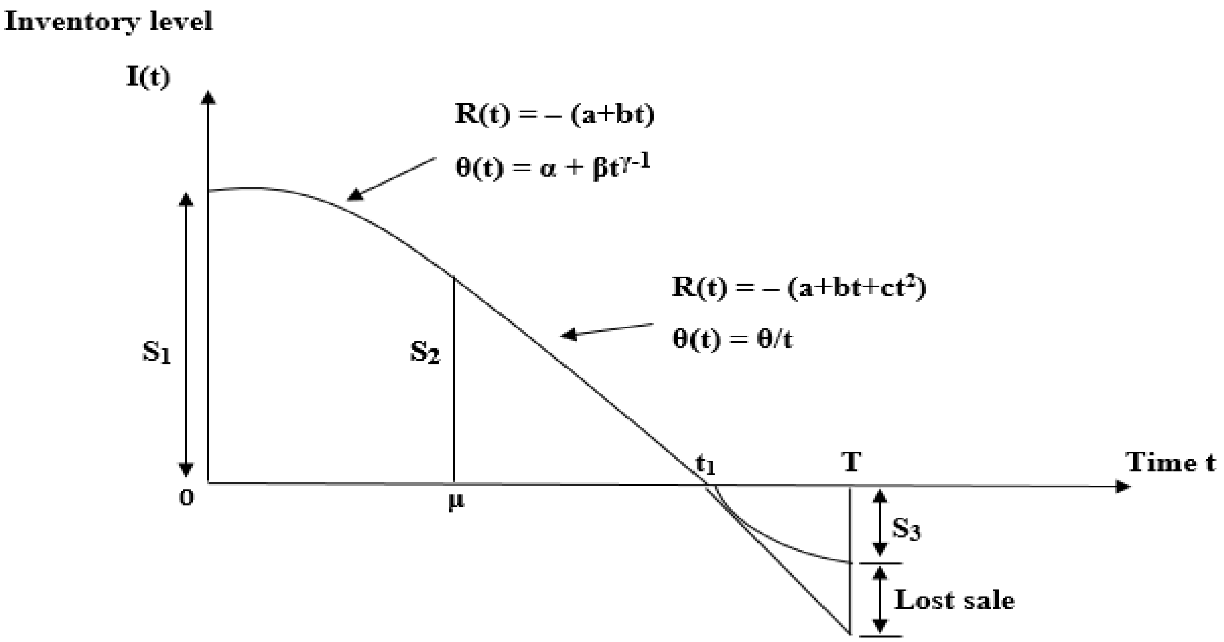


Figure 1: Graphical representation of the inventory system

During the period $[0, \mu)$, the inventory depletes due to the deterioration and demand. Hence, the inventory level at any time during $[0, \mu)$ is described by differential equation

$$\frac{dI(t)}{dt}+(\alpha+\beta\gamma t^{\gamma-1})I(t)=-(a+bt), \qquad (0\le t\le\mu) \tag{1}$$

With the boundary condition $I(0) = S_1$, the solution of equation (1) is

$$I(t)=S_1\left(1-\alpha t-\beta t^{\gamma}\right)-\left(at+\frac{bt^2}{2}\right)+\alpha\left(\frac{at^2}{2}+\frac{bt^3}{6}\right)+\beta\gamma\left(\frac{at^{\gamma+1}}{\gamma+1}+\frac{bt^{\gamma+2}}{2(\gamma+2)}\right), \qquad (0\le t\le\mu) \tag{2}$$

(neglecting higher powers of α, β)

During the period $[\mu, t_1)$, the inventory depletes due to the deterioration and demand. Hence, the inventory level at any time during $[\mu, t_1)$ is described by differential equation

$$\frac{dI(t)}{dt}+\frac{\theta}{t}I(t)=-(a+bt+ct^2), \qquad (\mu\le t\le t_1) \tag{3}$$

With the boundary condition $I(t_1) = 0$, the solution of equation (3) is

$$I(t)=-\left(\frac{at}{\theta+1}+\frac{bt^2}{\theta+2}+\frac{ct^3}{\theta+3}\right)+\xi t^{-\theta}, \qquad (\mu\le t\le t_1) \tag{4}$$

During the interval $[t_1, T]$ the inventory level depends on demand and a fraction of demand is backlogged. The State of inventory during $[t_1, T]$ can be represented by the differential equation

$$\frac{dI(t)}{dt} = -(a+bt+ct^2)e^{-\delta(T-t)}, \qquad (t_1 \le t \le T) \tag{5}$$

With the boundary condition $I(t_1) = 0$, the solution of equation (3) is

$$I(t) = -e^{-\delta(T-t)}\left[\frac{(a+bt+ct^2)}{\delta} - \frac{(b+2ct)}{\delta^2} + \frac{2c}{\delta^3}\right] + e^{-\delta(T-t_1)}\left[\frac{(a+bt_1+ct_1^{\,2})}{\delta} - \frac{(b+2ct_1)}{\delta^2} + \frac{2c}{\delta^3}\right], \qquad (t_1 \le t \le T) \tag{6}$$

Putting $I(t_1) = 0$ in equation (4), we get

$$S_1 = \frac{1}{1-\alpha t_1 - \beta t_1^{\,\gamma}}\left[\left(at_1 + \frac{bt_1^{\,2}}{2}\right) - \alpha\left(\frac{at_1^{\,2}}{2} + \frac{bt_1^{\,3}}{6}\right) - \beta\gamma\left(\frac{at_1^{\,\gamma+1}}{\gamma+1} + \frac{bt_1^{\,\gamma+2}}{2(\gamma+2)}\right)\right] \tag{7}$$

Putting $I(T) = -S_3$ in equation (6), we get

$$S_3 = \left[\frac{(a+bT+cT^2)}{\delta} - \frac{(b+2cT)}{\delta^2} + \frac{2c}{\delta^3}\right] - e^{-\delta(T-t_1)}\left[\frac{(a+bt_1+ct_1^{\,2})}{\delta} - \frac{(b+2ct_1)}{\delta^2} + \frac{2c}{\delta^3}\right] \tag{8}$$

Order costs or Setup Costs or Replenishment costs include the fixed cost associated with obtaining goods through placing of an order or purchasing or manufacturing or setting up a machinery before starting production.

The cost of placing order or replenishment costs is as follows:

$$OC = A \tag{9}$$

Inventory is available in the system during the time interval $[0, t_1]$. Hence, the deterioration cost for inventory in stock is computed for time period $[0, t_1]$ only.

The number of deteriorating items in the interval $[0, \mu)$ is

$$DC_1 = \int_0^{\mu} (\alpha + \beta r t^{\gamma-1}) I(t)\,dt$$

$$DC_1 = S_1\left[\alpha\mu + \beta\mu^{\gamma}\right] - \alpha\left[\frac{a\mu^2}{2} + \frac{b\mu^3}{6}\right] - \beta\gamma\left[\frac{a\mu^{\gamma+1}}{(\gamma+1)} + \frac{b\mu^{\gamma+2}}{2(\gamma+2)}\right]$$

and the number of deteriorating items in the interval $[\mu, t_1]$ is

$$DC_2 = \int_0^{t_1} \frac{\theta}{t}\, I(t)\,dt$$

$$DC_2 = \left[\mu^{-\theta} - t_1^{-\theta}\right]\xi - \theta\left[\frac{a}{\theta+1}(t_1-\mu) + \frac{b}{\theta+2}\left(\frac{t_1^{\,2}-\mu^2}{2}\right) + \frac{c}{\theta+3}\left(\frac{t_1^{\,3}-\mu^3}{3}\right)\right]$$

Therefore the total deterioration cost during the time interval [0, t_1] is

$$DC = C_d\,(DC_1 + DC_2)$$

$$\begin{aligned}\Rightarrow DC = &\left\{S_1\left[\alpha\mu+\beta\mu^{\gamma}\right]-\alpha\left[\frac{a\mu^2}{2}+\frac{b\mu^3}{6}\right]-\beta\gamma\left[\frac{a\mu^{\gamma+1}}{(\gamma+1)}+\frac{b\mu^{\gamma+2}}{2(\gamma+2)}\right]\right\}\\ &+\left\{\left[\mu^{-\theta}-t_1^{-\theta}\right]\xi-\theta\left[\frac{a}{\theta+1}(t_1-\mu)+\frac{b}{\theta+2}\left(\frac{t_1^2-\mu^2}{2}\right)+\frac{c}{\theta+3}\left(\frac{t_1^3-\mu^3}{3}\right)\right]\right\}\end{aligned} \tag{10}$$

Inventory is available in the system during the time interval [0, t_1]. Hence, the cost for holding inventory in stock is computed for time period [0, t_1] only.

Holding cost is as follows:

$$IHC=\int_0^{\mu}(h+rt)I(t)dt+\int_{\mu}^{t_1}(h+rt)I(t)dt$$

$$\begin{aligned}\Rightarrow IHC = &\left\{\begin{aligned}&h\left[S_1\left(\mu-\frac{\alpha\mu^2}{2}-\frac{\beta\mu^{\gamma+1}}{\gamma+1}\right)-\left(\frac{a\mu^2}{2}+\frac{b\mu^3}{6}\right)+\alpha\left(\frac{a\mu^3}{6}+\frac{b\mu^4}{24}\right)+\beta\gamma\left(\frac{a\mu^{\gamma+2}}{(\gamma+1)(\gamma+2)}+\frac{b\mu^{\gamma+3}}{2(\gamma+2)(\gamma+3)}\right)\right]\\ &+r\left[S_1\left(\frac{\mu^2}{2}-\frac{\alpha\mu^3}{3}-\frac{\beta\mu^{\gamma+2}}{\gamma+2}\right)-\left(\frac{a\mu^3}{3}+\frac{b\mu^4}{8}\right)+\alpha\left(\frac{a\mu^4}{8}+\frac{b\mu^5}{30}\right)+\beta\gamma\left(\frac{a\mu^{\gamma+3}}{(\gamma+1)(\gamma+3)}+\frac{b\mu^{\gamma+4}}{2(\gamma+2)(\gamma+4)}\right)\right]\end{aligned}\right\}\\ &+\left\{\begin{aligned}&h\left[\left(\frac{at_1^{\theta+1}}{\theta+1}+\frac{bt_1^{\theta+2}}{\theta+2}+\frac{ct_1^{\theta+3}}{\theta+3}\right)\left(\frac{t_1^{1-\theta}-\mu^{1-\theta}}{1-\theta}\right)-\left(\frac{a\left(t_1^2-\mu^2\right)}{2(\theta+1)}+\frac{b\left(t_1^3-\mu^3\right)}{3(\theta+2)}+\frac{c\left(t_1^4-\mu^4\right)}{4(\theta+3)}\right)\right]\\ &+r\left[\left(\frac{a(t_1^{\theta+1})}{\theta+1}+\frac{b(t_1^{\theta+2})}{\theta+2}+\frac{c(t_1^{\theta+3})}{\theta+3}\right)\left(\frac{t_1^{2-\theta}-\mu^{2-\theta}}{2-\theta}\right)-\left(\frac{a\left(t_1^3-\mu^3\right)}{3(\theta+1)}+\frac{b\left(t_1^4-\mu^4\right)}{4(\theta+2)}+\frac{c\left(t_1^5-\mu^5\right)}{5(\theta+3)}\right)\right]\end{aligned}\right\}\end{aligned} \tag{11}$$

Shortage due to stock out is accumulated in the system during the interval [t_1, T]. The optimum level of shortage is presented at t = T; therefore the total shortage cost during this time period is as follows:

$$SC=-C_S\int_{t_1}^{T}I(t)dt$$

$$\Rightarrow SC = C_S\left\{\begin{aligned}&\frac{a+bT+cT^2}{\delta^2}-\frac{2b+4cT}{\delta^3}+\frac{6c}{\delta^4}\\ &-e^{-\delta(T-t_1)}\left[\frac{\left(a+bt_1+ct_1^2\right)\left(1+\delta[T-t_1]\right)}{\delta^2}-\frac{\left(b+2ct_1\right)\left(2+\delta[T-t_1]\right)}{\delta^3}+\frac{(2c)\left(3+\delta[T-t_1]\right)}{\delta^4}\right]\end{aligned}\right\} \tag{12}$$

Due to stock out during [t_1, T], shortage is accumulated, but not all customers are willing to wait for the next lot size to arrive. Hence, this results in some loss of sale which accounts to loss in sale.

Lost sale cost is accumulated as follows:

$$LSC=C_p\int_{t_1}^{T}(a+bt+ct^2)(1-e^{-\delta(T-t)})dt$$

$$\Rightarrow LSC=C_p\left\{\begin{array}{l} a(T-t_1)+\frac{b}{2}(T^2-t_1^2)+\frac{c}{3}(T^3-t_1^3) \\ -\left(\frac{a+bT+cT^2}{\delta}-\frac{b+2cT}{\delta^2}+\frac{2c}{\delta^3}\right)+e^{-\delta(T-t_1)}\left(\frac{a+bt_1+ct_1^2}{\delta}-\frac{b+2ct_1}{\delta^2}+\frac{2c}{\delta^3}\right)\end{array}\right\} \tag{13}$$

The cost of purchasing a unit of an item is known as purchase cost which consist of units at the beginning of the cycle period (the on hand inventory level is S_1 units) and shortage units (S_3).

The purchase cost is as follows:

$$PC = P_c\left(S_1 + S_3\right)$$

$$PC = P_c\left\{\begin{array}{l} \frac{1}{1-\alpha t_1-\beta t_1^{\gamma}}\left[\left(at_1+\frac{bt_1^2}{2}\right)-\alpha\left(\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)-\beta\gamma\left(\frac{at_1^{\gamma+1}}{\gamma+1}+\frac{bt_1^{\gamma+2}}{2(\gamma+2)}\right)\right] \\ +\left[\frac{(a+bT+cT^2)}{\delta}-\frac{(b+2cT)}{\delta^2}+\frac{2c}{\delta^3}\right]-e^{-\delta(T-t_1)}\left[\frac{(a+bt_1+ct_1^2)}{\delta}-\frac{(b+2ct_1)}{\delta^2}+\frac{2c}{\delta^3}\right]\end{array}\right\} \tag{14}$$

Hence the total cost per unit time is given by

$$TC = \frac{1}{T}\left(OC + IHC + DC + SC + LSC + PC\right) \tag{15}$$

Now, our objective is to determine optimum values μ^*, t_1^* and T^* of μ, t_1 and T respectively to minimize the total cost TC. Using mathematical software, the optimal values μ^*, t_1^* and T^* can be obtained by solving $\frac{\partial TC}{\partial \mu}=0$, $\frac{\partial TC}{\partial t_1}=0$, $\frac{\partial TC}{\partial T}=0$, which can satisfy the following sufficient conditions:

$$\left.\begin{array}{l} \left[\left(\frac{\partial^2 TC}{\partial \mu^2}\right)\right]_{\mu=\mu^*,t_1=t_1^*,T=T^*} > 0 \\ \left[\left(\frac{\partial^2 TC}{\partial \mu^2}\right)\left(\frac{\partial^2 TC}{\partial t_1^2}\right)-\left(\frac{\partial^2 TC}{\partial \mu\partial t_1}\right)^2\right]_{\mu=\mu^*,t_1=t_1^*,T=T^*} > 0 \\ \left|\begin{array}{ccc} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu\partial t_1} & \frac{\partial^2 TC}{\partial \mu\partial T} \\ & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\partial T} \\ & & \frac{\partial^2 TC}{\partial T^2}\end{array}\right|_{\mu=\mu^*,t_1=t_1^*,T=T^*} > 0 \end{array}\right\} \tag{16}$$

5. Numerical Example:

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking A = 500, $\alpha = 0.0003$, $\beta = 0.0001$, $\gamma = 5$, $\delta = 0.1$, $C_d = 1.3$, $C_S = 1.06$, $P_c = 2.1$, $C_p = 1.2$, h = 2, r = 0.5, a = 20, b = 15, c = 10 and θ =15 (with appropriate units), optimal values of μ, t_1 and T are $\mu^* = 0.9895539446$, $t_1^* = 0.9895539501$, $T^* = 2.537943974$ units, optimal inventory level (Q) = 142.7123218 and optimal total cost per unit time TC = 361.6052474 units.

6. Sensitivity Analysis:

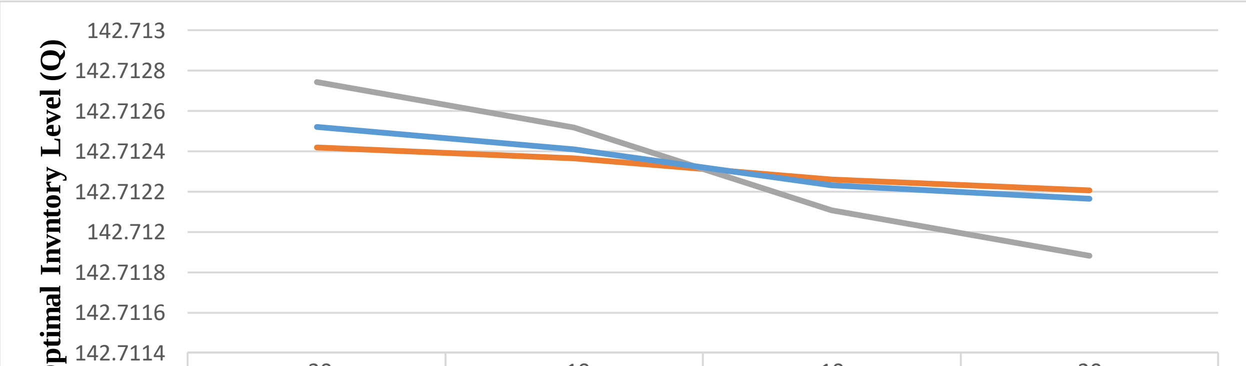
Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. In this section, we study the sensitivity for the time (μ), time (t_1), cycle time (T) and total cost per time per unit (TC) with respect to the changes in the values of the parameters α, β, and γ.

The sensitivity analysis is performed by considering 10% and 20% increase or decrease in each one of the above parameters keeping all other remaining parameter as fixed. The results are presented in **Table**.

Table: Partial Sensitivity Analysis

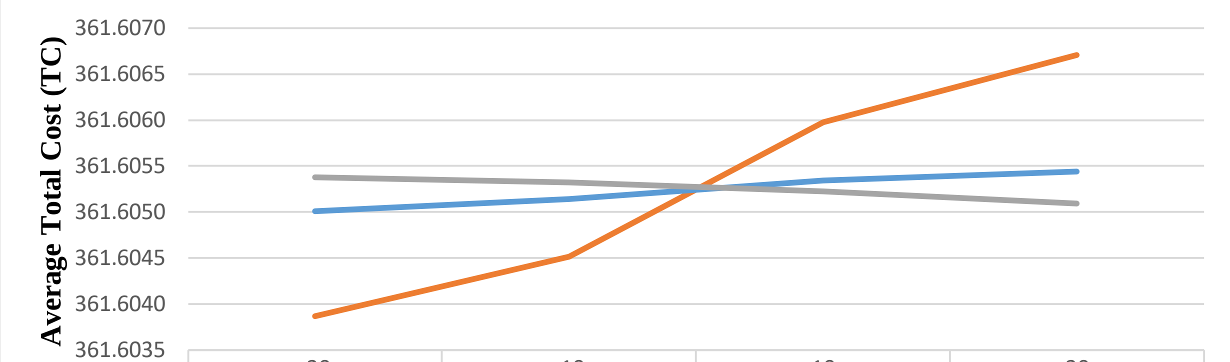
	% Change	μ	t_1	T	Q	TC
α	-20%	0.9897021317	0.9897021359	2.537957361	142.7124181	361.6039
	-10%	0.9896280354	0.9896280402	2.537950667	142.7123649	361.6045
	10%	0.9894798594	0.9894798657	2.537937282	142.7122588	361.6060
	20%	0.9894057799	0.9894057868	2.537930591	142.7122060	361.6067
β	-20%	0.9896100264	0.9896100311	2.537950496	142.7127440	361.6050
	-10%	0.9895819818	0.9895819869	2.537947234	142.7125178	361.6051
	10%	0.9895259148	0.9895259207	2.537940714	142.7121060	361.6053
	20%	0.9894978924	0.9894978986	2.537937456	142.7118805	361.6054
γ	-20%	0.9895556120	0.9895556169	2.537944937	142.7125221	361.6054
	-10%	0.9895545696	0.9895545749	2.537944400	142.7124103	361.6053
	10%	0.9895536412	0.9895536469	2.537943635	142.7122314	361.6052
	20%	0.9895535903	0.9895535964	2.537943367	142.7121652	361.6051

7. Graphical Presentation of Sensitivity Analysis:



	-20	-10	10	20
α	142.7124181	142.7123649	142.7122588	142.712206
β	142.712744	142.7125178	142.712106	142.7118805
ν	142.7125221	142.7124103	142.7122314	142.7121652

Figure 1



	-20	-10	10	20
α	361.6039	361.6045	361.6060	361.6067
β	361.6050	361.6051	361.6053	361.6054
ν	361.6054	361.6053	361.6052	361.6051

Figure 3

8. Conclusions:

- We may conclude that the Total Cost per time per unit (TC) increases due to increment in the value of the parameters α, β and TC is decrease due to increment in the value of parameter γ.
- It is observed from table that the decision variables μ, t_1 and T are decreases due to increment in the value of the parameters α, β and γ.
- We also conclude that optimal inventory level (Q) decreases due to increment in the value of the parameters α, β and γ.

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