IMPACT OF HIGH FREQUENCY FLUCTUATING SUCTION VELOCITY ON HEAT TRANSFER RATE UNDER MHD FREE CONVECTION FLOW

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ABSTRACT

The effects of suction parameter on the transient and fluctuating fields for fluids of different Prandtl numbers have been discussed with the help of Tables. Transfer is obtained and its variation is shown by Tables, also.

KEYWORDS : Heat Transfer, MHD, Hydromagnetic.

INTRODUCTION

The effects of harmonic oscillations of low and high frequencies on free convection laminar boundary layer flow past a semi-infinite flat plate has been first analysed by Stuart [7]. Pop [5] studied the unsteady hydromagnetic free convection flow past a vertical infinite porous plate when the plate temperature fluctuates with time about a non-zero constant mean. Borjini et al. [1], Raptis et al. [6], Kolsi et al. [3], Ghaly [2] and Yao et al. [8] have studied heat transfer problems of free convective flow.

We, in this paper have discussed the hydromagnetic free convection flow past a vertical infinite plate with variable suction velocity at the plate. Also, plate temperature fluctuates with time about a non zero constant mean. For large values of frequency Stuart [7] has also applied similar type of approximations in the temperature terms. We have assumed the variable suction velocity at the plate and derived the expressions for velocity and temperature fields in the medium.

BASIC EQUATIONS

We consider an unsteady two dimensional flow electrically conducting, incompressible, viscous fluid past an infinite vertical porous plate in the presence of a constant uniformly distributed transverse magnetic field B_0. Let $\underline{X}$ and $\underline{y}$ denote the distances measured along and perpendicular to the plate, respectively. By virtue of MHD boundary layer, the free convection flow under Boussinesq approximation is governed by the equations

$$\frac{\partial \underline{V}}{\partial \underline{y}} = 0, \tag{1}$$

$$\frac{\partial \underline{u}}{\partial \underline{t}} + \underline{v}\frac{\partial \underline{u}}{\partial \underline{y}} = g_x \beta(\underline{T} - \underline{T}_\infty) + \vartheta \frac{\partial^2 \underline{u}}{\partial \underline{y}^2} - \frac{\sigma B_0^2}{\underline{\rho}} \underline{u} \tag{2}$$

$$\underline{\rho} c_p \left(\frac{\partial \underline{T}}{\partial \underline{t}} + \underline{v}\frac{\partial \underline{T}}{\partial \underline{y}^2}\right) = k \frac{\partial^2 \underline{T}}{\partial \underline{y}^2} \tag{3}$$

On integrating (1), we take $\underline{V}$ in the present problem as function of $\underline{t}$. Hence, we write

$$\underline{v} = -V_0\left(1 + \in A e^{i\tilde{\omega} t}\right) \tag{4}$$

In view of equation (4) and non-dimensional transformations

$$y = \frac{\bar{y} V_0}{\nu}, \quad t = \frac{\bar{t} V_0^2}{4\nu}, \quad \omega = \frac{4\nu\bar{\varpi}}{V_0^2}, \quad u = \frac{\bar{u}}{U_0}, \quad U = \frac{\bar{U}}{U_0},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty} \quad G_r = \frac{\nu g_x \beta(\bar{T}_w - \bar{T}_\infty)}{U_0 V_0^2} \quad \text{(The Grashoff Number)},$$

$$E = \frac{U_0^2}{c_p(\bar{T}_w - \bar{T}_\infty)} \text{ (The Eckert Number)}, \quad P_r = \frac{\mu c_p}{k} \text{ (The Prandtl Number)},$$

$$M = \frac{\sigma B_0^2 \nu}{\bar{\rho} V_0^2} \quad \text{(The M}$$

The equation (2) and (3) reduce to

$$\frac{\partial^2 u}{\partial y^2} + \left(1 + \in A e^{i\omega t}\right)\frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = -G_r\theta + Mu, \tag{5}$$

$$\frac{\partial^2 \theta}{\partial y^2} + P_r\left(1 + \in A e^{i\omega t}\right)\frac{\partial \theta}{\partial y} - \frac{\partial u}{\partial t} = P_r\frac{\partial \theta}{\partial t}. \tag{6}$$

The above equations are to be solved with the use of following boundary conditions

$$\left.\begin{array}{lll} y=0; & u=0, & \theta=1+\epsilon e^{i\omega t}, \\ y\to\infty; & u=0, & \theta=0 \end{array}\right\} \quad (7)$$

SOLUTION OF THE EQUATION

To solve the equations (5) and (6) subject to the conditions (7), we assume

$$\left.\begin{array}{l} u(y,t)=u_0(y)+\epsilon e^{i\omega t}u_1(y), \\ \theta(y,t)=\theta_0(y)+\epsilon e^{i\omega t}\theta_1(y). \end{array}\right\} \quad (8)$$

Substituting (8) into (5) and (6), equating the coefficients of harmonic and non-hormonic terms and neglecting the coefficients of ϵ^2, we get

$$u_0''+u_0'-Mu_0=-G_r\theta_0, \quad (9)$$

$$u_1''+u_1'-(M+i\omega)u_1=-G_r\theta_1-Au_0', \quad (10)$$

$$\theta_0''+P_r\theta_0'=0, \quad (11)$$

$$\theta_1''+P_r\theta_1'-i\omega P_r\theta_1=-P_rA\theta_0', \quad (12)$$

Where primes denote differentiation with respect to y.

The corresponding boundary conditions are

$$\left.\begin{array}{lll} y=0; & u_0=u_1=0, & \theta_0=\theta_1=1, \\ y\to\infty; & u_0=u_1=0, & \theta_0=\theta_1=0, \end{array}\right\} \quad (13)$$

$$u_0(y)=\frac{G_r}{P_r(P_r-1)-M}\left(e^{-my}-e^{-P_r y}\right), \tag{14}$$

$$u_1(y)=\frac{G_r\left(\dfrac{iP_rA}{\omega}-1\right)}{nP_r(nP_r-1)-(M+i\omega)}\left(e^{-nP_r y}-e^{-qy}\right)$$
$$+\frac{G_rP_rA}{P_r(P_r-1)-(M+i\omega)}\left(\frac{1}{P_r(P_r-1)-M}+\frac{i}{\omega}\right)\left(e^{-qy}-e^{-P_r y}\right)$$
$$+\frac{G_r mA}{\{P_r(P_r-1)-M\}\{m(m-1)-(M+i\omega)\}}\left(e^{-my}-e^{-qy}\right), \tag{15}$$

$$\theta_0(y)=e^{-P_r y} \tag{16}$$

$$\theta_1(y)=e^{-nP_r y}+\frac{iP_rA}{\omega}\left(e^{-P_r y}-e^{-nP_r y}\right) \tag{17}$$

where, $m=\frac{1}{2}\left[1+(1+4M)^{\frac{1}{2}}\right]$, $q=\frac{1}{2}\left[1+\{1+4(M+i\omega)\}^{\frac{1}{2}}\right]$, $n=\frac{1}{2}\left[1+\left(1+\frac{4i\omega}{P_r}\right)^{\frac{1}{2}}\right]$.

Solving the equations (9) – (12) subject to the boundary conditions (13), we get

$$u(y,t)=\frac{G_r}{P_r(P_r-1)-M}\left(e^{-my}-e^{-P_r y}\right)$$
$$+\in e^{i\omega t}\left[a\left(e^{-nP_r y}-e^{-qy}\right)+b\left(e^{-qy}-e^{-P_r y}\right)+c\left(e^{-my}-e^{-qy}\right)\right], \tag{18}$$

$$\theta(y,t)=e^{-Py}+\in e^{i\omega t}\left[e^{-nP_r y}+\frac{iP_rA}{\omega}\left(e^{-P_r y}-e^{-nP_r y}\right)\right] \tag{19}$$

where, $a=\dfrac{G_r\left(\dfrac{iP_rA}{\omega}-1\right)}{nP_r(nP_r-1)-(M+i\omega)}$, $b=\dfrac{G_rP_rA}{P_r(P_r-1)-(M+i\omega)}\left(\dfrac{1}{P_r(P_r-1)-M}+\dfrac{i}{\omega}\right)$,

$c=\dfrac{G_r mA}{\{P_r(P_r-1)-M\}\{m(m-1)-(M+i\omega)\}}$.

Putting the values of u_0, u_1, □$_0$ and □$_1$ in the equations (8), the expressions for velocity and temperature fields are

When the magnetic parameter M is fixed and □□is too large , we consider the following approximations :

$$n\approx\left(\frac{i\omega}{P_r}\right)^{\frac{1}{2}},\ q\approx(i\omega)^{\frac{1}{2}},\ a\approx\frac{G_r}{\sqrt{i\omega P_r}-i\omega(P_r-1)},\ b\approx\frac{iG_rP_rA}{\omega[P_r(P_r-1)-M]},\ c\approx\frac{iG_r mA}{\omega[P_r(P_r-1)-M]} \tag{20}$$

In view of the above approximations, equations (18) and (19) reduce to

$$u(y,t)=\frac{G_r}{P_r(P_r-1)-M}\left(e^{-my}-e^{-Py}\right)+\in e^{i\omega t}\left(u_{1r}+i\,u_{1i}\right) \tag{21}$$

$$\theta(y,t)=e^{-P_r y}+\in e^{i\omega t}\left(\theta_{1r}+i\,\theta_{1i}\right) \tag{22}$$

for, $\omega\, t=\frac{\pi}{2}$ the expressions for transient velocity and transient temperature are

$$u\left(y,\frac{\pi}{2\omega}\right)=\frac{G_r}{P_r(P_r-1)-M}\left(e^{-my}-e^{-P_r y}\right)-\in u_{1i} \tag{23}$$

$$\theta\left(y,\frac{\pi}{2\omega}\right)=e^{-P_r y}-\in\theta_{1i} \tag{24}$$

$$+\frac{G_r A(P_r-m)}{\omega[P_r(P_r-1)-M]}e^{-y\sqrt{\frac{1}{2}\omega}}\sin y\sqrt{\frac{1}{2}\omega},$$

$$u_{1i}=\frac{G_r}{\omega(P_r-1)}\left(e^{-y\sqrt{\frac{1}{2}P_r\omega}}\cos y\sqrt{\frac{1}{2}P_r\omega}-e^{-y\sqrt{\frac{1}{2}\omega}}\cos y\sqrt{\frac{1}{2}\omega}\right)+\frac{G_r P_r A}{P_r(P_r-1)-M}\left(e^{-y\sqrt{\frac{1}{2}\omega}}\cos y\sqrt{\frac{1}{2}\omega}-e^{-P_r y}\right)$$

$$+\frac{G_r\, m A}{\omega[P_r(P_r-1)-M]}\left(e^{-my}-e^{-y\sqrt{\frac{1}{2}\omega}}\cos y\sqrt{\frac{1}{2}\omega}\right),\ \theta_{1r}=e^{-y\sqrt{\frac{1}{2}P_r\omega}}\left(\cos y\sqrt{\frac{1}{2}P_r\omega}-\frac{P_r A}{\omega}\sin y\sqrt{\frac{1}{2}P_r\omega}\right),$$

$$\theta_{1i}=e^{-y\sqrt{\frac{1}{2}P_r\omega}}\left(\frac{P_r A}{\omega}\cos y\sqrt{\frac{1}{2}P_r\omega}+\sin y\sqrt{\frac{1}{2}P_r\omega}\right)+\frac{P_r A}{\omega}e^{-Py}.$$

From the equation (21) we obtain the non-dimensional skin friction τ which is given by

$$\tau=\left(\frac{du}{dy}\right)_{y=0}=\frac{G_r(P_r-m)}{P_r(P_r-1)-M}+\in|B|\cos(\omega t+\propto) \tag{25}$$

where , $|B|=\left(B_r^{\,2}+B_i^{\,2}\right)^{\frac{1}{2}}$, $\propto=\tan^{-1}\left(\frac{B_i}{B_r}\right)$,

$$B_r=\frac{G_r A}{\sqrt{2\omega}[P_r(P_r-1)-M]}\left(\frac{P_r(P_r-1)-M}{A(1+\sqrt{P_r})}+P_r-m\right),$$

$$B_i=\frac{G_r A}{\sqrt{2\omega}[P_r(P_r-1)-M]}\left(m-P_r-\frac{P_r(P_r-1)-M}{A(1+\sqrt{P_r})}\right).$$

From the equation (22) the non-dimensional rate of heat transfer □□□can be calculated as

$$\kappa = -G_r\left(\frac{d\theta}{dy}\right)_{y=0} = G_r P_r + \in |D|\cos(\omega t + \delta) \tag{26}$$

where, $|D| = \left(D_r^{\,2} + D_i^{\,2}\right)^{\frac{1}{2}}$, $\delta = \tan^{-1}\left(\frac{D_i}{D_r}\right)$, $D_r = G_r\sqrt{\frac{1}{2}P_r\omega}\left(1 + \frac{AP_r}{\omega}\right)$,

$$D_i = G_r\left[\sqrt{\frac{1}{2}P_r\omega}\left(1 - \frac{AP_r}{\omega}\right) + \frac{AP_r^{\,2}}{\omega}\right].$$

RESULTS AND DISCUSSION

For the purpose of discussion, we have chosen the value of the Prandtl number as 0.71 and 7 approximately which are representative values of air and water respectively at 20^0c. The other values of P_r are chosen arbitrarily. Here, we have discussed the variations of respective quantities when the plate is being cooled ($G_r < 0$) by the free convection currents only. Results are discussed with the help of tables.

From the tables, we have discussed the variations of u_{1r}, u_{1i}, u, D_r, D_i and □□with respect to A and P_r.

From Tables – I and II we see that with A increasing, the real part u_{1r} of the fluctuating velocity decreases and imaginary part u_{1i} increases. Further we observe that for constant increase in suction parameter A, the variations of u_{1i} for small values of P_r are greater than that obtained for large values,

For, $\omega t = \frac{\pi}{2}$ and $\in = \frac{1}{2}$, the values of transient velocity u with respect to A and P_r are increased, Table – III. From the table we observe that when the plate is being cooled by natural convection, velocity fields both for air and water decrease slowly with respect to A. we further conclude that at constant A, the velocity field for air is greater than that obtained for water.

From Tables – IV and V we see that real part Dr of the rate of heat transfer increases with A increasing while imaginary part D_i decreases with A increasing. Values of □□are entered in Table – VI and it has been observed that for $\omega t = \frac{\pi}{2}$ and $\in = \frac{1}{2}$, the transient temperature □□decreases with A increasing. At constant A, □□decreases with P_r increasing.

TABLE – I

Variation of u_{1r} with respect to A and P_r (G_r =5, □□=50, y = 1, m = 2)

P_r \ A	0	0.2	0.4	0.6	0.8	1.0
0.71	0.00230	0.00224	0.00218	0.00212	0.00206	0.00200
3	0.00033	0.00029	0.00027	0.00023	0.00020	0.00017
7	0.00011	0.00009	0.00007	0.00006	0.00004	0.00003

TABLE – II

Variation of u_{1i} with respect to A and P_r (G_r =5, □□=50, y = 1, m = 2)

P_r \ A	0	0.2	0.4	0.6	0.8	1.0
0.71	0.00310	0.00376	0.00442	0.00502	0.00574	0.00640
3	0.00010	0.00068	0.000146	0.00224	0.00292	0.00380
7	0.00003	0.00010	0.00024	0.00037	0.00051	0.00065

TABLE – III

Variation of u with respect to A and P_r (G_r =5, □□=50, y = 1, m = 2, □t = □/2, ϵ = 1/2)

P_r \ A	0	0.2	0.4	0.6	0.8	1.0
0.71	0.8174	0.8170	0.8167	0.8164	0.8160	0.8157
3	0.1069	0.1065	0.1061	0.1057	0.1054	0.1050
7	0.0168	0.0167	0.01668	0.01661	0.0165	0.0164

TABLE – IV

Variation of D_r with respect to A and P_r (G_r =5, □□=50)

P_r \ A	0	0.2	0.4	0.6	0.8	1.0
0.71	21.0000	21.0588	21.01176	21.1764	21.2016	21.2942
3	43.3000	43.8196	44.3392	44.8588	45.3784	45.8980
7	66.1500	68.0622	69.8544	71.7066	73.5588	75.4100

TABLE – V

Variation of D_i with respect to A and P_r (G_r =5, □□=50)

P_r \ A	0	0.2	0.4	0.6	0.8	1.0
0.71	21.0000	20.9510	20.9020	20.8536	20.8384	20.7516
3	43.3000	42.9600	42.6200	42.2800	41.9400	41.6000
7	66.1500	64.4056	64.4056	63.5334	62.6612	61.7800

TABLE – VI

Variation of □with respect to A and P_r (□t = □/2 ϵ = 1/2, y = 1)

P_r \ A	0	0.2	0.4	0.6	0.8	1.0
0.71	0.4900	0.4894	0.4886	0.4879	0.4872	0.4864
3	0.0498	0.0492	0.0491	0.0488	0.0486	0.0483
7	0.00091	0.00090	0.00089	0.00088	0.00086	0.00085

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