# A NOTE ON $(s, S)$ INVENTORY POLICY WITH/WITHOUT RETAIL AND INTERRUPTION OF SERVICE OR PRODUCTION

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**Abstract:** We took for the following results the Markov chain $\Omega$ is stable if and only if $\frac{\lambda}{\mu}\frac{(\delta_1+\delta_2)}{\delta_2} < 1$, Expected Number of interruptions encountered by a customer, Expected duration of an interrupted service, Expected amount of time a customer is served during his (possibly interrupted) service and System behavior with variational in parameters.

**Keywords:** Stochastic process, Queuing theory And Inventory models.

## 1. Introduction

The results in this chapter turn out to be a particular case of what was discussed in chapter. Nevertheless the sharper assumption of zero lead time has produced several stronger results in this section. One consequence is the explicit expression for the stability of the system and the closed form expression for the system state distribution. Further our investigation of the optimal reorder level (s) and the maximum number(S) of items that could be stored in the inventory could be made analytical.

The section introduces the concept of interruption to an inventory system where the processing of inventory requires a random time, which leads to a queue of customers waiting for inventory. The arrival process is assumed to be Poisson and service time follows an exponential distribution. During the processing of inventory, the service may be interrupted due to breakdown of the server. The failure time of a busy server is assumed as exponentially distributed and the failed server is taken for repair immediately, where the repair time also follows an exponential distribution. Inventory is managed according to an $(s, S)$ policy with zero lead time. As in the assumption of instantaneous replenishment leads to an explicit steady state analysis under the stability condition. The optimal values for reorder level $s$ and maximum inventory level $S$ is also analyzed based on a cost function. This chapter is arranged as follows. In section, we do the mathematical modeling of the above system; in section we obtain the stability condition and the explicit steady state probability vector under stability. Explicit expressions for several important performance measures are obtained in section and their behavior, as different parameters vary, is discussed in section. A cost function is also constructed in that section and its nature studied numerically.

## 2. MATHEMATICAL MODEL

The system is described as under. Customer arrive to a single server counter according to a Poisson process of rate $\lambda$ where inventory is served. Duration of service are iid exponential random variables with parameter $\mu$. Inventory is replenished according to $(s, S)$ policy, the replenishment being instantaneous. Further no shortage is permitted. While the server serves a customer, the service can be interrupted, the inter occurrence time of interruption being exponentially distributed with parameter $\delta_1$. Following a service interruption the service restarts according to an exponentially distributed time with parameter $\delta_2$. For the model under discussion, we make the following assumptions:

1. No inventory is lost due to service interruption.
2. The customer being served when interruption occurs, waits there until his service is completed.

At time $t$ let $N(t)$ be the number of the customers in the system including the one being served, $L(t)$ be the inventory level and set

$$S(t) = \begin{cases} 0 \text{ if the server is idle} \\ 1 \text{ if the server is busy} \\ 2 \text{ if the server is on interruption} \end{cases}$$

Then $\Omega\{X(t), t \geq 0\} = \{(N(t), S(t), L(t)), t \geq 0\}$ will be a Markov chain with state space $E = \{(0,0,k) | s \leq k \leq S-1\} \cup \{(i,j,k) | i \geq 1, j = 1,2, s+1 \leq k \leq S\}$. The state space of the Markov chain is partitioned into levels $\tilde{i}$ defined as $\tilde{0} = \{(0,0,s), \dots\dots, (0,0,S-1)\}$, and $\tilde{i} = \{(i,1,s+1), \dots\dots, (i,1,S), (i,2,s+1), \dots\dots, (i,2,S)\}$, for $i \geq 1$. This makes the Markov chain under consideration, a level independent Quasi Birth Death (QBD) process. In the following sequel, $Q$ stands for $S - s$, $I_n$ denotes an identity matrix of order $n$ and $e$ denotes a column matrix of 1's of appropriate order. Now the infinitesimal generator matrix of the process is

$$T = \begin{bmatrix} B_0 & B_1 & 0 & 0 & 0 & 0 & 0 \\ B_2 & A_1 & A_0 & 0 & 0 & 0 & 0 \\ 0 & A_2 & A_1 & A_0 & 0 & 0 & 0 \\ 0 & 0 & A_2 & A_1 & A_0 & 0 & 0 \\ & & & & & & \end{bmatrix}$$

Where $B_0 = \lambda I_Q$; $B_1 = [D_1 \quad 0]_{Q\times 2Q}$; $D_1 = \begin{bmatrix} 0 & 0 & & & \lambda \\ \lambda & 0 & . & & 0 \\ 0 & \lambda & 0 & & . \\ . & . & & . & . \\ 0 & 0 & & \lambda & 0 \end{bmatrix}_{Q\times Q}$ ; $B_2 = \begin{bmatrix} \mu I \\ 0 \end{bmatrix}_{Q\times 2Q}$ $A_1 =$

$\begin{bmatrix} -(\lambda+\mu+\delta_1)I & \delta_1 I \\ \delta_2 I & -(\lambda+\delta_2)I \end{bmatrix}_{2Q\times 2Q}$, where each block is a $Q \times Q$ matrix $A_0 = \lambda I_{2Q}$ and $A_2 = \begin{bmatrix} D_2 & 0 \\ 0 & 0 \end{bmatrix}_{2Q\times 2Q}$,

where each block is a $Q \times Q$ matrix.

## 3. ANALYSIS OF THE MODEL

### 3.1 Stability Condition

Define $A = A_2 + A_1 + A_0$ and $\pi = (\pi(1,s+1), \dots, \pi(1,s), \pi(2,s+1), \dots . \pi(2,s))$ be the steady state vector of the generator matrix $A$. The relations $\pi A = 0$ and $\pi e = 1$ when solved result in the values of various components of $\pi$ as

$$\pi(1,s+1) = \dots\dots = \pi(1,s) = \frac{\delta_2}{Q(\delta_1+\delta_2)} \text{ and } \pi(2,s+1) = \dots\dots = \pi(2,s) = \frac{\delta_2}{Q(\delta_1+\delta_2)}$$

The QBD process with generator $T$ is stable if and only if the rate of drift to the left is larger than the rate of drift of the level to the right: that is $\pi A_0 e < \pi A_2 e$. That is if and only if

$$\lambda < \frac{\delta_2 \mu}{(\delta_1+\delta_2)}$$

Thus we have the following theorem for stability of the system under study.

**Theorem: 3.1**

The Markov chain $\Omega$ is stable if and only if $\frac{\lambda}{\mu}\frac{(\delta_1+\delta_2)}{\delta_2} < 1$

**Note:** since the lead-time is assumed as zero, the absence of the inventory parameters $s$ and $S$ is expected. The $\frac{(\delta_1+\delta_2)}{\mu\delta_2}$ is actually the expected duration of an effective service, which is subject to interruptions at a rate $\delta_1$ and to rate $\delta_2$. Therefore, $\frac{\lambda}{\mu}\frac{(\delta_1+\delta_2)}{\delta_2}$ is the number of arrivals during a service, which should be less than 1 for stability of the system under study.

### 3.2 Computation of steady state vector

We find the steady state vector of $\Omega$ explicitly. Let $\pi = (\pi_0, \pi_1, \pi_2, \dots)$, be the steady state vector, where $\pi_0$ is partitioned as $\pi_0 = \big(\pi_0(0, s+1), \dots, \pi_0(0, S)\big)$ and $\pi_i$'s are partitioned as $\pi_i = (\pi_i(1, s+1), \dots, \pi_i(1, S), \pi_i(2, s), \pi_i(2, s+1), \dots., \pi_i(1, S))$

Then from $\pi T = 0$ and $\pi e = 1$ we get

$$-\lambda\pi_0(0,j) + \mu\pi_1(1, j+1) = 0, s \le j \le S-1 \quad (3.1)$$

$$\lambda\pi_0(0,j) - (\lambda+\mu+\delta_1)\pi_1(1,j) + \delta_2\pi_1(2,j) + \mu\pi_2(1,j+1) = 0, s+1 \le j \le S-1$$

$$\lambda\pi_0(0,s) - (\lambda+\mu+\delta_1)\pi_1(1S), +\delta_2\pi_1(2,S) + \mu\pi_2(1,s+1) = 0 \quad (3.2)$$

$$\lambda\pi_1(1,j) - (\lambda+\mu+\delta_1)\pi_{i+1}(1,j) + \delta_2\pi_{i+1}(2,j) + \mu\pi_{i+2}(1,j+1) = 0, s+1 \le\ j \le S-1$$

$$\lambda\pi_1(1,S) - (\lambda+\mu+\delta_1)\pi_{i+1}(1,S) + \delta_2\pi_{i+1}(2,S) + \mu\pi_{i+2}(1,s+1) = 0, i \ge 1 \quad (3.3)$$

$$\delta_1\pi_1(1,j) - (\lambda+\delta_2)\pi_1(2,j) = 0, s+1 \le j \le S-1 \quad (3.4)$$

$$\lambda\pi_i(2,j) + \delta_1\pi_{i+1}(1,j) - (\lambda+\delta_2)\pi_{i+1}(2,j) = 0, i \ge 1, s+1 \le j \le S-1 \quad (3.5)$$

For solving the above system of equations, we first consider an $M/PH/1$ queue with arrival process Poisson with parameter $\lambda$ and service time for each customer having PH distribution with representation $(\alpha, K)$, where the initial probability vector is $\alpha = (1,0)$ and the matrix

$$K = \begin{bmatrix} -(\mu+\delta_1) & \delta_1 \\ \delta_2 & -\delta_2 \end{bmatrix}$$

Then the generator matrix of this system (namely, the M/PH/1 queue) has the form:

$$\hat{T} = \begin{bmatrix} -\lambda & \lambda\,\alpha & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ K^0 & K-\lambda\,\mathrm{I} & \lambda\,\mathrm{I} & 0 & 0 & \vdots & \cdot & \cdot \\ 0 & K^0\alpha & K-\lambda\,\mathrm{I} & \lambda\,\mathrm{I} & 0 & 0 & \cdot & \cdot \\ 0 & 0 & K^0\alpha & K-\lambda\,\mathrm{I} & \lambda\,\mathrm{I} & \vdots & \vdots & \vdots \\ & & & & \cdot & \vdots & \vdots & \vdots \\ & & & & & & \cdot & \cdot \end{bmatrix}$$

Let $x = (x(0), x(1), x(2), \dots.)$ be the steady state vector of $\hat{T}$. Partitioning $x(i)$s' as $x(0) = x(0,0), x(i) = \big(x(i,1), x(i,2)\big), i \ge 1$, the steady state relation $x\hat{T} = 0$, gives us the following equations.

$$-\lambda x(0,0) + \mu x(1,1) = 0 \qquad 3.2.a$$

$$\lambda x(0,0) - (\lambda + \mu + \delta_1)x(1,1) + \delta_2 x(1,2) + \mu x(2,1) = 0 \qquad 3.2.b$$

$$\delta_1 x(1,1) - (\lambda + \delta_2)x(1,2) = 0 \qquad 3.2.c$$

$$\lambda x(i,1) - (\lambda + \mu + \delta_1)x(i+1,1) + \delta_2 x(i+1,2) + \mu x(i+2,1) = 0, i \geq 1 \qquad 3.2.d$$

$$\lambda x(i,2) + \delta_1 x(i+1,1) - (\lambda + \delta_2)x(i+1,2) = 0, i \geq 1 \qquad 3.2.e$$

If we assume that

$$\left.\begin{array}{l} \pi_0(0,s) = \pi_0(0,s+1) = \ \ldots\ldots.\ldots = \pi_0(0,S-1)\text{and} \\ \pi_i(j,s+1) = \pi_i(j,s+2) = \ \ldots = \pi_i(j,s+1)(j,S), i \geq 1, j = 1,2 \end{array}\right\} \qquad (3.2.I)$$

then the $S$-s equations in 3.1 for each value of $I$ will be the same as the single equation 3.2.a and similarly equation 3.2 to 3.5 reduces to 3.2.b to 3.2.e respectively and therefore the probabilities $\pi_i(j,k)$ can be obtained from the corresponding $x(i,j)$ as

$$\left.\begin{array}{l} \pi_0(0,k) = \dfrac{1}{Q}x(0,0), s \leq k \leq S-1 \\ \pi_1(j,k) = \dfrac{1}{Q}x(i,j), j = 1,2, s+1 \leq k \leq S, \end{array}\right\} \qquad (3.2.\text{II})$$

The intuition behind the assumption 3.2.I is that, since replenishment is instantaneous, in the steady state, there will be equal chance for each inventory level to be visited. It can be verified that the $\pi_i(j,k)$'s obtained from 3.2.II, satisfies the steady state equations 3.2.1 to 3.2.5 and so are the unique steady state probabilities of the system under the stability condition. Now for the steady state probabilities $x(i,j)$, we have results available for the standard M/PH/1 queue, which give

$x(i) = x(1)R^{i-i}, i \geq 1$, where

$$R = \begin{bmatrix} \dfrac{\lambda}{\mu} & \dfrac{\lambda\delta_1}{\mu(\lambda+\delta_2)} \\ \dfrac{\lambda}{\mu} & \dfrac{\lambda(\mu+\delta_1)}{\mu(\lambda+\delta_2)} \end{bmatrix} \text{and}$$

$$x(1) = x(0)\begin{bmatrix} \dfrac{\lambda}{\mu} & \dfrac{\lambda\delta_1}{\mu(\lambda+\delta_2)} \end{bmatrix}; x(0) = x(0,0) - \frac{\lambda(\delta_1+\delta_2)}{\mu\delta_2}$$

**4.1 Expected Number of interruptions encountered by a customer**

For computing expected number of interruptions encountered by a customer we proceed in the same line as in Krishnamoory et. Al. by considering a Markov process $\{X_1(t), t \geq 0\} = \{(N_1(t), S_1(t)), t \geq 0\}$, where $N_1(t)$ denoted the number of interruptions that has occurred up to time $t = S_1(t) = 0$ or 1 according as the service is under interruption or not at time $t$. The Markov process $\{X_1(t), t \geq 0\}$ has state space $\{0,1,2, \ldots.\} \times \{0,1\} \cup \{\Delta\}$, where $\Delta$ is an absorbing state which denotes service completion. The infinitesimal generator of the process is the same.

$$\hat{U} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & . & . & . \\ \hat{B}_{00} & \hat{A}_{00} & \hat{A}_{01} & 0 & 0 & & . & . \\ \hat{A}_2 & 0 & \hat{A}_1 & \hat{A}_0 & 0 & & . & . \\ \hat{A}_2 & 0 & 0 & \hat{A}_1 & \hat{A}_0 & . & . & . \\ \hat{A}_2 & 0 & 0 & 0 & \hat{A}_1 & \hat{A}_0 & . & . \\ & & & & & & . & . \\ & & & & & & . & . \end{bmatrix}, \text{where in the present case,}$$

$$\hat{B}_{00} = [\mu], \hat{A}_{00} = [-(\mu+\delta_1)], \hat{A}_{01} = [\delta_1 \quad 0], \hat{A}_2 = \begin{bmatrix} 0 \\ \mu \end{bmatrix}, \hat{A}_1 = \begin{bmatrix} -\delta_2 & \delta_2 \\ 0 & -(\mu+\delta_1) \end{bmatrix} \text{ and } \hat{A}_0 = \begin{bmatrix} 0 & 0 \\ \delta_1 & 0 \end{bmatrix}$$

If $y_k$ is the probability that absorption occurs with exactly $k$ interruptions, then

$$y_0 = -\hat{A}_{00}{}^{-1}\hat{B}_{00} = \frac{\mu}{\mu+\delta_1}$$

$$y_k = \left(-\hat{A}_{00}{}^{-1}\hat{A}_{01}\right)\left(-\hat{A}_1{}^{-1}\hat{A}_0\right)\left(-\hat{A}_1{}^{-1}\hat{A}_2\right) = \frac{\mu}{\mu+\delta_1}\left(\frac{\mu}{\mu+\delta_1}\right)^k, k = 1,2,3,\ldots$$

The expected number of interruptions before absorption is given by

$$E_1 = \sum_{k=0}^{\infty} k y_k = \left(-\hat{A}_{00}{}^{-1}\hat{A}_{01}\right)\left[I_2 - \left(-\hat{A}_1{}^{-1}\hat{A}_0\right)\right]^{-1} e = \frac{\delta_1}{\mu}$$

### 4.2 Expected duration of an interrupted service

Here we calculated the average duration of an interrupted service. The procedure for this is again similar to that in [29]. The service process with interruption can be viewed as a Markov process with two transient states 0 and 1, which denote whether the server is interrupted or is busy respectively, and a single absorption state $\Delta$. The absorption state $\Delta$ denotes the completion of the service after the intervening interruptions and repairs. The process can be represented by $\hat{X}(t) = \{0,1,\Delta\}$. Let $T$ be the time until absorption in the process $\hat{X}(t)$. The infinitesimal generator matrix of the process is given by

$$\hat{H} = \left[\hat{B}, \hat{B}_0\right], \text{where } \hat{B} = \begin{bmatrix} -\delta_2 & \delta_2 \\ \delta_1 & -(\mu+\delta_1) \end{bmatrix} \text{ and } \hat{B}_0 = \begin{bmatrix} 0 \\ \mu \end{bmatrix}$$

The probability distribution $F(\cdot)$ of $T$ is given by $F(x) = 1 - \xi \exp(\hat{B}x)\, e, x \geq 0$. Its density function $F'(x)$ in $(0,\infty)$ is given by $F'(x) = \xi \exp(\hat{B}x)\,\hat{B}_0$. The Lapalce-Stieltjes transform $f(s)$ of $F(\cdot)$ is $f(s) = .\,\xi(sI - \hat{B})^{-1}\hat{B}_0$. The expected time $E_s$ for service completion is

$$E_s = \xi(-\hat{B})^{-1}e = \frac{\delta_1+\delta_2}{\mu\delta_2}$$

### 4.3 Expected amount of time a customer is served during his (possibly interrupted) service

Though we can find the expected waiting time using Little's formulae we do so otherwise and verify the result obtained using the above. For any M/G/1 queue the mean waiting time of a customer in the system is given by $E\{W\} = E\{W_Q + s\} = E(s) + E\{W_Q\}$

$$= E_s + \frac{\lambda}{2(1-\rho)}E(s^2) = \frac{\delta_1+\delta_2}{\mu\delta_2} + \frac{\lambda}{2\left\{1 - \frac{\lambda(\delta_1+\delta_2)}{\mu\delta_2}\right\}} 2\xi(-\hat{B})^{-2}e$$

$$= \frac{\delta_1 + \delta_2}{\mu\delta_2} + \frac{\lambda}{2\left\{1 - \frac{\lambda(\delta_1+\delta_2)}{\mu\delta_2}\right\}} \frac{2}{\mu^2\delta_2^2}\{\mu\delta_1 + (\delta_1 + \delta_2)^2\} = \frac{\delta_2^2 + \delta_1\lambda + \delta_1\delta_2}{\delta_2\{\mu\delta_2 - \lambda(\delta_1 + \delta_2)^2\}}.$$

We have obtained the expression for the expected number of customers in the system as

$$EN = \frac{\delta_2^2 + \delta_1\lambda + \delta_1\delta_2}{\mu\delta_2 - \lambda(\delta_1 + \delta_2)}$$

Hence Little's theorem us verified

**4.5 Busy Period**

We have the expected duration of a busy period $T$ is given by

$$E(T) = E(s) + \frac{\lambda}{2(1-\rho)}E(s^2) = \frac{\delta_1 + \delta_2/\mu\delta_2}{\{1 - \lambda(\delta_1 + \delta_2)/\mu\delta_2\}} = \frac{\delta_1 + \delta_2}{\mu\delta_2 - \lambda(\delta_1 + \delta_2)}$$

**4.6 Other performance measures**

1. Probability that server is busy is given by
$$P_\beta = \sum_{i=1}^{\infty}\sum_{j=s+1}^{S}\pi_i(1,j) = \frac{\lambda}{\mu}$$
2. Probability that server is on interruption is given by
$$P_\alpha = \sum_{i=1}^{\infty}\sum_{j=s+1}^{S}\pi_i(2,j) = \frac{\delta_1}{\delta_2}\frac{\lambda}{\mu}$$
3. Probability that server is idel is given by
$$P_\gamma = 1 - P_\alpha - P_\beta = 1 - \frac{\lambda}{\mu}\left(1 + \frac{\delta_1}{\delta_2}\right)$$
4. Expected inventory level is given by
$$EIL = \sum_{j=s+1}^{s}\pi_0(0,j) + \sum_{i=1}^{\infty}\sum_{j=s+1}^{S}\pi_i(1,j) + \pi_i(2,j) = \frac{s+S}{2}$$
5. Expected number of customers in the system is given by
$$EN = \sum_{i=1}^{\infty}\sum_{j=s+1}^{S}\{i\pi_i(1,j) + i\pi_i(2,j)\} = \pi_1(I-R)^{-2}e = \frac{\lambda}{\delta_2}\frac{\delta_2^2 + \delta_1\lambda + \delta_1\delta_2}{\mu\delta_2 - \lambda(\delta_1 + \delta_2)}$$
6. Expected rate of ordering is given by
$$E_{OR} = \sum_{i=1}^{\infty}\mu\pi_i(1, s+1) = \frac{\lambda}{Q}$$
7. Expected interruption rate is given by
$$E_{INTR} = \delta_1\sum_{i=1}^{\infty}\sum_{j=s+1}^{S}\pi_i(1,j) = \frac{\delta_1\lambda}{\mu}$$

**5. System behavior with variational in parameters**

The explicit expressions for all the system performance measures make the analysis of their dependence on various parameters more transparent. The maximum inventory level $S$ and the reorder level $s$ affects the expected inventory level and expected reorder rate only. The other performance measures are independent of $s$ and $S$. This can be attributed to the fact that replenishment is instantaneous. The expression fro server interruption probability $P_\alpha$ show that if we take $\delta_1 = \delta_2$, the probability $P_\alpha$ is just $\frac{\lambda}{\mu}$ which is independent of both $\delta_1$ and $\delta_2$. The expected number of customers in the system increases with increase in arrival rate $\lambda$ and decrease with decrease in service rate $\mu$; both these facts are clear from the expression for expected number of customers. However, since the effect of the parameters $\delta_1$ and $\delta_2$ on the expected number of customers is not that clear from the expression for $EN$, we studied this numerically. Table 1(a) and 1(b) show the effect of $\delta_1$ and $\delta_2$ respectively on $EN$. Table 1(a) shows that as $\delta_1$, the interruption rate increases, $EN$ also increases, which is expected as the interruptions become more frequent, the effective service time of a customer increases and this leads to an increase in the queue length. Table 1(b) shows that an increase in the repair rate $\delta_2$, results in a decrease in the expected number of customers in the system. This is also expected as the repair rate increases; the server becomes active in a shorter time after an interruption which leads to an increase in the service completion rate and hence the queue length also.

| $\delta_1$ | $EN$ |
|---|---|
| 2 | 1.417 |
| 2.2 | 1.561 |
| 2.4 | 1.722 |
| 2.6 | 1.902 |
| 2.8 | 2.104 |
| 3 | 2.33 |
| 3.2 | 2.595 |
| 3.4 | 2.897 |
| 3.6 | 3.25 |
| 3.8 | 3.667 |
| 4 | 4.267 |

**(a)**

| $\delta_2$ | $EN$ |
|---|---|
| 2 | 2.5 |
| 2.2 | 2.129 |
| 2.4 | 1.869 |
| 2.6 | 1.678 |
| 2.8 | 1.532 |
| 3 | 1.416 |
| 3.2 | 1.324 |
| 3.4 | 1.248 |
| 3.6 | 1.184 |
| 3.8 | 1.13 |
| 4 | 1.083 |

**(b)**

**Table 1 :** Effect of $\delta_1$ and $\delta_2$ on $E(\sigma)$ with $\lambda = 1, \mu = 3, s = 6, S = 20$; we have taken for table 1(a), $\delta_2 = 3$, and for table 1(b), $\delta_1 = 2$.

## REFERENCE

1. J R Artalejo and A Gomez-Corral; Retrial Queueing Systems: A Computational Approach; Springer-Verlag, Berlin(2008).
2. J R Artalejo, A Krishnamoorthy and M J Lopez-Herrero; Numerical analysis of $(s, S)$ inventory systems with repeated attempts; Ann Oper Res; (2006) 141 : 67-83.
3. O. Berman and E. Kim, Stochastic models for inventory management at service facilities, Communications in Statistics. Stochastic Models 15 (1999), no.4, 695-718.
4. S R Chakravarthy, A Krishnamoorthy and V C Joshua; Analysis of a multi-server retrial queue with search of customers from the orbit; Performance Evaluation; Vol.63, Issue 8,2006, pp. 776-798.

5. Y Chen and X Li; The effect of customer segmentation on an inventory system in the presence of supply disruptions; Proceedings of the 2009 Winter Simulation Conference; Eds. M.D. Rossetti, R. R. Hill, B. Johansson, A. Dunkin, and R. G. Ingalls, eds. Pp 2343-2352.
6. T G Deepak, C.N.Viswanath and V C Joshua; On an Inventory with positive service time having optimal processing time; Bulletin of Kerala Mathematics Association, Vol. 4, No.2, 2007, December, 75-86.
7. A Fedegruen and L Green; Queuing systems with service interruptions; Operations Research, vol. 34(5); 752-768, 1986.
8. A Krishnamoorthy, Chakravarthy, P K Pramod; Queues with Interruption and Repeat/Resumption of Service- A Survey and Further Results, Fourth International Conference on Neural, Parallel & Scientific Computations; August 11-14, 2010, Morehouse College, Atlanta, Georgia, USA.
9. H Masuyama and T Takine. Stationary queue length in a FIFO single server queue with service interruptions and multiple batch Markovian arrival streams; Journal of the Operations Research Society of Japan vol.46(3) , 319-341, 2003
10. J.Medhi; Stochastic process (Second edition). New age International (P),LTD Publishers, New Delhi.