# A NOTE ON MATHEMATICAL PROGRAMMING APPROACH IN RELIABILITY AND MULTIVARIATE SAMPLING PROBLEMS

[1]Dr. V. SHANMUGA SUNDARAM & [2]M. SIVALINGAM

1. Head & Assistant Professor, Department of Statistics, Mahendra Arts & Science College, (Autonomous), Kalippatti, Namakkal-637501.

&

2. M.Phil., Research Scholar, Department of Mathematics, Mahendra Arts & Science College, (Autonomous), Kalippatti, Namakkal-637501.

**Abstract:** We took for the following result for multivariate sampling problems in Mathematical programming.

**Keywords:** Sampling Theory, Linear programming, Non linear programming, integer Programming and Stochastic programming.

## I INTRODUCTION

Optimization is the science of selecting the best of many possible decisions in a complex real life situation. The ultimate target of all such decisions is to either maximize the desire benefit or to minimize the effort required, incurred in a certain course of action.

The systematic approach to decision making generally involves three closely interrelated stages. The first stage towards optimization is to express the desired benefits, the required efforts and collecting the other relevant data, as a function of certain variables that may be called “decision variables”. The second stage continues the process with an analysis of the mathematical model and selection of appropriate numerical technique for finding the optimal solution. The third stage consists of finding an optimal solution, generally with the help of computer.

The existence of optimization problem can be traced to the middle of eighteenth century. The work of Newton, Lagrange and Cauchy in solving certain type of optimization problems arising in physics and geometry by using differential calculus methods and calculus of variations is pioneering. These optimization techniques are known as classical optimization techniques and can be generalized to handle cases in which the variables are required to be non-negative and constrains may be inequalities, but these generalizations are primarily of theoretical value and do not usually constitute computational procedure. However, in some simple situations they can provide solutions, which are practically acceptable.

The optimization problem that have been posed and solved in the recent years have tended to become more and more elaborate, not to say abstract. Perhaps, the most outstanding example of the rapid development of the optimization techniques occurred with the introduction of dynamic programming by Bellman in 1957 and of the maximum principle by Pontryagin in 1958, and the techniques were designed to solve the problems of the optimal control of dynamic systems.

## II Mathematical Programming

A large number of real-life optimization problems that are usually not solvable by classical optimization methods are formulated as mathematical programming problems. There has been considerable advancement towards the development of the theory and algorithms for solving various types of mathematical programming problems.

The first mathematical programming problem was considered by economists (Von Neumann) in early 1930s, as the problem of optimum allocation of limited resources. Leontief in 1951 showed a practical solution method for linear type problems when demonstrated his input-output model of an economy. These economic solution procedures did not provide optimal solution, but only a feasible solution, providing the model's linear constraints. In 1941, Hitchcock formulated and solved the transportation type problem, which was also accomplished by Koopmans in 1949. In 1942, Kantorovitch also formulated the transportation problem but did not solve it. In 1945, the economist G.J. Stigler formulated and solved the "minimum cost diet" problem. During World War II a group of researchers under the direction of Marshall K. Wood sought to solve allocation type problem for the United States Air Force team SCOOP (Scientific Computation of Optimum Porgrams). One of the members or this group, George B.Dantzig, formulated and devised a solution procedure in 1947 for Linear Programming Problems (LPP). This solution procedure, called the Simplex method, marked the beginning of the field of study called mathematical programming. During the 1950s other researchers such as David Gale, H.W. Kuhn and A.W. Tucker contributed to the theory of duality in L.P. Others such a Charnes and Cooper contributed numerous LP applications illustrating the use of mathematical programming in managerial decision-making.

A general Mathematical Programming Problem can be stated as following:

$$Max\ (or\ Min) = Z = f(X) \qquad (2.1)$$

$$Subect\ to\ g_i(X) \leq or = or \geq b_i \forall\ i = 1,2, \ldots \ldots, m \qquad (2.2)$$

Where

$X =$ Vector of unknown variables that are subject to the control of decision maker.

$Z =$ Value of the objective function which measures the effectiveness of the

Decision choice.

$g_i(X) =$ The function representing the $i^{th}$ constraint, $i = 1, \ldots \ldots, m$

$b_i =$ available $i^{th}$ productive resource in limited supply, $i = 1, \ldots \ldots, m$

The objective function (2.1) is a mathematical equation describing a functional relationship between various decision variables and the outcome of the decisions. The outcome of managerial decision-making is the index of performance, and is generally measured by profits, sales, costs, or time. Thus, the value of the objective function in mathematical programming is expressed in monetary, physical, or some other terms, depending on the nature of the problem. The objective function $f(X)$ and the constraining functions $g_i(X)$ may be either linear or nonlinear functions of variables. The objectives of the decision makes is to select the values of the variables so as to optimize the value of the objective function $Z$ under the given constraints. If $f(X)$ and $g(X)$ both are linear functions of $X$, then the problem (2.1)-(2.2) represents a linear programming problem (LPP). When the objective function to be minimized (maximized) is convex (concave) and the set defined by the constraining inequalities (2.2) is also convex, the problem is called a convex programming problem (C.P.P), otherwise it is a non-convex programming problem. If some or all the components of the vector $X$ are required to be integer, then we call it an integer programming problem (IPP). The methods of linear programming, non-linear programming and Integer Programming are discussed below.

## III Linear Programming Techniques

The general approach to the modeling and solution to linear mathematical models, and more specifically those models that seek to optimize a linear measure of performance, under linear constraints is called as linear programming or more appropriately as linear optimization. The general form of the (single objective) Linear Programming Problem (LPP) is given as to find the decision variables $x_1, x_2, \ldots \ldots, x_p$, which maximize or minimize a linear function, subject to some linear constraints and the non-negativity restrictions on the decision variables. Mathematical model for a general linear programming problem is stated as follows:

$$Max\ (or\ Min)\ Z = \sum_{j=1}^{p} c_j x_j$$

$$Subject\ to\ \sum_{j=1}^{p} a_{ij} x_j \leq or = or \geq b_i \forall\ i = 1,2, \dots \dots, m$$

$$x_j \geq 0 \qquad\qquad \forall\ j = 1,2, \dots \dots, p$$

where $c_j, a_{ij}$ and $b_i$ (called parameters of the LPP) are known constants for all $i$ and $j$.

**IV Nonlinear Programming Techniques**

The mathematical model that seeks to optimize a non-linear measure of performance is called non-linear program. Every real world optimization problem has always a non-linear form which becomes a linear programming problem after a slight modification. Non linear programming emerges as an increasingly important tool in economic studies and in operations research. Non linear programming problems arise in various disciplines as engineering, business administration, physical sciences and in mathematics or in any other area where decision must be taken in some complex situation that can be represented by a mathematical model:

$$Minimize\ f(x), Subject\ to\ g_i(x) \geq 0, \quad i = 1,2, \dots., m$$

where all or some of the functions $f(x)$ and $g_i(x), i = 1, \dots., m$ are non-linear.

Interest in nonlinear programming problem developed simultaneously with the growing interest in linear programming. In the absence of general algorithms for nonlinear programming problem, it lies near at hand to explore the possibilities of approximate solution by linearization. The nonlinear functions of a mathematical programming problem were replaced by piecewise linear functions. These approximations mat be expressed in such a way that the whole problem is turned into linear programming.

**V Integer Programming**

Any decision problem (with an objective to be maximized or minimized) in which the decision variables must assume non fractional or discrete values may be classified as an integer optimization problem. In general, an integer problem may be constrained or unconstrained and the functions representing the objective and constraints any be linear or non linear. An integer problem is classified as linear if by relaxing the integer restriction on the variables, the resulting functions are strictly linear.

The general mathematical model of an integer-programming problem can be stated as:

Maximize (or Minimize)

$$Z = f(X)$$

Subject to $g_i(X)\{\leq or = or \geq\} b_i, i = 1,2, \dots \dots, m$

$x_j \geq 0 \qquad\qquad j = 1,2, \dots \dots \dots n.$

$x_j$ is an integer for $j \in J \subseteq I = (1,2, \dots \dots., n)$

where $X = (x_1, \dots., x_n)$ is $n$-component vector of decision variables

If $J = I$, that is, all the variables are restricted to be integers we have an all (or pure) integer programming problem (AIPP) Otherwise, if $J \subset I$, i.e., not all the variables are restricted to be integers, we have a mized integer-programming problem (MIPP).

## VI Stochastic Programming

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problem are formulated with known parameters, real world problems almost invariably include some unknown parameters. When the parameters are known only within certain bounds, one approach of tackling such problems is called robust optimization. Here is a goal to find a solution, which is feasible for all such data and optimal in some sense. Stochastic programming models are similar in style but take advantage of the fact that probability distribution governing the data are known or can be estimated. The goal here is to find some policy that is feasible for all (or almost all) the possible data; for instance we maximize the expectation of some function of the decision and random variables. More generally, such models are formulated, solved analytically or numerically, and analyzed in order to provide useful information to a decision maker.

The most widely applied and studied stochastic programming models are two stage linear programs. Here the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first stage decision. A recourse decision can then be made in the second state that compensates for any bad effects that might have been experienced as a result of first stage decision. The optimal policy from such a model is a single first stage policy and a collection of recourse decisions (a decision rule) defining which second stage action should be taken in response to each random outcome.

A Stochastic linear programming problem can be stated as:

$$Maximize\ f(x) = \sum_{j=1}^{n} c_j x_j$$

$$Subject\ tp \sum_{j=1}^{n} a_{ij} x_j \geq b_i, i = 1,2, \dots, m$$

and $x_j \geq 0; j = 1,2, \dots, n$

where some or all the coefficients $c_j, a_{ij}$ and $b_j$ are random variables with known probability distributions. The decision variables $x_j$ are assumed to be deterministic for simplicity.

## VII Optimization in Multivariate Stratified Sampling with a Probabilistic Cost Constraint

Optimum allocation of sample size to various strata in univariate stratified random sampling is well defined in the literature. But usually in real life problems more than one population characteristics are to be estimated, which may be of conflicting nature. There are situations where the cost of measurement varies from stratum to stratum. Also the cost of enumerating various characters is generally much different. Further the strata variances for the various characters may not be distributed in the same way. Allocation based on one character may not be optimum for the others. One way to resolve this problem is to search for a compromise allocation, which is in some sense optimum for all character.

Kokan and Khan (1967), Chatterjee (1968), Huddleston et al. (1970), Bethel (1985, 1989), Chromy (1987) all discussed the use of convex programming in relation to multivariate optimal allocation problem. The above convex programming approach gives the optimal solution to the problem with given tolerance limits on variances but the resulting cost may not be acceptable so that a further search is usually required for an optimal solution which falls within the budgetary constraints limit.

The problem of optimal allocation in stratified sampling is generally stated in two ways. Either one minimizes the cost of survey for a desired precision or the variance of the sample estimate is minimized for a given budget of the survey. Kokan and Khan (1967) formulated the minimization of the cost of the survey for desired precisions on various characters as the following convex integer programming problem;

$$\left.\begin{array}{c}\underset{n}{Min}\sum_{i=1}^{L} c_i n_i \;\; Subject\; to \;\sum_{i=1}^{L}\frac{a_{ij}}{n_i}-\sum_{i=1}^{L}\frac{a_{ij}}{N_i}\le k_j, j=1,\ldots p \\ and\; a_i \le n_i \le N_i, n_i \in I, i=1,\ldots\ldots,L)\end{array}\right\} \qquad (7.1)$$

where $L$ is the number of strata, $N_i$ are the strata sizes, $p$ is the number of characters to be estimated in the survey, $I$ is the set of integers and $c_i, a_i, a_{ij}$ and $k_j$ are all positive constants.

If the budget of the survey is fixed in advance, then the multivariate allocation problem was stated to minimize the variances for various characters for a desired precision as the following $p$ convex integer programming problems:

$$\left.\begin{array}{c}\underset{n}{\min} V = \sum_{i=1}^{L}\frac{a_{ij}}{n_i}-\sum_{i=1}^{L}\frac{a_{ij}}{N_i}, j=1,\ldots p \\ Subject\; to \\ \sum_{i=1}^{L} c_i n_i + c_0 \le C \; and \;\; a_i \le n_i \le N_i, n_i \in I, i=1,\ldots\ldots,L)\end{array}\right\} \qquad (7.2)$$

Further, in a survey the costs for enumerating a character in various strata are not know exactly, rather these are being estimated from sample costs. As such the formulated allocation problem should be considered as stochastic programming problem. Stochastic programming problem was first formulated by Dantzig (1955), who suggested a two stage programming technique for its solutions. Later, Charnes & Cooper (1959) developed the chance constrained programming technique in which the chance constraints where converted into equivalent deterministic non-linear constraints.

When the constants $c_i$ and $a_{ij}, (i=1,\ldots.,L, j=1\ldots.,p)$ are fixed, the problem (7.1) we solved Kokan and Khan (1967) by using analytical procedure. Prekopa (1995) developed a method from stochastic point of view. The case when sampling variances are random in the constraints (i.e. $a_{ij}$ random in (1.1)) has been dealt with Diaz-Garcia et al. (2007). Javaid and Bakhshi (2009) applied modified E-model for solving this problem when the costs were considered random in the objective function.

In this chapter, we consider the case of random costs in the formulation (2.2). The probabilistic cost constraint is converted to an equivalent deterministic constraint by using chance constrained programming. The problem (2.2) with multiple objectives is then treated for searching an optimum solution. In section 2.2 the cost functions in various strata are assume to be linear. This work has been published in international journal of Mathematics and Applied Statistics, see Bakshi,Z.H. et al (2010). The case of non-linear cost functions among various strata is treated in section 2.5.

## VIII Formulation of the problem for linear cost functions

We consider a multivariate population consisting of $N$ units which is divided into $L$ disjoint strata of sizes $N_1, N_2, \ldots, N_L$ such that $N=\sum_{i=1}^{L} N_i$. Suppose that $p$ characteristics $(j=1,\ldots.,p)$ are measured on each unit of the population. We assume that the strata boundaries are fixed in advance. Let $n_i$ units be drawn without replacement from the $i^{th}$ stratum $i=1,\ldots\ldots.,L$.

For $j^{th}$ character, an unbiased estimate of the population mean $\bar{Y}_j$ $(j=1,\ldots,p)$, denoted by $\bar{Y}_{jst}$, has its sampling variance

$$V(\bar{y}_{jst})=\sum_{i=1}^{L}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)W_i^2 S_{ij}^2, j=1,\ldots.,p, \qquad (8.1)$$

Where

$$W_i = \frac{N_i}{N}, S_{ij}^2 = \frac{1}{N_i - 1}\sum_{h=1}^{N_i}\left(y_{ijh} - \bar{Y}_{ij}\right)^2$$

are the variances, $y_{ijh}$ are the population values and $\bar{Y}_{ij}$ is the population mean for the $j^{th}$ character in the $i^{th}$ stratum. The estimated variance of $j^{th}$ character in $i^{th}$ stratum is

$$S_{ij}^2 = \frac{1}{n_i - 1}\sum_{h=1}^{n_i}\left(y_{ijh} - \bar{Y}_{ij}\right)^2 \qquad (8.2)$$

Where $\bar{Y}_{ij}$ is the sample mean for $j^{th}$ character in $i^{th}$ stratum.

Let $C_{ij}$ be the cost of enumerating the $j^{th}$ character in the $i^{th}$ stratum and let the overhead cost $c_0$ be constant. Let $C$ be the upper limit on the total cost of the survey.

Then assuming linear cost function one should have

$$\sum_{i=1}^{L}\sum_{j=1}^{p} c_{ij} n_i + c_0 \le C \;\; or \;\; \sum_{i=1}^{L} c_i n_i + c_0 \le C, \qquad (8.3)$$

Where $c_i = \sum_{j=1}^{p} c_{ij}$ is the cost of enumeration of all the $p$ character in the $i^{th}$ stratum. For minimizing the variances it is clear from (8.2) that we should have $n_i \ge 2$. Further, the sample size should not exceed the stratum size, i.e, $n_i \le N_i$. So the restrictions on the sample from various strata are

$$2 \le n_i \le N_i, \quad and\; n_i \in I, \qquad (8.4)$$

Where $I$ is the set of integers. The survey is to be conducted in such a way that the variances for all the $p$ characters are minimized for a fixed budget. Combining (8.1), (8.2) and (8.4) the multivariate allocation problem in stratified sampling can be stated as the following non-linear programming problem with multiple objectives:

$$\left.\begin{array}{ll} \min_n V = \sum_{i=1}^{L}\frac{W_i^2 S_{ij}^2}{n_i} - \sum_{i=1}^{L}\frac{W_i^2 S_{ij}^2}{N_i}, j = 1, \dots p & (i) \\ \text{Subject to} & \\ \sum_{i=1}^{L} c_i n_i + c_0 \le C & (ii) \\ 2 \le n_i \le N_i, n_i \in I, i = 1, \dots L & (iii) \end{array}\right\} \quad (8.5)$$

Ignoring the constant terms in {8.5(i)}, the problem (8.5) is reduced to the following $p$ convex programming problems:

$$\min_n V = \sum_{i=1}^{L}\frac{W_i^2 S_{ij}^2}{n_i}, j = 1, \dots p$$

Subject to (8.6)

$$\sum_{i=1}^{L} c_i n_i + c_0 \leq C$$

$$and\ 2 \leq n_i \leq N_i, n_i \in I, i = 1, \dots L$$

In many practical situations the costs $c_i$ in the various strata are not fixed and vary from on unit to the other. Let us assume that $c_i, i = 1, \dots L$ are independently normally distributed random variables.

So, we write the problem (8.6) in the following chance constrained programming form:

$$\left.\begin{array}{ll} \underset{n}{\text{Min}}\, V = \sum_{i=1}^{L} \frac{W_i^2 S_{ij}^2}{n_i}, j = 1, \dots p & (i) \\ \text{Subject to} & \\ P\left(\sum_{i=1}^{L} c_i n_i + c_0 \leq C\right) \geq p_0 & (ii) \\ 2 \leq n_i \leq N_i, n_i \in I, i = 1, \dots L & (iii) \end{array}\right\} \quad (8.7)$$

Where $p_0, 0 \leq p_0 \leq 1$ is a specified probability.

**XI SOLUTION USING CHANCE CONSTRAINED PROGRAMMING**

The costs $c_i = i = 1, \dots, L$ in the constraint 8.7 (ii) are assumed to be independently and normally distributed random variables. Let $c' = (c_1, \dots, c_L)$ and $n' = (n_1, \dots, n_L)$. Then the function $(c'n + c_0)$, will also be normally distributed with mean $E(c'n + c_0)$ and variance $V(c'n + c_0)$.

If $c_i \sim N(\mu_i, \sigma_i^2)$, then its p.d.f will be

$$f(c_i) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{1}{2\sigma_i^2}(c_i - \mu_i)^2}, i = 1, \dots, L$$

The Joint distribution of $(c_1, \dots, c_L)$ is given by

$$f(\underline{c}') = \frac{1}{(2\pi)^{L/2} \prod_{i=1}^{L} \sigma_i} \exp\left(-\frac{1}{2} \sum_{i=1}^{L} \frac{(c_i - \mu_i)^2}{\sigma_i^2}\right),$$

Then, the mean of the function $(c'n + c_0)$ is obtained as

$$E(c'n + c_0) = E\left(\sum_{i=1}^{L} c_i n_i\right) + c_0 = \sum_{i=1}^{L} n_i E(c_i) + c_0 = \sum_{i=1}^{L} n_i \mu_i + c_0 \quad (9.1)$$

Where $\mu_i = E(c_i), i = 1, \dots, L$.

The variance is obtained as

$$V(c'n + c_0) = V(c'n) = V\left(\sum_{i=1}^{L} c_i n_i\right) = \sum_{i=1}^{L} n_i^2 V(c_i) = \sum_{i=1}^{L} n_i^2\, \sigma_i^2, \qquad (9.2)$$

Where $\sigma_i^2 = V(c_i)$.

Now let $f(c) = \sum_{i=1}^{L} c_i n_i + c_0$, then {2.9(ii)} is given by

$P(f(c) \le C) \ge p_0$,

$$or\ P\left\{\frac{f(c) - E(f(c))}{\sqrt{V(f(c))}} \le \frac{C - E(f(c))}{\sqrt{V(f(c))}}\right\} \ge p_0$$

Where $\left[\frac{f(c)-E(f(c))}{\sqrt{V(f(c))}}\right]$ is a standard normal variate with mean zero and variance one. Thus the probability of realizing $\{f(c)\}$ less than or equal to $C$ can be written as

$$P(f(c) \le C) = \phi\left[\frac{C - E(f(c))}{\sqrt{V(f(c))}}\right] \qquad (9.3)$$

Where $\phi(z)$ represents the cumulative density function of the standard normal variable evaluated at $z$. If $K_\alpha$ represents the value of the standard normal variate at which $\phi(K_\alpha) = p_0$, then the constraint (9.3) can be written as

$$\phi\left[\frac{C - E(f(c))}{\sqrt{V(f(c))}}\right] \ge \phi(K_\alpha) \qquad (9.4)$$

The inequality will be satisfied only if

$$\left[\frac{C - E(f(c))}{\sqrt{V(f(c))}}\right] \ge (K_\alpha),$$

Or equivalently,

$$E(f(c)) + K_\alpha\sqrt{V(f(c))} \le C \qquad (9.5)$$

Substituting from (9.1) and (9.2) in (9.5), we get

$$\left(\sum_{i=1}^{L} \mu_i n_i + c_0\right) + K_\alpha\sqrt{\sum_{i=1}^{L} n_i^2\, \sigma_i^2} \le C \qquad (9.6)$$

The constants $\mu_i$ and $\sigma_i$ in (9.6) are unknown (by hypothesis). So we will use the estimators of mean $E(c'n + c_0)$ and variance $V(c'n + c_0)$ given by

$$\hat{E}(c'n + c_0) = \sum_{i=1}^{L} n_i E(c_i) + c_0 = \sum_{i=1}^{L} n_i \bar{c}_i + c_{,0}\, , say$$

$$\hat{V}(c'n + c_0) = \hat{V}(c'n) = \sum_{i=1}^{L} n_i^2 E(\sigma_i^2) + c_0 = \sum_{i=1}^{L} n_i^2 \sigma_{c_i}^2\, , say$$

Where $\bar{c}_i$ and $\sigma_{c_i}^2$ are the estimated means and variances from the sample.

Thus, an equivalent deterministic constraint to the stochastic constraint is given by

$$\left(\sum_{i=1}^{L} \bar{c}_i n_i + c_0\right) + K_\alpha \sqrt{\sum_{i=1}^{L} n_i^2 \sigma_{c_i}^2} \leq C$$

The equivalent deterministic non-linear programming problem to the stochastic programming (8.7) problem is given by

$$\min_n V = \sum_{i=1}^{L} \frac{W_i^2 S_{ij}^2}{n_i}, j = 1, \dots p$$

*Subject to* (9.7)

$$\left(\sum_{i=1}^{L} \bar{c}_i n_i + c_0\right) + K_\alpha \sqrt{\sum_{i=1}^{L} n_i^2 \sigma_{c_i}^2} \leq C$$

$$2 \leq n_i \leq N_i, n_i \in I, i = 1, \dots L.$$

## X COMPROMISE SOLUTION

A compromise solution to the $p$ problems (9.7) can be obtained by assigning the weight to various characters according to some measure of their importance, see khan, E.A. et al. (2003).

Let the respective weights be $b_j, j = 1, \dots, p$.

Then the equivalent deterministic non-linear programming problem is

$$\min_n V = \sum_{i=1}^{L} \frac{W_i^2}{n_i} \sum_{j=1}^{p} b_j S_{ij}^2\, , j = 1, \dots p \qquad (i)$$

Subject to (10.1)

$$\left(\sum_{i=1}^{L} \bar{c}_i n_i + c_0\right) + K_\alpha \sqrt{\sum_{i=1}^{L} n_i^2 \sigma_{c_i}^2} \leq C \qquad (ii)$$

$$2 \leq n_i \leq N_i, n_i \in I, i = 1, \dots L \qquad (iii)$$

The non-linear programming problem in (10.1) is convex as the objective function in {10.1 (i)} is convex, see Kotan & Khan (1967) and the left hand side in {10.1(ii)} is also convex. So it is possible to solve the convex programming problem (CPP) (10.1) by using any standard convex programming algorithm. The optimal sample numbers thus obtained may turn out to be fractional. However, it is known that the variance functions are flat at the optimum solution. So for large sample size it is enough to round the fractional values to the nearest integers. However, for small $n$ the branch and bound method should be applied for finding the optimal integer solution.

## XI Numerical Illustration

Consider an allocation problem in which the population is divided into three strata ($L = 3$) with two characters ($p = 2$) under study. The weight and variance in each stratum are as given in Table- 1. Let the relative weights of the two characters under estimation be $b_1 = 3$ and $b_2 = 5$.

**TABLE – 1**

Data for three strata and two characteristics

| Stratum $i$ | $N_i$ | $W_i$ | $S_{i1}$ | $S_{i2}$ |
|---|---|---|---|---|
| | | | $b_1 = 3$ | $b_2 = 5$ |
| 1 | 16 | 0.26 | 12.13 | 16.65 |
| 2 | 25 | 0.42 | 7.89 | 11.93 |
| 3 | 19 | 0.32 | 16.13 | 24.91 |

We assume that the costs of measurement, $c_i$, ($i = 1, \dots, 3$) in the various strata are independently normally distributed with the following means and variances

$$E(c_1) = 25, \quad E(c_2) = 30, \quad E(c_3) = 40 \text{ and overhead cost } c_0 = 75$$

$$V(c_1) = 6,\ V(c_2) = 5,\ V(c_3) = 7$$

The total amount available for the survey $C = 1450$ units.

The chances constraint is required to be satisfied with 99% probability.

On using this information in (8.7) the allocation problem turns into the following stochastic non-linear programming problem:

$$\underset{n}{\text{Min}}\, V = \sum_{i=1}^{3} \frac{W_i^2}{n_i} \sum_{j=1}^{2} b_j S_{ij}^2 , j = 1,2$$

Subject to (11.1)

$$P\left(\sum_{i=1}^{L} c_i n_i + c_0 \le 1450\right) \ge 0.99$$

$$2 \le n_i \le N_i, i = 1,2,3.$$

The value of standard normal variate $K_\alpha$ corresponding to 99% confidence limits is 2.33. substituting the given values in (10.1), the equivalent deterministic (non-linear programming) problem to (11.1) is obtained is:

$$\underset{n}{\text{Min.}V} = \frac{123.54}{n_1} + \frac{158.47}{n_2} + \frac{399.1877}{n_3}$$

Subject to

$$(25n_1 + 30n_2 + 40n_3 + 75) + 2.33\sqrt{3n_1^2 + 5n_2^2 + 4n_3^2} \leq 1450 \qquad (11.2)$$

$$2 \leq n_1 \leq 16$$

$$2 \leq n_2 \leq 25$$

$$2 \leq n_3 \leq 19$$

The non-linear deterministic problem in (11.2) is solved by using LINGO computer program, a package for constrained optimization by LINDO System Inc, see user guide (2001). The solution obtained is $n_1 = 6.321$, $n_2 = 5.968$ and $n_3 = 8.617$ with objective function value

$$f(n) = 92.417$$

The integer solution obtained by branch and bound method in 3 iteration is obtained as $n_1 = 7$, $n_2 = 6$ and $n_3 = 8$ with value of the objective function $f(n) = 93.958$.

## REFERENCES


1. Arthanari, T.S., and Dodge Y. (1981). Mathematical Programming in Statistics, John Wiley & Sons, New York.
2. Haimes, Y.Y. and D. Li (1988). Hierarchical multiobjective analysis for large scale systems: Review and current status. Automation 24(1), 53-69.
3. Kall, P (1976). Stochastic Linear Programming, Springer Verlag, Berlin.
4. Kall, P. and Wallace, S.W., (1994). Stochastic Programming, Wiley, New York.
5. Piene, L. Echyer and Felisa Vazguez-ab ad (2000). Global Stochastic optimization with low-dispersion point sets. Operations Research, 48, (6), pp.939-950.
6. Pewkopa A. (1978). The use of stochastic programming for the solution of some problems in statistics and probability, Technical Summary Report #1834. University of Wisconsin-Madison, Mathematical research center.
7. Schoen, F.(1994). On a new stochastic global optimization algorithm based on censored observation. Jour. Global Optim., 4, (1), pp. 17-35.
8. Sengupta, J.K. (1970). Stochastic Linear Programming with Chance-Constraints, Int. Econ. Rev. 11, pp. 287-304.
9. Tintner, G. (1955). Stochastic linear programming with applications to agricultural economics. In H.A. Anotsiewicz (Ed.), Proceedings of the Second Symposium in Linear Programming (pp. 197-228), National Bureau of Standards, Washington, DC.
10. Utyuzhnikov, S.(2010). "Multi-Objective Optimization: Quasi-Evengeneration of Pareto frontier and its local approximation" Handbook of Optimization Theory: Decision Analysis and Application, Eds: Varela, J., Acuna S. Nova Science Publisher.