# RELIABILITY ANALYSIS OF *N* COMPONENT SYSTEM USING MARKOVIAN MODEL

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**Abstract:** In this paper probability of n component system with m number of components failed with different failure rates has been derived by markovian approach and also obtained the reliability expressions for n component system which is to be worked with at least k number of components should work.

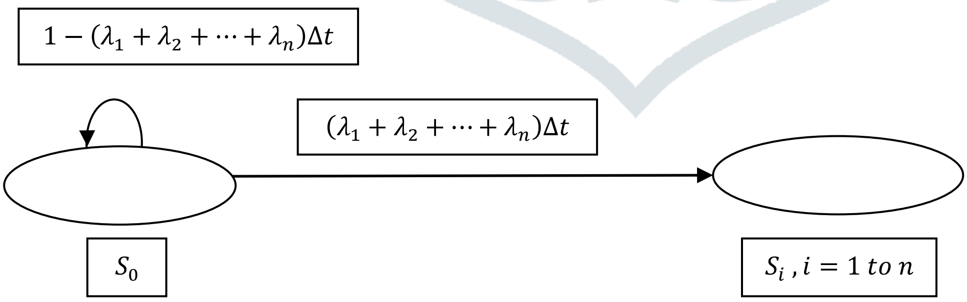
Key Words: Probability, Reliability, Markovian approach, Component

## I.INTRODUCTION

In a *k* out of *n* system *(n-k)* units are redundant and are added for the purpose of improving the system reliability. Such systems are known as partial redundant systems when *k >1*. The k units are called basic units whose survival is must for the survival of the system. For instance two of the four generators may be necessary to supply the required power. Another example is four engine air craft, only two engines are required for successful operation.
Reliability is defined as the probability that a system or component will function over intended time period[1].
Markov model is a technique that works well when failure hazards and repair hazards are constant. Markov process involves different states. The fundamental assumption in a markov process is that the transition probability from i to j and is independent of all previous states[2]. Markov models are widely used in Reliability and Availability. Reni Sagayaraj et al [3] discussed the markov model in system reliability with applications. Lan Wu et al [4] studied reliability evaluation of the solar power system based on the Markov chain method. Feng Ding and Shuai Han [5] described multi-state reliability analysis of rotor system using semi Markov model and UGF. S.Kalaiarasi et al [6] discussed the analysis of system reliability using markov technique. Vidhya G Nair and M.Manoharan [7] studied reliability analysis of a multi state system with common cause failures using Markov regenerative process.

## II.MATHEMATICAL MODEL

Consider a system consisting of *n* no. of components $(x_1, x_2, \dots, x_n)$ . Suppose that at time *t*, the system has *m* no. of components fail with failure rate of $m^{th}$ component is $\lambda_m$ $(m = 1,2,\dots,n)$. In markovian analysis the assumption is taken that, the chance of more than one component fail in small time is negligible. Hence there are $2^n$ no. of states occur. These being, (i) all components good state i.e., $S_0 = (x_1, x_2, \dots, x_n)$, (ii) one component failed states are
$S_i = i^{th}$ component failed state $=(x_1, x_2, \dots, \overline{x_i}, x_{i+1}, \dots, x_n)$ , $i = 1$ *to* $n$
No. of states occur for one component fail are *n*




Then the Probability expression for state $S_0$ at time $(t + \Delta t)$ is

$$P_{S_0}(t + \Delta t) = [1 - (\lambda_1 + \lambda_2 + \cdots + \lambda_n)\Delta t]P_{S_0}(t) \quad (2.1)$$

The difference equation is

$$P_{S_0}{}'(t) = -(\lambda_1 + \lambda_2 + \cdots + \lambda_n)P_{S_0}(t) \qquad (2,.2)$$

Solving the above equation, we get

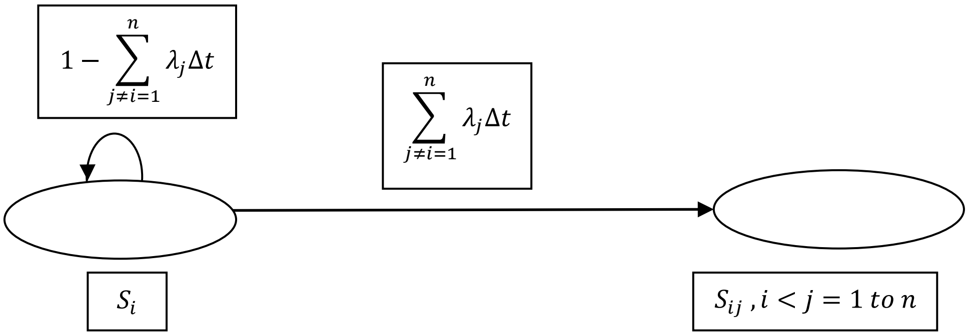
$$P_{S_0}(t) = exp\left(-\left(\sum_{i=1}^{n} \lambda_i\right)t\right) \qquad (2.3)$$

(iii) Two components failed states is

$S_{ij}$ = state of $i$ and $j^{th}$ component fail

$$= (x_1, x_2, \ldots, \overline{x_i}, \overline{x_j}, \ldots, x_n) \text{ where } i < j = 1 \text{ to } n$$

No. of states occur for two components fail is $\binom{n}{2}$




Probability of $i^{th}$ component at $(t + \Delta t)$ is

$$P_{S_i}(t + \Delta t) = P_{S_0}(t)\lambda_i \Delta t + P_{S_i}(t)\left[1 - \sum_{j \neq i = 1}^{n} \lambda_j \Delta t\right] \text{ where } i = 1,2, \ldots n \quad (2.4)$$

The difference equation is

$$P_{S_i}{}'(t) = \lambda_i P_0(t) - \left(\sum_{j \neq i = 1}^{n} \lambda_j\right) P_i(t) \quad (2.5)$$

Solving the above equation, we get

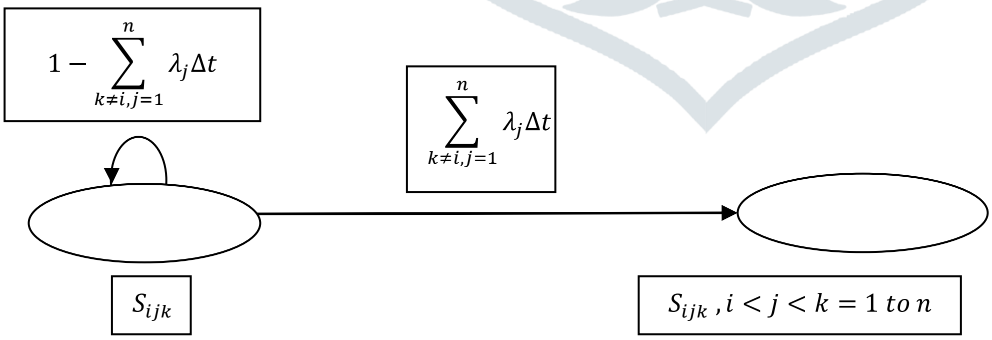
$$P_{S_i}(t) = exp\left(-\left(\sum_{j \neq i = 1}^{n} \lambda_j\right) t\right) \text{ where } i = 1 \text{ to } n \quad (2.6)$$

(iv). Three components failed states are

$S_{ijk} = (x_1, x_2, \ldots, \overline{x_i}, \overline{x_j}, \overline{x_k} \ldots, x_n) \text{ where } i < j < k = 1 \text{ to } n.$

No. of states occur for three components fail are $\binom{n}{3}$

In this manner no. of states in which $k$ components failed are $\binom{n}{k}$




Probability of 2-components i.e., $i^{th}$ and $j^{th}$ components fail at time $(t + \Delta t)$is

$P_{S_{ij}}(t + \Delta t)$ = The transition probability of $i^{th}$ and $j^{th}$ components are fail

$$= P_{S_i}(t)\lambda_i \Delta t + P_{S_j}(t)\lambda_j \Delta t + P_{S_{ij}}(t)\left[1 - \sum_{k \neq i, j = 1}^{n} \lambda_k \Delta t\right] \text{ wherei} < j = 1,2, \ldots n \quad (2.7)$$

The difference equation is

$$P_{S_{ij}}'(t) = \lambda_j P_i(t) + \lambda_i P_j(t) - P_{ij}(t)\left(\sum_{k\neq i<j=1}^{n} \lambda_k\right) \quad (2.8)$$

The solution of the above equation is

$$P_{S_{ij}}(t) = e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i-\lambda_j)t} - e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i)t} - e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_j)t} + e^{-(\sum_{l=1}^{n}\lambda_l)t}$$

$$where\ i < j = 1\ to\ n \quad (2.9)$$

The probability of three components *i*, *j*, *k* fail is

$$P_{S_{ijk}}(t+\Delta t) = P_{S_{ijk}}(t)\lambda_k\Delta t + P_{S_{jk}}(t)\lambda_i\Delta t + P_{S_{ik}}(t)\lambda_j\Delta t + P_{S_{ijk}}(t)\left[1 - \sum_{l\neq i,j,k=1}^{n} \lambda_l\Delta t\right]$$

$$where\ i < j < k = 1\ to\ n \quad (2.10)$$

The difference equation is

$$P_{S_{ijk}}'(t) = P_{S_{ij}}(t)\lambda_k + P_{S_{jk}}(t)\lambda_i + P_{S_{ik}}(t)\lambda_j + P_{S_{ijk}}(t)\left[1 - \sum_{l\neq i,j,k=1}^{n} \lambda_l\Delta t\right] \quad (2.11)$$

And the solution is

$$P_{S_{ijk}}(t) = e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i-\lambda_j-\lambda_k)t} - e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i-\lambda_j)t} - e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_j-\lambda_k)t} - e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i-\lambda_k)t} + e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i)t}$$
$$+ e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_j)t} + e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_k)t} - e^{-(\sum_{l=1}^{n}\lambda_l)t}$$

$$where\ i < j < k = 1\ to\ n \quad (2.12)$$

In similar way, the probability of *m* components fail is

$$P_{S_{12\ldots m}}(t+\Delta t) = P_{S_{23\ldots m}}(t)\lambda_1\Delta t + P_{S_{13\ldots m}}(t)\lambda_2\Delta t + \cdots + P_{S_{12\ldots m}}(t)[1-(\lambda_{m+1}+\lambda_{m+2}+\cdots+\lambda_n)\Delta t]$$
$$= \sum_{j=1}^{m} P_{S_{12\ldots j\ldots m}}(t)\lambda_j\Delta t + P_{S_{12\ldots m}}(t)\left[1 - \sum_{j=1}^{n-m} \lambda_{m+j}\Delta t\right] \quad (2.13)$$

The difference equation is

$$P_{S_{12\ldots m}}'(t) = \sum_{j\neq i=1}^{m} P_{S_{12\ldots i\ldots m}}(t)\lambda_j - P_{S_{12\ldots m}}(t)\left[\sum_{j=1}^{n-m} \lambda_{m+j}\right] \quad (2.14)$$

And the solution is

$$P_{S_{12\ldots m}}(t) = (-1)^m e^{-(\sum_{i=1}^{n}\lambda_i)t} + (-1)^{m-1}\sum_{i=1}^{m} e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i)t}$$
$$+(-1)^{m-2}\sum_{i<j=1}^{m} e^{-(\sum_{l=1}^{n}\lambda_l-\lambda_i-\lambda_j)t} + \cdots + e^{-(\sum_{l=1}^{n}\lambda_l-\sum_{i=1}^{m}\lambda_i)t}$$

$$where\ m = 1\ to\ n \quad (2.15)$$

The probability of all components fail i.e., *n* no. of components fail is $P_{S_{12\ldots n}}(t)$

And

$$P_{S_{12\ldots n}}(t) = 1 - \left[P_{S_0}(t) + \sum_{i=1}^{n} P_{S_i}(t) + \sum_{i<j=1}^{n} P_{S_{ij}}(t) + \cdots + P\big((n-1)components\ fail\big)\right] \quad (2.16)$$

Then

The total probability of one component fail = $\sum_{i=1}^{n} P_{S_i}(t)$

The total probability of two components fail = $\sum_{i<j=1}^{n} P_{S_{ij}}(t)$

The total probability of three components fail = $\sum_{i<j<k=1}^{n} P_{S_{ijk}}(t)$

and

The total probability of m components fail F(t)= $\sum_{i<j<k\ldots m=1}^{n} P_{S_{123\ldots m}}(t)$

If the system is to be worked when at least k components should be worked, then the system reliability $R = P(X \geq k) = 1 - F(t)$.

## III.NUMERICAL EXAMPLE

For *n* = 4,if the system works when at least 2 components good, then the system fails $\big((n-k) = 4-2\big)$ when more than two components fail.

Hence total probability of three components fail is

$$F(t) = \sum_{i<j<k=1}^{4} P_{S_{ijk}}(t)$$

$$= P_{S_{123}}(t) + P_{S_{124}}(t) + P_{S_{134}}(t) + P_{S_{234}}(t)$$

Let $\lambda_1 = 0.1, \lambda_2 = 0.2, \lambda_3 = 0.3, \lambda_4 = 0.4$

Table 1

| | $t$ =2 | $t$ =3 | $t$ = 4 | $t$ = 5 |
|---|---|---|---|---|
| $P_{S_{123}}(t)$ | 0.012115 | 0.020902 | 0.025614 | 0.02615 |
| $P_{S_{124}}(t)$ | 0.018061 | 0.033224 | 0.043641 | 0.047986 |
| $P_{S_{134}}(t)$ | 0.03019 | 0.058987 | 0.082618 | 0.097233 |
| $P_{S_{234}}(t)$ | 0.067063 | 0.13861 | 0.205869 | 0.257542 |

Failure probability and Reliability at time $t$

Table 2

| | $t$ = 2 | $t$ = 3 | $t$ = 4 | $t$ = 5 |
|---|---|---|---|---|
| $F(t)$ | 0.127429 | 0.251723 | 0.35774 | 0.428911 |
| $R(t)$ | 0.872571 | 0.748277 | 0.64226 | 0.571089 |

Fig.1


## IV.CONCLUSION

Probability of n component system with m number of components with different failure rates has been derived by markovian approach and also derived the reliability expressions for n component system which is to be worked with at least n-k number of components existed to work. The failure probability and reliability have been calculated for 4 component system in which at least 2 components should operate successfully. It is observed from the tables that the reliability decreases as t increases and failure probability increases as t increases.

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