ROBUST WATERMARKING TECHNIQUE BASED ON TRANSFORM BASED DECOMPOSITION AND PRINCIPAL COMPONENT ANALYSIS

[1]GurdeepKaur, [2]Anupam Garg
[1]Student, [2]Assistant Professor,
[1]Department of Computer Science and Engineering
[1]BhaiGurdas Institute of Engineering and Technology, Sangrur, India

Abstract :This work introduces a hybrid technique combining RIDGLET, RPCA and SVD. The watermark logo i.e. a gray-scale image is scrambled using Arnold transform to enhance the robustness and security. The cover image is decomposed into two components using RPCA. The significant low-rank component is further decomposed to transform coefficients to embed the processed watermark using SVD into the original image. The perceptual quality of the extracted watermarks is measured using two parameters, Peak-Signal-to-Noise-Ratio (PSNR) and Normalized Correlation (NC) and it is also verified by performing various signal processing operations and geometric attacks. Results of proposed method using ridgelet transform are compared with curvelet transform as both methods are used for the decomposition of the image and peppers image is used as host image to brief the results. It has been found that proposed methods gives better PSNR results in almost all cases when different attacks applied such that only Additive Gaussian attack gives less PSNR value when proposed method is used. When NC is compared, its value falls in between .95 and .99 and for Additive Gaussian and sharpening attacks it comes 0.8732 and .9383 respectively which is less than existed method. The reason behind this is the random addition of Gaussian noise and sharpening. So from the results, we can confirm that Ridgelet is better than Curvelet in proposed technique. Also, it has high robustness and good imperceptibility that could be analysed through calculating the NC and PSNR values among extracted watermark and cover images.

***IndexTerms* - Robust watermarking Fragile watermarking, Principal components, Singular value decomposition, Arnold transform etc.**

I. Introduction

The technological advancement leads to widespread availability of digital data on the world-wide-web. A major portion of this data is in the form of digital images. Now a day, Digital Image Processing (DIP) is among rapidly growing technologies. As these images are very easily available on internet, illegal copying and false ownership claims are two major issues of concern [1].In addition to that, tampering of digital image also becomes a big threat to individuals as well as to organizations. Image manipulation has become very common these days for defaming individuals/ organizations. In such manipulations, important parts of digital data are re-placed with the help signal processing tools. With the easy availability of powerful image processing tools, image manipulation and copyright infringement becomes a serious issue [1]. In order to protect the rightful owner-ship of images, researchers have developed different protection techniques. Digital image watermarking is one of the most widely used techniques, which can protect the ownership as well as well locate the tampered region of host image. Fragile watermarking is used to protect the image manipulation and tampering, whereas robust watermarking is used to protect the ownership and copyright of host image. Robust watermark is inserted in such a way that it remains intact even after attacks. On the other hand, fragile watermark is inserted in such a way that even a small change can destroy it [2]. Due to this fragile nature of watermark, tamper localization becomes possible in the host. Another important aspect of image watermarking is the self- recovery of tampered region without severely affecting the image visibility. Digital image watermarking can be divided into visible and in-visible domain [1]. In visible watermarking techniques, visible watermark is embedded into the host image in such a way that wa-termark remains perceptible to the user. On the other hand, invisible watermark is embedded in such a way that alterations made to the host image remains unnoticeable by human eye and the water- mark can only be extracted with a watermark extraction algorithm. Invisible watermarking can further be divided into time domain and transform domain methods [1] . In time domain method, the pixel values are changed in accordance with the watermark data. On the contrary, transform domain method uses some transform on the host before embedding the watermark data. Transform do- main shows higher robustness than the spatial domain methods [2] . Transform domain methods can again be sub-classified into DCT , DWT , curvelet transform , moments transform, IWT and SVD based image watermarking methods. On contrary to robust watermarking, fragile watermark vanishes from the host image as soon as there is a tampering in host. This feature of fragile image water-marking helps us to locate the tampered region as tampered portion loses the watermark [3]. Multi- purpose image watermarking is used to provide features of both robust and fragile watermark in one scheme itself [4]. The advantage of using such scheme is to figure out different type of false claim in a single check. In the proposed work, two watermarks are utilized (one after another) to provide multipurpose feature to watermarking scheme. The robust watermark is inserted first followed by the fragile watermark insertion. The insertion order is kept in such a way that the insertion of dual watermarks affects each other minimally.A watermarking scheme must be secured towards security out- breaks. Along with that, it must also provide high value of capacity, imperceptibility and robustness [1][2] .

II. Existed work

In the literature, different watermarking schemes using various types of image transform have been proposed. Garg et al. [5] proposed a blind DWT based watermarking scheme where the PN sequences are inserted to wavelet coefficients of the medical image according to watermark bits. Das et al. [6]proposed a blind DCT based watermarking scheme where the watermark is inserted based on the correlation between the DCT coefficients of two adjacent blocks. Wang and Lin [7]proposed the DWT based blind watermarking scheme where watermark bits are inserted into quan- tized wavelet coefficients of the cover image. Feng et al. [8] proposed DWT –DCT, and Arnold scrambling based watermarking scheme. In this scheme, the scrambled watermark logo is embedded in mid-frequency coefficients of DCT blocks of approximation wavelet coefficients using spread spec- trum watermarking. The scrambled watermark logo is generated using Arnold scrambling. Sahraee and Ghofrani[9] proposed the DWT based watermarking scheme, in which distance measurement between quantization of adjacent wavelet coefficients of the cover image.Singh and Singh [10] proposed DWT-SVD and DCT based blind watermarking scheme. In this scheme, DCT coefficients of the watermark logo are inserted into the middle singular value of wavelet coefficients of the hybrid block with a size of 4 ×4 of the cover image. This scheme is robust against common signal process- ing watermarking attacks.Hien et al. [11] proposed a blind image watermarking scheme using RDWT and independent component analysis (ICA). Here, watermark embedding is performed in RDWT domain while ICA is used for blind recovering of watermark logo. Mankar et al. [12] proposed RDWT and spread spectrum modulation based blind watermarking scheme. In this scheme, wavelet coefficients of cover image are modified by PN sequences. Oskooei et al. [13] proposed RDWT, ICA and principal component analysis (PCA) based blind watermarking scheme. In this scheme, watermark embedding is performed in RDWT domain while hybridization of ICA-PCA is used for blind recovering of watermark logo. Lagzian et al. [14] proposed RDWT –SVD based non-blind watermarking scheme. Here, S matrix values of selected wavelet coefficients of cover image are modified by the watermark logo and gain factor.Bajaj et al.[15] proposed RDWT-DCT-SVD based hybrid and non- blind watermarking scheme. In this scheme, S matrix values of all DCT coefficients of selected wavelet coefficients of cover image are modified by the watermark logo and gain factor. Singh et al. [16] proposed RDWT-SVD and Arnold scrambling based multi-biometric watermarking scheme. Here, scramble biometric information is taken as watermark and is inserted into S matrix values of selected wavelet coefficients of the cover image. The scrambled biometric information is generated using Arnold scrambling in this scheme. Thanki et al. [17] proposed RDWT-SVD and compressive sensing based multiple watermarking scheme. Here, S matrix values of three encrypted watermark images are inserted into S matrix values of selected wavelet coefficients of each channel of the color cover image to generate a color watermarked image with multiple watermark logos.Roy and Pal [18] proposed RDWT-DCT and Arnold scrambling based blind watermarking scheme. Here, the scrambled watermark logo is inserted into mid frequency DCT coefficients of horizontal wavelet coefficients of the cover image to generate a watermarked image. Singh et al. [19] proposed NSCT-RDWT-SVD based non-blind watermarking scheme. Here, hybrid coefficients of cover image are modified by hybrid coefficients of the watermark logo and gain factor. Ernawan and Kabir[20] proposed RDWT-SVD based blind watermarking scheme for a grayscale image. In this scheme, U matrix value of LL subband of cover image is modified by an encrypted watermark image. The encrypted watermark image is generated using Arnold scrambling. Zhang et al. [21] proposed the first DCuT based blind watermarking scheme for grayscale images. In this scheme, medium frequency curvelet coefficients of cover image are modified by a binary image to get watermarked image. This scheme is robust against various types of watermarking attacks. Hien et al. [22] proposed DCuT based non-blind watermarking scheme for grayscale images. Here, watermark logo is inserted into selected curvelet co- efficients of the cover image to get watermarked image. Tao et al. [23] proposed DCuT based blind watermarking scheme where PN sequences are inserted into selected curvelet coefficients of the cover image according to watermark bit. In this scheme, the selection of curvelet coefficients and the gain factor was done using just noticeable distortion (JND) model. Gu et al. [24] proposed a curvelet based adaptive watermarking scheme for color images. In this scheme, the water- mark is embedded into the middle-frequency curvelet coefficients to generate a watermarked color image. This scheme is robust against various types of watermarking attacks. The main limitation of these existing schemes using DCuT is that it is unable to fulfill trade off between robustness and imperceptibility. So that, take advantage of hybridization of transforms, a new combination of RPCA and Ridglet for watermarking is designed and proposed in this paper. The reason behind the choice of this combination is due to the fact that this combination to enhance imperceptibility of the proposed scheme for providing better robustness against various watermarking attacks. Here, Arnold scrambling provides security to the watermark logo before inserted into the cover image.

III. Proposed work

The details on Ridglet transform, Robust PCA, SVD and Arnold scrambling are given in this section.

3.1 Ridgelet transform

In many image processing tasks, a sparse representation of an image is used in order to compact the image into a small number of samples. Wavelets are a good example of sparse geometrical image representation. But regardless of the success of the wavelets, they exhibit strong limitations in terms of efficiency when applied in more than one dimension. Wavelets show good performance for piecewise smooth functions in one dimension, but fail to efficiently represent objects with highly anisotropic elements such as lines or curvilinear structures (e.g. edges). The reason is that wavelets are non-geometrical and do not exploit the regularity of the edge curve. Wavelets are therefore good at representing zero-dimensional or point singularities. However, two-dimensional piecewise smooth signals (such as face images) have one-dimensional singularities meaning that wavelets will not accurately represent the smoothness of the image along the curve [25]. Candies and Donoho recently developed ridgelets: a new method of representation to deal with line singularities of the image in 2D [25]. The idea is to map a line singularity into a point singularity using the Radon transform. A wavelet transform can then be used to effectively handle the point singularity in the Radon domain. Their initial proposal was intended for functions defined in the continuous R2 space [26]. The Continuous Ridgelet Transform (CRT) in R2 is defined as

$$CRT_f(a,b,\theta)=\int_{\mathbb{R}^2}\psi_{a,b,\theta}(x)\,f(x)\,dx \quad (3.1)$$

where the ridgelets Ψa,b,θ (x) in 2D are defined from a wavelet-type function in 1D, ψ (x), as

$$\psi_{a,b,\theta}(x)=\frac{1}{\sqrt{a}}\psi\left(\frac{x_1\cos\theta+x_2\sin\theta-b}{a}\right) \quad (3.2)$$

where x = (x1, x2)T . In Equations (3.1) and (3.2), the parameters a, b and θ relate to scaling, shift and rotation transforms respectively.

In 2D, ridgelets and wavelets are linked by the Radon transform. The Radon transform is denoted as

$$R_f(\theta,t)=\int_{\mathbb{R}^2}\delta(x_1\cos\theta+x_2\sin\theta-t)\,f(x)dx \quad (3.3)$$

where δ is the Dirac distribution. The ridgelet transform is then equivalent to the application of a 1D wavelet transform to the slices (projections) of the Radon transform:

$$CRT_f(a,b,\theta)=\int_{\mathbb{R}^2}\psi_{a,b}(t)\,R_f(\theta,t)\,dt \quad (3.4)$$

So the ridgelet function can be realised by applying a 1D wavelet transform in Radon space. The Finite Radon Transform (FRAT) is defined as summations of image pixels over a certain set of "lines" in the image:

$$r_k[l]=FRAT_f(k,l)=\frac{1}{\sqrt{P}}\sum_{i,j\in L_{k,l}} f(i,j) \quad (3.5)$$

3.2 Principal Component Analysis (PCA)

Principal Component Analysis was put forward by Karl Pearson (1901) and Hotelling (1933) as a technique for simplifying a data set. It is a method used for dimension reduction of multivariable data set. PCA transforms the raw data into principal components which are essentially the functions of the original variable [27]. The PCA transformation in this work, was carried out using the inbuilt Matlab functions [28]. The basis for this analysis is the calculation of that axis which has the greatest covariance with the data points. This axis is known as the principal component[29]. To do this, the routine shifts the data points by calculating mean and then it finds the covariance matrix which is ultimately used to find the eigen values and thus the eigenvectors.

3.3 Singular Value Decomposition (SVD)

Singular Value Decomposition is a subspace analysis mathematical tool used for diagonalizing the input matrix. It is commonly being used in image processing domain for decomposing the image to obtain the relevant elements. Let the host image be represented by I. The SVD of I is given by Equation 3.6.

$I=U * S * V^T$ (3.6)

(1) Where U and V are the unitary matrices and S is a diagonal matrix. The diagonalized matrix S represents the singular values in descending order.

$$\begin{bmatrix}x'\\y'\end{bmatrix}=\begin{bmatrix}1 & a\\a & ab+1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}\bmod N \quad (3.7)$$

Where x and y represent a spatial point of the image. N represents the dimensions of the image (square),x' and y' represent the spatial point of the scrambled image. The positive integers a and b are taken as 1. When the above equation is used for transformation, the position of spatial points gets changed. Due to the periodicity of Arnold transform, the spatial points get back to the original position after T_N iterations. T_N represents the periodicity of the image under consideration. For the watermark chosen for this work, T_N was found to be 24. Various iterations are depicted in Fig. 1.

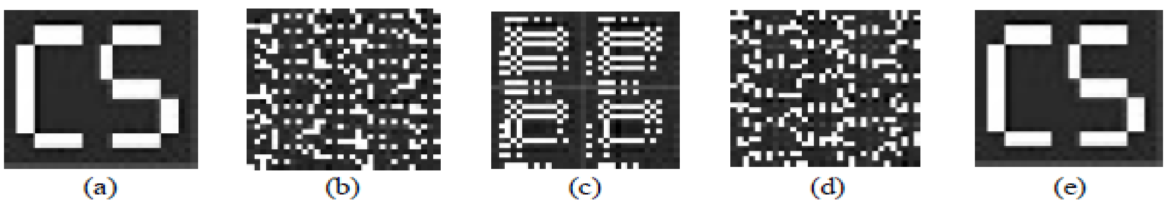


Figure 1: Scrambled Image using Arnold Transform: (a) Original Watermark; (b) 6th Iteration; (c) 12th Iteration; (d) 18th Iteration; (e) 24th Iteration

After obtaining T_N, choose any integer N such that it is less than T_N and use it to scramble the watermark. During extraction process the original watermark can be obtained by subjecting the extracted watermark to(T_N - N) iterations.

3.4 Algorithm description of proposed watermarking scheme

The algorithm for embedding and extraction of the watermark and the quality assessment parameters are explained in this section.

3.4.1 Embedding Process

The embedding process can be further divided into 3 important steps..

- Processing of the Host Image:

Host image (size: 512 x 512) is subjected HSV tcolor space conversion and Robust PCA is applied.ThenLow rank matrix Lisused to carry out 2 level Ridgelet decomposition. The Low frequency band (size: 128 x 128) is extracted and The low frequency band corresponds to the large scale values. These are preferred over the small scale values as small scale values are affected by noise components. It is observed that when the large-scale values are used for watermarking, the likelihood of watermarking coefficients interfering with the visual quality of the image is low. Also, most of the image processing attacks aimed at destroying the watermark is directed towards the high frequency coefficients of the image, as a result low frequency coefficients are preferred. The coefficients of ridglet are used further using SVD to obtain Singular Values (denoted as S).

- Processing of the Watermark: Watermark (size: 128 x 128) is subjected to an algorithm that determines the periodicity of Arnold Transform. A key based on the periodicity is determined and is then used to scramble the image using Arnold Transform. Scrambled image is subjected to SVD to obtain Singular Values (denoted as S').

- Embedding: Cox's natural formula for embedding the watermarking coefficients is used. The formula is given by Equation 3.8.

$$S' = S + \delta S_W \quad (3.8)$$

Where,

S' : Modified singular values

S : Singular values obtained from the host image after processing

δ : Single scaling factor

S_W : Singular values obtained from the watermark after processing

Modified score matrix of host image is reconstructed using the unitary matrices obtained in the processing of the host image. Using the modified score matrix and the eigen values so obtained using decomposition,the low frequency sub – band is obtained. The modified low frequency sub – band replaces the original low frequency band followed by an inverse 2 level Ridgelet to obtain the signed image.

3.4.2 Extraction Process

The extraction process carried out in this work is known as the informed extraction, wherein both the signed and the watermarked images are used for extracting the scrambled watermark. The process is depicted in Fig. 4.Both the host and signed image are subjected to the same techniques that were used for watermarking. After obtaining the singular values from both the signed and host images, the modified singular values of the watermark were obtained using Equation 3.9.

$$S_W' = \frac{(S_{Si} - S)}{\delta} \quad (3.9)$$

Where,

S_W' : Modified singular values

S: Singular values obtained from the host image after processing

δ : Single scaling factor

S_{Si} : Singular values obtained from the signed image after processing

After obtaining the singular values reconstruct the score matrix. Subtract the chosen key from the periodicity of the watermark and scramble the reconstructed watermark matrix using Arnold transform to obtain the Watermark.Perform matrices multiplication for U component of watermarked image, new S component received in previous step and V component of watermarked image which results in extracted watermark and Evaluate MSE and NC and PSNR values of recovered watermark.

IV. Results and Analysis

Results have been carried out on number of images in which different types of geometric and image processing attacks has been applied. The analysis is done to understand the effect of different types of attacks on proposed watermarking technique. The results here showed are on Peppers image which is widely used in image processing applications. The results are also shown for curvelet based watermarking system.

4.1 Performance evaluation of proposed watermarking system

The robustness of the watermarking algorithm is evaluated using the StirMark benchmarks [30][31]. The signed images are subjected to various image processing tasks aimed at destroying the watermark. The watermark is then extracted and compared with the original version. The quality is assessed by following parameters:

- Peak Signal to Noise Ratio (PSNR): The process of watermarking will change the coefficients of the image so that it will deviate from the original version. Similarly, when an image is attacked, the coefficients will further deviate from the original value and the mean square error (Equation 4.1) will increase resulting in a decrease in PSNR value (Equation 4.1).

$$MSE = \frac{1}{rc}\sum_{x=0}^{r-1}\sum_{y=0}^{c-1}\left[I(x,y) - I^{'}(x,y)\right]^2 \quad (4.1)$$

$$PSNR = 10\log_{10}\left(\frac{I_{max}^{2}}{MSE}\right) \quad (4.2)$$

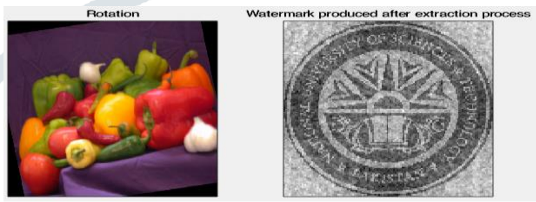
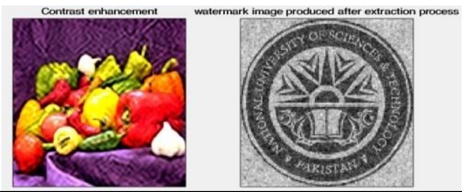
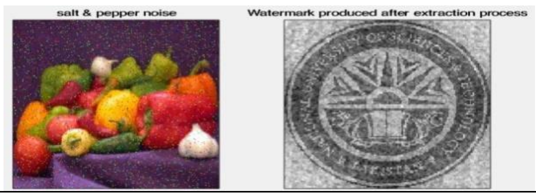
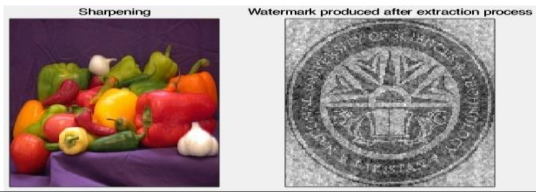
Where 'r' and 'c' represent the dimensions of the image under consideration. The host image is represented byI(x,y) and signed image by $I^{'}$. I_{max} corresponds to the maximum intensity value in the image. For grayscale images the value is 255.

- Normalized Cross – Correlation (NC): The extracted watermark can be compared with the one embedded using correlation. The NC value is calculated as given in Equation 4.3.

$$NC(W,W^{'}) = \frac{\sum_{x=1}^{rw}\sum_{y=1}^{cw}\left[W(x,y) * W^{'}(x,y)\right]}{\sum_{x=1}^{rw}\sum_{y=1}^{cw}[W(x,y)]^2} \quad (4.3)$$

Where'rw' and 'cw' represent the dimensions of the watermark. The original watermark is represented by W(x,y) and the extracted watermark is represented by $W^{'}$(x,y).

Table 1: Images and Watermark obtained after implementing different attacks

Attacks and extracted watermark using Proposed Method	
	
	
	

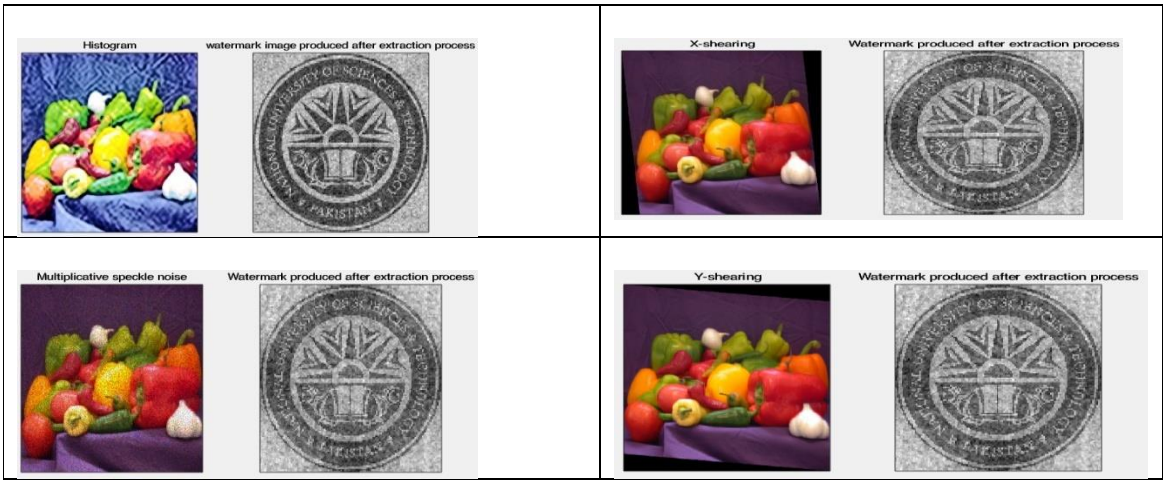


Table 2: PSNR and NC for Peppers image

Peppers Image	Existed Method		Proposed Method	
Attack name	**PSNR**	**NC**	**PSNR**	**NC**
Affine	7.3254	0.9299	14.8851	0.9676
Contrast Enhancement	10.8290	0.9545	13.0647	0.9503
Additive Gaussian	10.0762	0.9533	9.0564	0.8732
Histogram	10.5391	0.9490	18.9014	0.9875
Multiplicative Speckle Noise	9.0229	0.9614	13.2010	0.9519
Rotation	9.5108	0.9457	14.9179	0.9680
Salt & Pepper noise	11.0724	0.9615	13.4636	0.9548
Sharpening	11.6758	0.9650	12.1442	0.9383
X-Shearing	9.1600	0.9532	14.4430	0.9641
Y-Shearing	9.1598	0.9556	14.4137	0.9638

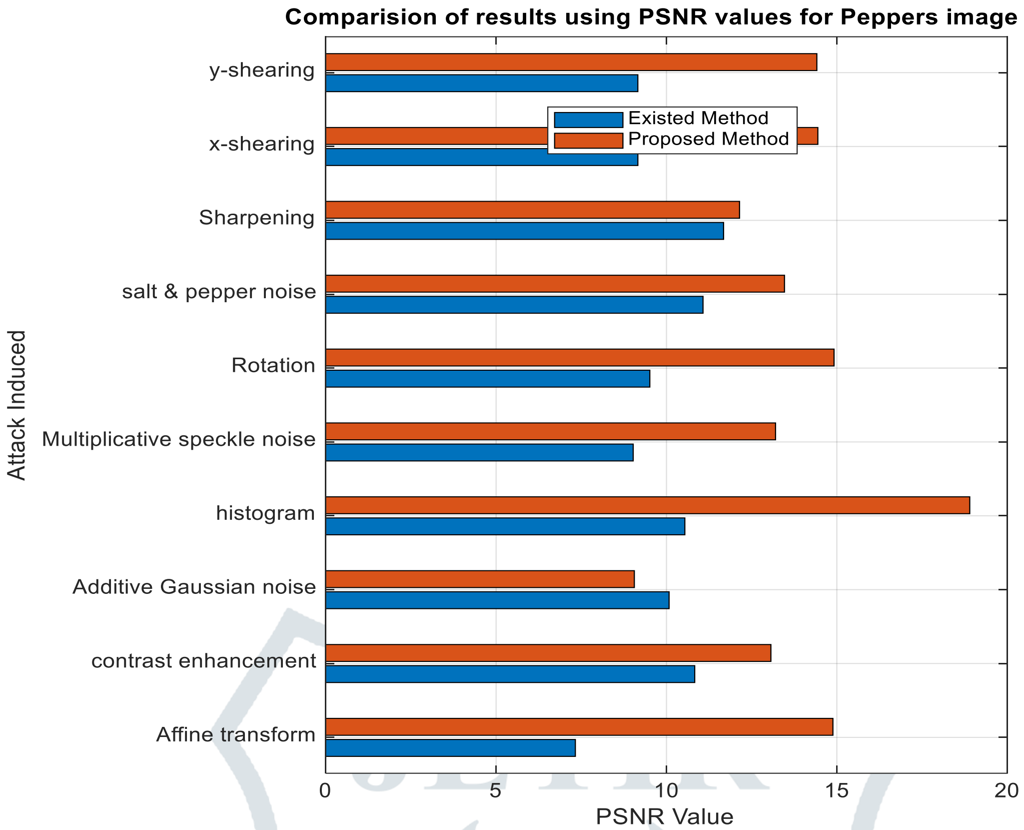


Figure 2: PSNR values for extracted watermark with different attacks

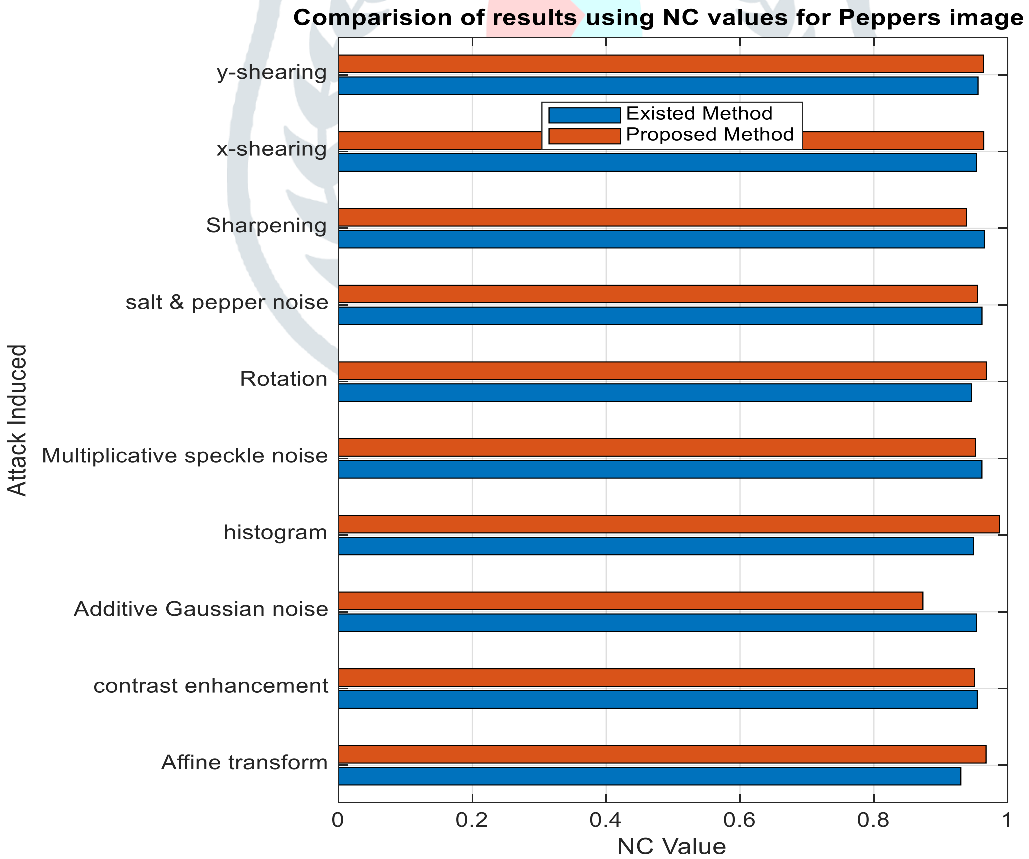


Figure 3: NC values for extracted watermark with different attacks

Numbers of experiments are performed to investigate the consequences of embedding and extraction using 512*512 dimension of cover image and 128*128 dimension of watermark image as shown in Table 1. Table 1-2 and figure 2-3 given above shows the results based on PSNR and NC values. It has been found that proposed methods gives better PSNR results in almost all cases when different attacks applied such that only Additive Gaussian attack gives less PSNR value when proposed method is used. When NC is compared, its value falls in between .95 and .99 and for Additive Gaussian and sharpening attacks it comes 0.8732 and .9383 respectively which is less than existed method. The reason behind this is the random addition of Gaussian noise and sharpening. So from the results, we can confirm that Ridgelet is better than Curvelet in proposed technique. Also, it has high robustness and good imperceptibility that could be analysed through calculating the NC and PSNR values among extracted watermark and cover images.

V. Conclusion

It is observed from the results presented in this paper, that when the watermarked images are subjected to image processing attacks, the embedded watermark gets affected to a little extent. This paper presents watermarking scheme using the Ridglet – PCA – SVD hybrid structure using a scrambled image as watermark. The scrambled watermark is obtained by employing the Arnold transform. This results in improved normalized cross correlation (NC) and peak signal to noise ration (PSNR) values there by maintaining high perceptibility. It is evident from the results presented in Table 1 and Table 1 that when a scrambled watermark is embedded, the image processing attacks have little effect on the watermark. The normalized cross correlation values retrieved using proposed approaches are high as compared to the other techniques. The PSNR value obtained are higher when compared with the existed technique. Moreover, the extracted watermarks using proposed watermarking technique after image processing attacks have high degree of similarity with the ones embedded during preliminary watermarking procedure.

References:

[1] A. Haouzia , R. Noumeir , Methods for image authentication: a survey, Multimedia Tools Appl. 39 (1) (2008) 1–46 .

[2] V.M. Potdar , S. Han , E. Chang , A survey of digital image watermarking techniques, in: INDIN'05. 2005 3rd IEEE International Conference on Industrial In- formatics, August, 2005, IEEE, 2005, pp. 709–716.

[3] I.A. Ansari , M. Pant , C.W. Ahn ,SVD based fragile watermarking scheme for tamper localization and self-recovery, Int. J. Mach. Learn. Cybern. 7 (6) (2016) 1225–1239 .

[4] I.A. Ansari , M. Pant , C.W. Ahn , J. Jeong , PSO optimized multipurpose image watermarking using SVD and chaotic sequence, in: Bio-Inspired Computing-Theories and Applications, Springer, Berlin Heidelberg, 2015, September, pp. 1–17 .

[5]GargS , Singh R . An efficient method for digital image watermarking based on PN sequences. Int J ComputSciEng 2012;4(9):1550–61 .

[6] Das C ,Panigrahi S , Sharma VK , Mahapatra KK . A novel blind robust image wa- termarking in DCT domain using inter-block coefficient correlation. AEU-Int J Electron Commun 2014;68(3):244–53 .

[7] Wang SH , Lin YP . Wavelet tree quantization for copyright protection water- marking. IEEE Trans Image Process 2004;13(2):154–65 .

[8]FengLP ,Zheng LB , Cao P . A DWT-DCT based blind watermarking algorithm for copyright protection. In: Computer science and information technology (ICC- SIT), 2010 3rd IEEE international conference on, 7. IEEE; July 2010. p. 455–8 .

[9]SahraeeMJ ,Ghofrani S . A robust blind watermarking method using quanti- zation of distance between wavelet coefficients. Signal Image Video Process 2013;7(4):799–807 .

[10] Singh D , Singh SK . DWT-SVD and DCT based robust and blind water- marking scheme for copyright protection. Multimed Tools Appl 2017;76(11): 13001–13024 .

[11]HienTD ,Nakao Z , Chen YW . RDWT domain watermarking based on indepen- dent component analysis extraction. In: Applied soft computing technologies: the challenge of complexity. Berlin, Heidelberg: Springer; 2006. p. 401–14 .

[12]MankarVH , Das TS , Sarkar S , Sarkar SK . Redundant Wavelet Watermarking us- ing Spread Spectrum Modulation. ELCVIA Electron Lett Computer Vision Image Anal 2008;7(2):1–10 .

[13]Oskooei, S.G., Dadgostar, M., Rad, G.R., &Fatemizadeh, E. (2009).Adaptive wa- termarking scheme based on ICA and RDWT.

[14]LagzianS ,Soryani M , Fathy M . A new robust watermarking scheme based on RDWT-SVD. Int J IntellInf Process 2011;2(1):22–9 .

[15] Bajaj A . Robust and reversible digital image watermarking technique based on RDWT-DCT-SVD. In: Advances in engineering and technology research (ICAETR), 2014 international conference on. IEEE; 2014. p. 1–5 .

[16] Singh P , Raman B , Roy PP . A multimodal biometric watermarking system for digital images in redundant discrete wavelet transform. Multimed Tools Appl 2017;76(3):3871–97 .

[17]ThankiR ,Dwivedi V , Borisagar K , Borra S . A Watermarking algorithm for mul- tiple watermarks protection using RDWT-SVD and compressive

[18] Roy S , Pal AK . A robust blind hybrid image watermarking scheme in RDWT-DCT domain using Arnold scrambling.Multimed Tools Appl 2017;76(3):3577–616 .

[19] Singh S ,Rathore VS , Singh R , Singh MK . Hybrid semi-blind image watermark- ing in redundant wavelet domain. Multimed Tools Appl 2017;76(18):19113–37 .

[20]ErnawanF ,Kabir MN . A block-based RDWT-SVD image watermarking method using human visual system characteristics. Visual Computer 2018:1–19 .

[21] Zhang ZY , Huang W , Zhang JL , Yu HY , Lu YJ . Digital image watermark algorithm in the curvelet domain. In: Intelligent information hiding and multimedia signal processing, 2006. IIH-MSP'06. international conference on. IEEE; December 2006. p. 105–8 .

[22]Hien TD , Kei I , Harak H , Chen YW , Nagata Y , Nakao Z . Curvelet-domain im- age watermarking based on edge-embedding. In: International conference on knowledge-based and intelligent information and engineering systems. Berlin, Heidelberg: Springer; September 2007. p. 311–17 .

[23] Tao P , Dexter S , Eskicioglu AM . Robust digital image watermarking in curvelet domain. In: Security, forensics, steganography, and watermarking of multime- dia contents X, 6819. International Society for Optics and Photonics; March 2008. p. 68191 .

[24] Song H ,Gu J . Curvelet based adaptive watermarking for images. In: Computer science and network technology (ICCSNT), 2012 2nd international conference on. IEEE; December 2012. p. 1101–5 .

[25] E. J. Candies, Ridgelets: Theory and applications, Ph.D. thesis, Department of Statistics, Stanford University (1998).

[26] J. L. Starck, E. J. Candies, D. Donoho, The curvelet transform for image denoising, IEEE Trans. Im. Proc. 11 (2002) 670–684.

[27] H. Hotelling. (1933) "Analysis of a Complex of Statistical Variables into Principal Components."Journal of Educational Psychology24: 417 – 441 and 498 – 520.

[28]MathworksMatlab. (2012) "PCA: Principal Component Analysis of Raw Data." MathworksMatlab Documentation. Available at: https://in.mathworks.com/help/stats/pca.html

[29]Ramsha Ahmed, M. MohsinRiaz, Abdul Ghafoor,"Attack resistant watermarking technique based on fast curvelet transform and Robust Principal Component Analysis" Multimedia Tools and Applications April 2018, Volume 77, Issue 8, pp 9443–945

[30] F. A. P. Petitcolas, R. J. Anderson, and M. G. Kuhn.(1998) "Attacks on Copyright Marking Systems."International Workshop on Information Hiding, Springer pp. 219 – 239.

[31] F. A. P. Petitcolas. (2000)"Watermarking Schemes Evaluation."IEEE Signal Processing Magazine17(5): 58 – 64.