# ELECTROMAGNETIC FIELD TENSOR IN MAGNETOHYDRODYNAMICAL APPROXIMATION

[1] **K.K.Tiwari,**[2] **Arti Tiwari**

[1]Ass. Professor,[2] Ass. Professor,

[1]*Deptt. of Basic Sciences*,[2]*Deptt. of Basic Sciences*

[1]S.R.M.S. College of Engg. & Tech., Bareilly, UP, India

[2]S.R.M.S. College of Engg.,Tech & Research., Bareilly, UP, India

***Abstract:*** The main purpose of this paper is to delineate the results for the electromagnetic field tensor in magnetohydrodynamical approximations. The fundamental governing equations for electromagnetic field tensor have been obtained under MHD approximations. The vorticity is also discussed under the same approximations.

***Index Terms*** **- electromagnetic field tensor, MHD, vorticity..**

## 1. INTRODUCTION:

In this paper we develop results for the electromagnetic field tensor $H_{ab}$ in the MHD approximation. Lichnerowicz [1] has shown that

$$\mathcal{L}_u H_{\alpha\beta} = 0 \tag{1.1}$$

wherein $L_u$
denotes the Lie derivative with respect to the velocity 4-vector $u^\alpha$.
And

$$\mathcal{L}_B H_{\alpha\beta} = 0 \tag{1.2}$$

wherein $L_B$ denotes the magnetic induction 4-vector $B^\alpha$.
It is to be noted that the dot indicates the covariant derivative along a particle world line is given by

$$\dot{A^\alpha} = A^\alpha{}_{,\beta}\, U^\beta$$

The following identities for the permutation tensor $\eta_{abcd}$ are defined as

$$\eta_{\alpha\beta\gamma\delta}\eta^{\alpha rst} = -3!\left\{\delta^r_\beta\delta^s_\gamma\delta^t_\delta - \delta^s_\gamma\delta^t_\delta\delta^r_\beta\right\} \tag{1.3}$$

$$\eta_{\alpha\beta\gamma\delta}\eta^{\alpha\beta st} = -2!\,2!\left\{\delta^s_\gamma\delta^t_\delta - \delta^t_\delta\delta^s_\gamma\right\} \tag{1.4}$$

In this paper we shall determine that equation (1.1) contracted with $H^{\alpha\beta}$ is equivalent to the divergence equation for $B^\alpha$. We shall also develop the physical significance of equation (1.2) by showing that it is equivalent to the propagation equation for $B^\alpha$.
The analogy between the equations governing $\Omega^\alpha = r\omega^\alpha$ and $B^\alpha$ in the magnetohydrodynamic approximation has been given in [4]. The analogy between magnetic field and the vorticity has given an application in the non-relativistic theory of magnetohydrodynamical turbulence and in the theory of gravitational collapse [1].

## 2. FUNDAMENTAL EQUATIONS:

For sufficiently large time, a porous disk of infinite radius has been rotating about a fixed axis through its centre with constant velocity $\omega$. The constitutive equation of a fluid is given by [9]

$$P^{ik} = -P\delta^{ik} + 2\mu g^{ik} - 2K_0\overline{g}^{ik} \tag{2.1}$$

$$\overline{g}^{ij} = g^{ij}_k v^k - g^{ik}v^j_k - g^{kj}v^i_k + g^{ij}v^k_k \tag{2.2}$$

wherein $P^{ik}$ is the stress tensor and $g^{ij}$ is the rate of strain tensor and is given by

$$2g^{ik} = v_{ik} + v_{ki} \tag{2.3}$$

In the magnetohydrodynamical approximation as $E^\alpha = 0$, then electromagnetic field tensor $H_{\alpha\beta}$ yields

$$H_{\alpha\beta} = \eta_{\alpha\beta\gamma\delta}B^\gamma U^\delta \tag{2.4}$$

Consequently

$$H_{\alpha\beta} = -H_{\beta\alpha} \tag{2.5}$$

$$H_{\alpha\beta}U^\beta = 0 \tag{2.6}$$

i.e.

$$H_{\beta\alpha}U^\alpha = 0$$

$$H_{\alpha\beta}B^\beta = 0 \tag{2.7}$$

i.e.

$$H_{\beta\alpha}B^\alpha = 0$$

By virtue of equation (1.4)

$$B_\alpha = \frac{1}{2}\eta^{\alpha\beta\gamma\delta}U_\beta H_{\gamma\delta} \tag{2.8}$$

## 3.MAXWELL'S EQUATIONS AND MHD APPROXIMATIONS:

Maxwell's equations in the magnetohydrodynamical approximation are given by [9].

$$\left(B^\alpha U^\beta - B^\beta U^\alpha\right)_{,\beta} = 0 \tag{3.1}$$

These may be split up with regards to the projection tensor.

$$h^\alpha_\beta = g^\alpha_\beta + U^\alpha U_\beta \tag{3.2}$$

And $U^{\alpha}$ into the propagation equation for $B^{\alpha}$.
The electromagnetic field tensor $H_{\alpha\beta}$ in the magnetohydrodynamical approximation in general theory of relativity is given by [7]

$$h^{\alpha}_{\beta}\dot{B}^{\beta} + \theta B^{\alpha} - U^{\alpha},_{\beta} B^{\beta} = 0 \qquad (3.3)$$

wherein $\theta = U^{\alpha}_{,\alpha}$
The divergence equation for $B^{\alpha}$ is given by

$$B^{\alpha}_{\beta} h^{\alpha}_{\beta} = 0 \qquad (3.4)$$

**REMARK 3.1:** It is noteworthy that the equation (1.2) is equivalent to the equation (3.3) and that the relation

$$H^{\alpha\beta}\left(\underset{B}{\mathcal{L}}H_{\alpha\beta}\right) = 0 \qquad (3.5)$$

is equivalent to equation (3.3).
First we define

$$A_{\alpha\beta} = \underset{U}{\mathcal{L}}H_{\alpha\beta}, \qquad (3.6)$$

On expressing $H_{\alpha\beta}$ in terms of $B^{\alpha}$ by means of equation (2.4) and multiplying equation (3.6) by $\frac{1}{2}\eta^{fg\alpha\beta}U_g$ on both sides and using the relation (1.2), we get

$$\frac{1}{2}\eta^{fg\alpha\beta}U_g A_{\alpha\beta} = h^{f}_{g}\dot{B}^{g} + \theta B^{f} - U^{f},_{g} B^{g} \qquad (3.7)$$

From the properties of $H_{\alpha\beta}$, we get $A_{\alpha\beta} = -A_{\beta\alpha}$ and $A_{\alpha\beta}U^{\beta} = 0$ , operating $\eta_{frst}U^{t}$ we get $\eta^{fg\alpha\beta}U_g A_{\alpha\beta} = 0$ if and only if

$$A_{\alpha\beta} = 0 \qquad (3.8)$$

Secondly we define

$$H^{\alpha\beta}\left(\underset{B}{\mathcal{L}}H_{\alpha\beta}\right) = \frac{1}{2}\left(H^{\alpha\beta}H_{\alpha\beta}\right)_{,\gamma} B^{\gamma} + 2H^{\beta\alpha}H_{\beta\alpha}B^{\gamma}_{\alpha} \qquad (3.9)$$

Express $H_{\alpha\beta}$ in terms of $B^{\alpha}$ by using equation (2.4) and the relation (1.2), we get

$$H^{\alpha\beta}\left(\underset{B}{\mathcal{L}}H_{\alpha\beta}\right) = 2B^{2}B^{\alpha},_{\beta} h^{\beta}_{\alpha,} \qquad (3.10)$$

wherein

$$B^{2} = B_{\alpha}B^{\alpha}$$

**Remark 3.2:** It is to be noted that if $B \neq 0$, then equation (3.5) is satisfied if and only if equation (3.4) is also satisfied.

## 4. VORTICITY IN MHD APPROXIMATIONS:

Let us consider the relation

$$U_{\alpha\beta} = \sigma_{\alpha\beta} + \frac{1}{3}\theta h_{\alpha\beta} + \omega_{\alpha\beta} - \dot{U}_{\alpha}U_{\beta}, \qquad (4.1)$$

where in $\sigma_{\alpha\beta}$ is the shear tensor.
Inserting equation (4.1) into the equation (3.2), we get the propagation equation for electromagnetic field tensor $H_{\alpha\beta}$ .

$$h^{\gamma}_{\alpha}h^{\delta}_{\beta}(H_{\gamma\delta}) + \frac{2}{3}\theta H_{\alpha\beta} - 2\left\{\left(\sigma^{\gamma}_{\alpha} + \omega^{\gamma}_{\alpha}\right)H_{\beta\gamma} - \left(\sigma^{\gamma}_{\beta} + \omega^{\gamma}_{\beta}\right)H_{\alpha\gamma}\right\} = 0 \qquad (4.2)$$

On comparing with the propagation equation for $\omega_{\alpha\beta}$ and taking the skew symmetric part of the propagation equation for $V_{\alpha\beta} = h^{\gamma}_{\alpha}h^{\delta}_{\beta}U_{\gamma\delta}$, we get

$$h^{\gamma}_{\alpha}h^{\delta}_{\beta}(\omega_{\gamma\delta}) + \left(\dot{U}_{\gamma\delta} - \dot{U}_{\delta\gamma}\right) + \frac{2}{3}\theta\omega_{\alpha\beta} - 2\left(\sigma^{\gamma}_{\alpha}\omega_{\beta\gamma} - \sigma^{\gamma}_{\beta}\omega_{\alpha\gamma}\right) = 0 \qquad (4.3)$$

In the MHD approximation, an acceleration potential $r$ is defined as [8]

$$\dot{U}_{\alpha} \overset{\text{def}}{\equiv} -h^{\beta}_{\alpha}(\log r)_{\beta,} \qquad (4.4)$$

We have

$$h^{\gamma}_{\alpha}h^{\delta}_{\beta}\left(\dot{U}_{\gamma\delta} - \dot{U}_{\delta\gamma}\right) = -h^{\gamma}_{\alpha}h^{\delta}_{\beta}\frac{\dot{r}}{r}\omega_{\gamma\delta} \qquad (4.5)$$

If we take $\Omega_{\alpha\beta} = r\omega_{\alpha\beta}$, then the equation (4.3) reduces in the form

$$h^{\gamma}_{\alpha}h^{\delta}_{\beta}\Omega_{\gamma\delta} + \frac{2}{3}\theta\Omega_{\alpha\beta} - 2\left(\sigma^{\gamma}_{\alpha}\Omega_{\beta\gamma} - \sigma^{\gamma}_{\beta}\Omega_{\alpha\gamma}\right) = 0 \qquad (4.6)$$

If $\omega^{\gamma}_{\alpha}\Omega_{\beta\gamma} - \omega^{\gamma}_{\beta}\Omega_{\alpha\gamma} = 0$ then this equation for $\Omega_{\alpha\beta}$ is the same equation (4.2) for $H_{\alpha\beta}$.
Multiplying equation (2.4) on both side by $U^{\beta}$, we get

$$H_{\alpha\beta}U^{\beta} = \eta_{\alpha\beta\gamma\delta}B^{\gamma}U^{\delta}U^{\beta} \qquad (4.7)$$

By virtue of equations (2.6) and (4.7), we get

$$\eta_{\alpha\beta\gamma\delta}U^{\beta}B^{\gamma}U^{\delta} = 0 \qquad (4.8)$$

Multiplying equation (3.2) by $H_{\alpha\beta}$, we obtain

$$H_{\alpha\beta}h^{\alpha}_{\beta} = H_{\alpha\beta}g^{\alpha}_{\beta} + (H_{\alpha\beta}U^{\alpha})U_{\beta} \qquad (4.9)$$

Inserting equation (2.6) in the above equation, we obtain

$$H_{\alpha\beta}h^{\alpha}_{\beta} = H_{\alpha\beta}g^{\alpha}_{\beta} \qquad (4.10)$$

Multiplying equation (2.8) by $U^{\gamma}$ on both sides, we get

$$\dot{B}_{\alpha}U^{\gamma} = \frac{1}{2}\eta^{\alpha\beta\gamma\delta}U_{\beta}\left(H_{\gamma\delta}U^{\gamma}\right) \qquad (4.11)$$

As a consequence of the equations (4.11) and (2.6), we obtain

$$\dot{B}^{\alpha}U^{\alpha} = 0$$

Equation (4.6) can be written as

$$h^{\gamma}_{\alpha}h^{\delta}_{\beta}\Omega_{\gamma\delta} + \frac{2}{3}U^{\alpha}_{\alpha}\Omega_{\alpha\beta} - 2\left(\sigma^{\gamma}_{\alpha}\Omega_{\beta\gamma} - \sigma^{\gamma}_{\beta}\Omega_{\alpha\gamma}\right) = 0 \qquad (4.12)$$

Multiplying equation (4.12) by $B_\alpha^\gamma$ on both sides, we get

$$(B_\alpha^\gamma h_\alpha^\gamma)h_\beta^\delta\Omega_{\gamma\delta}+\frac{2}{3}B_\alpha^\gamma\theta\Omega_{\alpha\beta}-2B_\alpha^\gamma\left(\sigma_\alpha^\gamma\Omega_{\beta\gamma}-\sigma_\beta^\gamma\Omega_{\alpha\gamma}\right)=0 \quad (4.13)$$

By virtue of equations (3.4) and (4.13), we obtain

$$\frac{2}{3}B_\alpha^\gamma\theta\Omega_{\alpha\beta}-2B_\alpha^\gamma\left(\sigma_\alpha^\gamma\Omega_{\beta\gamma}-\sigma_\beta^\gamma\Omega_{\alpha\gamma}\right)=0 \quad (4.14)$$

----------------

**REFERENCES**


1. A.lichnerowicz: Relativistic hydrodynamics and magnetohydrodynamics, Benjamin, new York Ch.4. (1967)
2. D.P. Mason: Vorticity of a perfect fluid undergoing a gravitational collapse, Aust. J. Phys., 29 (1976), 413-418.
3. E.N. Glass L: The Wyle tensor and shear-free perfect fluids, J. Math. Phys., 16 (1975) 2361-2363.
4. E.N. Parker: A kinematical theory of turbulent hydromagnetic fields, Ap. J. (138) (1963), 226-238.
5. E. Vorobev, O. Zikanov, P.A. Davidson & P. Knaepen :Anisotropy of magnetohydrodynamic turbulence at low magnetic Reynolds number.
   Physics of Fluids, 17: 125105 (1)-(12), (2005)
6. F. Alexei, & Cheviakov:Bogoyavlenskij Symmetries of Isotropic and Anisotropic MHD Equilibria as Lie Point Transformation, Proceedings
   Of Institute of Mathematics of Ukraine, Vol. 50, 69-76,(2004).
7. G.F.R. Ellis: Relativistic cosmology, Cargese Lectures in Physucs, ed. E. Schatzman, Gordon and Breach, New York. Vol. 6(1973), 1-60.
8. J.M. TenBarge, R.D., Hazeltine & S.M. Mahajan: Fluid model for relativistic magnetized plasmas, Physics of Plasma 15, 062112 (2008).
9. N.T.M., eldabe, A.A. Hassan and M.A.A. Mohamed : Effect of couple stresses on the MHD of a non-Newtonian Unsteady flow between two parallel porous plates Z. Naturfarch. 58a, 204-210 (2003).
10. S. Chandrasekhar: The invariant theory of isotropic turbulence in magnetohydrodynamics I and II, Proc. Roy. Soc. London, A, 204 (1950), 435-449 and 207 (1951), 301-306.