A study on Interpretation in three variables and its applications

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ABSTRACT:-

In this dissertation we divided into two chapters. In mathematics is the process of extracting the underlying essence of a mathematical concept removing any dependence on real world objects with which it might originally have been connected and generalizing it so that it has wider applications or matching among other descriptions of equivalent.

KEY WORDS:- Open set, system equations, invertable, matrix, determinant, continuously differentiable, differentiable, continuous function, one-one function, inverse function.

INTRODUCTION:-

The implicit functions defined by GM.Scarpello in 2002. All these this was matter for research for discards and other mathematics since the first half of the 17^{th} century for almost two hundred years were broadly defined an author is " the person who originated or gave existence to anything". The first known use of interpretation is 14^{th} century.

IMPLICIT FUNCTION:- In mathematics an implicit equation is a relation of the from $R(x_1,x_2,\ldots..x_n) = 0$ where R is a function of several variables.

For Example the implicit equation of the unit circle is $x^2+y^2=1$.
Ie, A function or relation in which the dependent variables is not isolated on one side of the equation.
Ex:- The Equation $x^2+xy-y^2=1$ represents an implicit function.

Interpretation:-
Definition:- The act or the result of interpreting, explanation. A particular or version of a work, method or style. A teaching technique that combines factual with stimulating explanatory information.

Implicit Function Theorem:-

Statement:- Let E C R^{n+m+k} be an open set f is a mapping from E into R^n such that f(a. b. c)=o for some point (a,b,c) € E put A = f^1 (a,b,c) and assume the A_x invertable then there exist an open sets U C R^{n+m+k} and W C R^m with (a,b,c) €U and bcw similarly y € R^k having the following property.

(1) To Every a€y corresponds a unique x such that (x,y,z) € u and f(x,y,z)=o →1

(2) If this x is defined to be g(y) then g is a mapping of y into R^n with g(b)=a or g(c)=b, f(g(y), y, z)=0 for each y€y →2

(3) $g^{-1}(c) = -(A_x)^{-1} A_y.A_z$ →3

Now will introduce the interpretation topic in three variables.

INTERPRETATION:-

The Equation f(x, y, z) = o can be written as a system of n equations with n+m+k variables.

$f_1 (x_1, x_2, \ldots\ldots, x_n ; y_1, y_2, \ldots\ldots y_m ; z_1, z_2 \ldots\ldots z_k) = 0$

$f_n (x_1, x_2, \ldots\ldots, x_n ; y_1, y_2, \ldots\ldots y_m ; z_1, z_2 \ldots\ldots z_k) = 0$

Assume A_x is invertable that the "n x n" matrix Evaluated at (a, b, c) defines as

$$\begin{pmatrix} D_1 f_1 & D_2 f_1 & \ldots\ldots D_n f_1 \\ D_1 f_2 & D_2 f_2 & \ldots\ldots D_n f_2 \\ D_1 f_n & D_2 f_n & \ldots\ldots D_n f_n \end{pmatrix}$$

ie.. The determinant of the above matrix at (a, b, c) = 0 also 1 holds at (a, b, c) € E.
Then the conclusion of the theorem is that 1 can be solved for $x_1, x_2, \ldots\ldots, x_n$ in terms of $y_1, y_2, \ldots\ldots y_m$ for every y nearby b and z nearby c that these solutions are continuously differentiable function of y and z.

PROOF OF THE ABOVE THEOREM: -

Proof: Define F : E →IR^{n+m+k} by F(x, y, z) = (f(x, y, z) y, z)

Since f is differentiable for Every (x, y, z) € E

There Exist B € L (R^{n+m+k}, R^n)

Such that
$$\lim_{(h, k, l) \to 0} \frac{\| f(x+h, y+k, z+l) - f(x, y, z) - B(h,k,l) \|}{\| (h, k, l) \|} \quad \to 2$$

Here B = f^1 (x, y, z)

Now

F (x + h, y + k, z + l) – F (x, y, z)

= (f (x + h, y + k, z + l), y +k, z + l) – (f (x, y, z), y, z)

=(f (x + h, y + k, z + l) – f(x, y, z), k, l)

From 2

$$\lim_{(h, k, l) \to 0} \frac{\| f(x+h, y+k, z+l) - F(x, y, z) - B(h, k, l), l \|}{\| (h, k, l) \|} = 0 \quad \to 3$$

Write c (h, k, l) = (B (h, k, l), l)

$C = f^{1}(x, y, z)$

and $C \in L(R^{n+m+k})$

Therefore F is differentiable at (x, y, z) and $C = f^{1}(x, y, z)$

Now we can show that f^{1} is continuous.

Let B and C as above and

$B_o = f^{-1}(x_o, y_o, z_o)$

$C_o = f^{-1}(x_o, y_o, z_o)$

Now consider $\| f^{-1}(x, y, z) - f^{-1}(x_o, y_o, z_o) \| = \| c - c_o \|$

But we have $c(h, k, l) = (B(h, k, l), l)$

And $c_o(h, k, l) = (B_o(h, k, l), l)$

So that $(C-C_o)(h, k, l) = ((B-B_o)(h, k, l), o)$

$= \| (C-C_o)(h, k, l) \| = \| ((B-B_o)(h, k, l), o) \|$

$= \| (B-B_o)(h, k, l) \|$

$< \| (B-B_o) \| \| (h, k, l) \|$

For all $(h, k, l) \in R^{n+m+k}$

$\rightarrow \| (C - C_o) \| < \| (B - B_o) \|$

Now $\| f^{-1}(x, y, z) - f^{-1}(x_o, y_o, z_o) \| < \| f^{-1}(x, y, z) - f^{-1}(x_o, y_o, z_o) \|$

$\rightarrow 0$ as $(x, y, z) \rightarrow (x_o, y_o, z_o)$

f^{-1} is continuous function on E

Thus $F \in e^{1}(E)$

Now write $F(a, b, c) = (f(a, b), b, c) = (o, b, c)$

Now let $T = f^{-1}(a, b, c)$ and $A = f^{-1}(a, b, c)$

From Equation No (3) $T(h, k, l) = 0$

$(A(h, k, l), l) = 0$

$(A(h, k, l)) = 0, l = 0$

$(A(h, k, 0)) = 0, l = 0$

A_x (h, k) = 0, l = 0

A (h, 0) = 0, l = 0

A_x (h) = 0, K=0, l = 0

(h, k, l) = (0, 0, 0)

This T is one – one and hence T is invertable Therefore f^{-1} (a, b, c) is invertable function.

We have F : E ---- lR^{n+m+k} is such that F € e (E); F(a, b, c) = (o, b, c) and
f^{-1} (a, b, c) is invertable. Thus F satisfies the hypothesis of inverse function theorem

Hence, there exist an open sets U, V and W of R^{n+m+k} such that (a, b, c) € U,
(o, b, c) € V and (o, o, c) € W with F is 1-1 on U, V and W

F (U) = V, F(V) = W

Write X = { y, z, € R^{m+k} / (o, y, z) € v }

Since (o, b, c) € v → b € x → b € x → x = Ø

Now the mapping y → (o, y) is continuous from R^m into R^{n+m}

And the mapping Z → (o, z) is continuous from R^k to R^{n+m+k}

Hence the proof of the Theorem.

Lemma1:- Every A€L(R^{n+m+k},R^n,R^m) can be split into three linear transformations A_x,A_y,A_z defined by A_x(h) = A(h,0,0), A_y(k)=A(0,k,0) A_z=A(0,0,l)

For any h€R^n , K€R^n, l€R^k then A_x€L(R^n), A_y€L(R^m,R^n), A_z€L(R^m,R^n,R^k) and

A(h,k,l)=A_x(h) + A_y(k) + A_z(l).

Proof:let z_1=(x_1,y_1,z_1) €R^{n+m+k}

z_2=(x_2,y_2,z_2) €R^{n+m+k}

and z_3=(x_3,y_3,z_3) €R^{n+m+k}

then A(z_1+z_2+z_3)=Az_1+Az_2+Az_3

and A(α z)=αAz for all z€R^{n+m+k} and α be any scalar with this observation,it is clear that A_x€L(R^n)

for A_x(h_1+h_2)=A(h_1+h_2,0,0)

=A(h_1,0,0)+A(h_2,0,0)+A(h_3,0,0)

=A_x(h_1)+A_x(h_2)+A_x(h_3)

Similarly $A_x(\alpha h)=\alpha A_x(h)$ for any h€R^n and scalar α.

Now A_y€$L(R^m,R^n)$

Let h€R^m,k€R^n and l€R^k then

$A_x(h)+A_y(k)+A_z(l)=A(h,0,0)+A(0,k,0)+A(0,0,l)$

$=A(h,k,l)$

Hence proved the lemma.

Lemma2:-If A€$L(R^{n+m},R^n)$ and if A_x is invertable,then there corresponds to every k€R^m is a unique h€R^n suchthat A(h,k)=0.this h can be computed from k by the formula $h=-(A_x)^{-1}A_y(k)$.

PROOF OF THE LEMMA:-

By lemma (1)

$A(h,k)=0$ if and only if $A_x(h)+A_y(k)=0$

If and only if $h=-(A_x)^{-1}A_y(k)$

Where A_x is invertable

NOTE:-The conclusion of the lemma 2 is in other words that the eqation A(h,k)=0 can be solved(uniquely)for h if k is given and that the solution h is linear function of k.

APPLICATIONS:-

1.From Implicit function theorem we have

Let ECR^{n+m+k} be an open set.f:E->R^{n+m},f€C^1(E) such that f(a,b,c)=0 for some point (a,b,c)€E.

Take n=m=k=1

Put $A=f^{-1}(a,b,c)$ and assume that A_x is invertable.then there exit open sets UCR^{n+m},WCR^m,VCR^k with (a,b,c)€U and b€W,C€V having the following property.

To every y€W,z€V corresponds a unique x suchthat (x,y,z)€U and f(x,y,z)=0.

If this x is defined to be g(y),then g is C^1-mapping of W into R^n,g(b)=a,f(g(y),y)=0,y€W and $g(b)=-(A_x)^{-1}A_y$.

2.Now take n=2,m=3,k=0 and consider the mapping $f=(f_1,f_2,f_3)$ from R^5toR^5 given by $F_1(x_1,x_2,y_1,y_2,y_3)=2e^{x_1}+x_2y_1-4y_2+3$.

$F_2(x_1,x_2,y_1,y_2,y_3)=x_2\cos x_1-6x_1+2y_1-y_3$.

If a=(0,1) and b=(3,2,7) then f(a,b)=0.

$$A=f^{-1}(a,b)=\begin{bmatrix} D_1f_1 & D_2f_1 & D_3f_1 & D_4f_1 & D_5f_1 \\ D_1f_2 & D_2f_2 & D_3f_2 & D_4f_2 & D_5f_2 \end{bmatrix}(a,b)$$

$$=\begin{bmatrix} 2e^{x1} & y_1 & x_2 & -4 & 0 \\ -x_2\sin x_1 & \cos x_1 & 2 & 0 & -1 \end{bmatrix}(0,1,3,2,7)$$

$$= \begin{bmatrix} 2 & 3 & 1 & -4 & 0 \\ -6 & 1 & 2 & 0 & -1 \end{bmatrix}$$

$[A_x] = \begin{pmatrix} 2 & 3 \\ -6 & 1 \end{pmatrix}$ and $[A_y] = \begin{pmatrix} 1 & -4 & 0 \\ 2 & 0 & -1 \end{pmatrix}$

and $[A_y] = [1 \ -4 \ 0]$

$\det[A_x] \neq 0$ so that A_x is invertable.

Hence by the implicit function theorem there exit open sets UCR^{n+m} and WCR^{m} with (a,b)€U and b€W with following property.

To every b€W corresponds a unique x such that (x,y)CU and f(x,y)=0.

If this x is defined to be g(y) then g is a C^1 mapping of W into R^n.with g(b)=a

'i.e. g(3,2,7) = (0,1) and f(g(y),y)=0 for any y€W also $g^{-1}(b)=-(A_x)^{-1}a=A_y$

$[(A_x)^{-1}]=[A_x]^{-1}=\begin{pmatrix} 1 & 1 & -3 \\ 20 & 6 & 2 \end{pmatrix}$

$g^{-1}(b)=g(3,2,7)= -1/20 \begin{pmatrix} 1 & -3 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} 1 & -4 & 0 \\ 2 & 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} 1/4 & 1/5 & -3/20 \\ -1/2 & 6/5 & 1/10 \end{pmatrix}$$

Therefore

$D_1g_1(3,2,7)=1/4, D_2g_1(3,2,7)=1/5, D_3g_1(3,2,7)=-3/20$.

$D_1g_2(3,2,7)=-1/2, D_2g_2(3,2,7)=6/5, D_3g_2(3,2,7)=1/10$.

CONCLUSION:-

Flaws in logic can also occur at the interpretation stage. We may have a well- designed study obscure the true meaning of the data by misreading the findings. practical significance (or)theoretical implications are discussed; guidance for future studies is offered. The study limitations are discussed. Alternative interpretations for the findings are considered.

I have already derived the interpretation in three variables. Similarly we can derive for n variables also in future. Develop their ability to work in a connected word.

Reference: Dr. M.Kavitha, St. Peter's Institute of Higher Education and Research, Avadi, Chennai-54