SUBDIGRAPHS OF NEARLY MOORE DIGRAPHS INITIATED BY FIXPOINTS OF AN AUTOMORPHISM

[1]Ms.M.Saranya, [2]Mr.M.V.Suresh
[1]Research scholar, Department of Mathematics, SPIHER, Avadi, Chennai-54
[2]Assistant Professor, Department of Mathematics, SPIHER, Avadi, Chennai-54

ABSTRACT:

The degree/distance across an issue for coordinated charts is the issue of deciding the biggest conceivable request for a digraph with given greatest out-degree d and width k. An upper bound is given by the Moore bound M (d, k) = $\sum_{i=0}^{k} d^i$ and nearly Moore digraphs will be digraphs with most extreme out-degree d, the distance across k, and request M (d, k)- 1.

In this paper, we will take a gander at the structure of subdigraphs of nearly Moore digraphs, which are initiated by the vertices fixed by some automorphism ϕ. In the event that the automorphism fixes at any rate three vertices, we demonstrate that the instigated subdigraph is either a nearly Moore digraph or an ordinary k-geodetic digraph of degree $d' \leq d-2$, request $M(d',k)+1$ and breadth k + 1.

As it is realized that nearly Moore digraphs have an automorphism r, these outcomes can enable us to decide basic properties of nearly Moore digraphs, for example, what number of vertices of each request there are concerning r. We decide this for d = 4 and d = 5, where we demonstrate that with the exception of in some uncommon cases, all vertices will have a similar request.

KEYWORDS: k-geodetic digraph; Moore digraph; the degree/breadth issue

1. INTRODUCTION:

Give G a chance to be a digraph and u be a vertex of greatest out-degree d in G, and let n_i indicate the quantity of vertices in separation i from u. At that point, we have $n_i \leq d^i$ for i = 0, 1, . . . , k, and hence the request n of G is limited by

$$n = \sum_{i=0}^{k} n_i \leq \sum_{i=0}^{k} d^i \quad (1)$$

On the off chance that equity is gotten in (1) we state that G is a Moore digraph of degree d and distance across k, what's more, the right-hand side of (1) is known as the Moore bound meant by $M(d,k) = \sum_{i=0}^{k} d^i$. Moore digraphs are known to be diregular and exist just when d = 1 (cycles of length (k + 1)) or k = 1 (complete digraphs with request d + 1). So we are keen on knowing how close the request can get to the Moore destined for d > 1 and k > 1. Give G a chance to be a digraph of most extreme out-degree d, distance across k and request M (d, k) - δ, at that point we state G is a (d, k, -δ)- digraph or then again a (d, k)- digraph of deformity δ. At the point when δ < M (d, k -1) we have out-normality, though by and large it isn't known whether we likewise have in-consistency. Of uncommon intrigue is the situation δ = 1, and a (d, k, -1)- digraph is additionally meant as a nearly Moore digraph. Nearly Moore digraphs do exist for k = 2 as the line digraphs of K_{d+1} for any $d \geq 2$, though (2, k, -1)- digraphs for k > 2, (3, k, -1)- digraphs for k > 2, (d, 3, -1)- digraphs for d > 1 and (d, 4, -1)- digraphs for d > 1 don't exist. We do realize that nearly Moore digraphs are diregular for d > 1 and k > 1.

In the last segment of the paper, we will require the accompanying hypothesis which abridges a portion of the above outcomes.

Theorem 1.1

Nearly Moore digraphs of degree 2 and 3 and measurement k > 2 don't exist.

Besides, nearly Moore digraphs fulfills the accompanying properties, where a≤ k-walk is a stroll of length at generally k.

Lemma 1.1(a) Give G a chance to be a nearly Moore digraph, at that point

(i) for each pair of vertices u, v ∈ V (G) there is all things considered one < k-stroll from u to v,
(ii) for each vertex u ∈V (G) there exist a novel vertex r(u) with the end goal that there are two ≤ k-strolls from u to r(u).

Proof:

The mapping r : V (G) →V (G) is in certainty an automorphism, and subsequently the two ≤ k-strolls from u to r(u) are inside disjoint. The vertex r(u) is said to be the rehash of u. In the event that we have u = r(u), in this way u has request 1 regarding r, u is said to be a self repeat. On the off chance that there is a self repeat in G, at that point there are actually k self repeats, which lie on a k-cycle.

In this paper we will give a few conditions for the presence of a nearly Moore digraph G as for some automorphism ϕ : V (G) → V (G). These outcomes would then be able to be utilized to explore the requests of the vertices as for the automorphism r. Before expressing the center aftereffect of this paper, we will present another sort of digraph which shows to be significant when portraying prompted sub digraphs of nearly Moore digraphs.

Give D a chance to be a digraph with the end goal that for each pair of vertices u, v ∈ V (D) we have all things considered one ≤ k-stroll from u to v, at that point we state D is k-geodetic. Give u a chance to be a vertex of least out-degree d, and let n_i be the quantity of vertices in separation i from u for i = 0, 1, . . . , k. At that point, $n_i \geq d^i$ and the request n of D is limited by

$$n \geq \sum_{i=0}^{k} n_i \geq \sum_{i=0}^{k} d^i \qquad (2)$$

Notice that the right-hand side is the Moore bound, M (d, k) and that the width for a k-geodetic digraph is at any rate k. As we definitely know, Moore digraphs exist just for d = 1 or k = 1, we wish to know how close the request of a k-geodetic digraph can get to the Moore bound. By a (d, k, s)- digraph we comprehend a k-geodetic digraph of least out-degree d and request M (d, k) + s. On the other hand we state that we have a (d, k)- digraph of overabundance s. The principal case which is fascinating is when s = 1. A (d, k, 1)- digraph has width k + 1, and for every vertex u there is actually one vertex, the anomaly o(u) with the end goal that dist(u, o(u)) = k + 1.

A (d, k, 1)- digraph is diregular if and just if the mapping o : V (D) →V (D) is an automorphism, we additionally have the accompanying hypothesis.

2. RESULTS

For straightforwardness, we will, in the rest of the piece of this paper, let a (d, k,1)-digraph (nearlyMoore digraphs) indicate any digraph which has degree d > 0, width k > 0 and request M (d, k)-1, in this way we will let k-cycles be incorporated into this class. Additionally, a (d, k, 1)- digraph will indicate any k-geodetic digraph of least out-degree d > 0 and request M (d, k) + 1.

The extent of this paper is to demonstrate the accompanying hypothesis.

Theorem 2.1: Give G a chance to be a nearly Moore digraph of degree $d \geq 4$ and distance across $k \geq 3$ and let H be a sub digraph actuated by the vertices which are fixed by some automorphism $\phi : V(G) \rightarrow V(G)$. At that point, H is either

(i) the void digraph,
(ii) two disengaged vertices,
(iii) an nearly Moore digraph of degree $d' \leq d$ and measurement k, or
(iv) a diregular (d', k, 1)- digraph where $d' \leq d - 2$.

In the rest of the piece of this paper we will expect G to be a nearly Moore digraph of degree $d \geq 4$ and breadth $k \geq 3$, and H to be a sub digraph of G prompted by the fix points of some automorphism $\phi : V(G) \rightarrow V(G)$.

We begin by expressing a few properties of the fixpoints of G.

Lemma 2.1: Give u and v a chance to be fixpoints of G regarding the automorphism ϕ, at that point

(i) r(u) is a fix point,
(ii) if there is a $\leq$ k-walk P from u to v and $v \neq r(u)$, all vertices $w \in P$ are fixpoints,
(iii) if v = r(u) and P and Q are the two $\leq$ k-strolls from u to v, either all inner vertices on P and Q are fixpoints, or none of them are. Moreover, on the off chance that dist(u, r(u)) < k, at that point all vertices on P and Q are fixpoints.

Proof:

(i) We know there are two $\leq$ k-strolls, P and Q, from u to r(u).
Presently, $\phi(P)$ and $\phi(Q)$ are two $\leq$ k-strolls from u to $\phi(r(u))$, and consequently $\phi(r(u))$ is a rehash of u. As u just has one rehash, the announcement pursues.

Let P be the unique k-stroll from u to v. At that point, $\phi(P)$ will likewise be a $\leq$ k-stroll from u to v, and consequently $P = \phi(P)$.

(ii) Assume not all vertices on the $\leq$ k-walk P are fixpoints, henceforth there exists a vertex $w \in P$ with the end goal that $w \neq \phi(w)$ and along these lines $\phi(P) = P$ is additionally a $\leq$ k-stroll from u to v = r(u). As there are just two $\leq$ k-strolls from u to v = r(u), we should have $\phi(P) = Q$ and along these lines none of the inside vertices of P are fixpoints, as P and Q are inside disjoint. Presently in the event that dist(u, r(u)) < k, at that point P and Q are clearly of various length, so we should have all vertices on P and Q as fixpoints.

Corollary 2.1:

Let ϕ be an automorphism of G, at that point all $\leq$ k-strolls among the fixpoints of ϕ in G are safeguarded to H, aside from perhaps the k-strolls from a vertex to its rehash.

Notice, that on the off chance that u and v are self repeats fixed by ϕ, at that point there are actually d inside disjoint (k + 1)- strolls from u to v, (u, u_i, . . . , v_i, v) for i = 1, 2, . . . , d. Hence forth if the request of u_i regarding ϕ is p, and the request of v_i as for ϕ is q, at that point (u, $u_i = \phi^p(u_i)$, . . . , $\phi^p(v_i)$, v) and (u, u $= \phi^q(u_i)$, . . . , $v_i = \phi^q(v_i)$, v) are both $\leq$ (k + 1)- strolls, and consequently we should have p = q. Said in another manner, the stage cycles regarding some automorphism ϕ of the vertices in $N^+(u)$ and $N^-(v)$ are a similar when u and v are self repeats.

Lemma 2.2: On the off chance that G has a self repeat which is fixed by ϕ, at that point H is a nearly Moore digraph with self repeats of degree d' $\leq$ d and width k.

Proof:

Let z = r(z) = ϕ(z), at that point as indicated by Lemma 2.1 we should have all vertices on the two $\leq$ k-strolls from z to r(z) as fixpoints, and all the self repeats lie on the non-paltry stroll from z to z, so H contains a k-cycle.

Notice that $d_H^+(z) = d_H^-(z) = d' \leq d$ for all z = r(z) ∈ V (H), as the stage cycles in $N^+(z)$ and $N^-(z)$ are the equivalent. Presently, in the event that we have a vertex u = ϕ(u) $\neq$ r(u), at that point we can pick a self respect z with the end goal that r(u) $\notin$ $N^-(z)$, as else we would have r(u) ∈ $N^-(z')$ for all self repeats z' of G, and in this manner r(r(u)) would be a self repeat, an inconsistency as u isn't a self repeat. In this way for this u and z we have d inside disjoint $\leq$ (k + 1)- strolls (u, u_i, . . . , z_i, z) in G. At that point, d' of the inside disjoint $\leq$ (k + 1)- strolls from u to z will likewise be in H, because of Lemma 2.1, and in this manner $d^+(u) \geq d'$. Expect that $d^+(u) > d'$, at that point there exists a j∈ { 1, 2, . . . d} to such an extent that $u_j = \phi(u_j)$ and $z_j \neq \phi(z_j)$. In any case, at that point (u_j, . . . , z_j, z) and (u_j, . . . , $\phi(z_j)$, z) are two particular $\leq$ k-strolls from u_j to z, an inconsistency as z is a self repeat.

So H is a diregular digraph of degree d'. Presently, expect H has breadth k + 1, this suggests there exists a vertex v with the end goal that $dist_H$ (v, r(v)) = k + 1 consequently the request of H is $n = 1 + d' + d'^2 + ... + d'^k + 1$ = M (d', k) + 1, as indicated by Corollary 2.1. In any case, taking a gander at a self repeat z ∈ H, we get the request as $n = 1 + d' + d'^2 + ... + d'^k - 1$ = M (d', k) -1, a logical inconsistency.

So H must be diregular with degree $d' \leq d$, width k and its request must be M (d, k)-1, thus it is a nearly Moore digraph with self repeats, as the bigness of H is k.

Lemma 2.3: Let ϕ fix at any rate three vertices, at that point H is diregular of degree d' and either

(i) H is a nearly Moore digraph of degree d' $\leq$ d and measurement k, or
(ii) (ii) H is a (d', k, 1)- digraph of degree d' $\leq$ d − 2.

Proof:

On the off chance that ϕ fixes a self repeat, at that point we have the principal instance of the announcement as per Lemma 2.2. Along these lines we can accept ϕ does not fix any selfrepeats.

Give u and v a chance to be any two fixed vertices in G, in this way they are not selfrepeats, and let $N^+(u) = \{u_1, u_2, . . . , u_d\}$ and $N^-(v) = \{ v_1, v_2, . . . , v_d\}$. Expect r(u) $\neq$ v_j for j = 1, 2, . . . , d. At that point, in

G we have inside disjoint ≤ (k + 1)- strolls (u, u_i, . . . , v_i, v) for i = 1, 2, . . . , d. As r is an automorphism, we get $r(u_i) \neq v$ for i = 1, 2, . . . , d. Presently, we have $u_i = \phi(u_i)$ if and just if $v_i = \phi(v_i)$ because of Lemma 2.1, consequently $d_H^+(u) = d_H^-(v)$. As we could have v = r(u), we see that every vertex in H is adjusted, as $d^+(u) = d^+(r(u))$ and $d^-(u) = d^-(r(u))$.

Presently, expect H isn't diregular, in this way for every vertex u ∈ V (H) we should have a vertex v ∈ $N^+(r(u)) \cap V(H)$ with the end goal that $d_H^+(u) \neq d_H^-(v)$. Let u ∈ V (G) be a vertex of least degree $d_1 \leq d$ in H, and let v ∈ V (H) be a vertex with $d_H^-(v) > d_1$. At that point, $d_H^-(v) = d_1 + 2$ as we should have v ∈ $N^+(r(u))$ with $dist_H(u, r(u)) = k + 1$ and $dist_H(r^-(v), v) \leq k$. Be that as it may, at that point there must be all things considered d_1 vertices of degree not the same as d_1 in H and at most $d_1 + 2$ vertices of degree unique in relation to $d_1 + 2$, henceforth $|V(H)| \leq d_1 + (d_1 + 2)$. This is an inconsistency to the way that $|V(H)| \geq d_1 + d_1^2 + ... + d_1^k$ as the distance across of H is at any rate k ≥ 3. Thus, clearly H is diregular. In the event that dist(u, r(u)) = k + 1, at that point every vertex in H must have at any rate two out-neighbors of request two concerning ϕ and consequently the announcement pursues.

3. Almost Moore digraphs of degree 4 and 5

In this area we will take a gander at nearly Moore digraphs of degree 4 and 5 and determine the request of the vertices as for the automorphism r.

Lemma 3.1: Let u ∈ V (G) be a vertex with ϕ(u) = u ≠ r(u), at that point if H is two segregated vertices or has measurement (k + 1) we should have two vertices in $N_G^+(u)$ which have request 2 as for ϕ.

Proof:

In G we have two ≤ k-ways, P and Q from u to r(u). In the event that H is either two disconnected vertices or has breadth k + 1, we should have that the inner vertices on P and Q are not in H. Accordingly, ϕ(P) = Q and ϕ(Q) = P , and subsequently $\phi^2(v) = v$ and ϕ(v) = v for all inward vertices v on P and Q.

Theorem 3.1: Give G a chance to be a nearly Moore digraph of degree 4, at that point the vertices of G have orders regarding the automorphism r as indicated by one of the accompanying:

(i) there are k vertices of request 1 and M (4, k) − 1 − k of request 3, or
(ii) all vertices are of a similar request p ≥ 2.

Proof:

Accept all through that not all vertices are of a similar request. Give u a chance to be a vertex of G of the littlest request p concerning r in G. Let $N^+(u) = \{u_1, u_2, u_3, u_4\}$, then we can part $N^+(u)$ into change cycles as for rp in one of the accompanying ways: $(u_1)(u_2)(u_3, u_4)$, $(u_1)(u_2, u_3, u_4)$, (u_1, u_2, u_3, u_4) or $(u_1, u_2)(u_3, u_4)$. Notice anyway that the part $(u_1)(u_2)(u_3, u_4)$ is beyond the realm of imagination, as there as indicated by Theorem 2.1 where $\phi = r^p$ would exist a (2, k, -1)- or (2, k, 1)- digraph as a prompted subdigraph of G, a logical inconsistency to Theorems 1.1 and 1.2.

First expect there is a vertex u of request 1, accordingly u is a selfrepeat and subsequently there are actually k vertices of request 1 inciting a k-cycle in G. In this way among the above methods for having stage cycles, the main probability is $(u_1)(u_2, u_3, u_4)$. At that point, all vertices which are not selfrepeats must have request 3 as per Lemma 2.2 by letting $\phi = r^3$.

Presently accept $u \in V(G)$ has the littlest conceivable request $p \geq 2$, at that point as per Lemma 3.1 the main conceivable change cycles are $(u_1, u_2)(u_3, u_4)$. Thusly, this is just conceivable if p = 2, as there will dependably be in any event p vertices of request p in G.

In this way G will contain M (4, k) - 3 vertices of request 4, subsequently 4 should separation M (4, k) - 3. Be that as it may, truth be told a logical inconsistency.

$$M(4, k) - 3 \equiv -2 + 4 + 4^2 + \ldots + 4^k \equiv 2 \quad \mod 4,$$

a logical inconsistency.

Theorem 3.2: Give G a chance to be a nearly Moore digraph of degree 5, at that point the vertices of G have orders regarding the automorphism r as indicated by one of the accompanying:

(i) there are M (3, k) + 1 vertices of request $p \geq 2$ and M (5, k) - M (3, k) -2 of request 2p
(ii) there are k + 2 vertices of request $p \geq 2$ and M (5, k)- 3 - k of request 2p
(iii) there are k vertices of request 1 and either M (5, k) -1- k of request 2 or M (5, k) -1- k of request 4
(iv) all vertices are of a similar request $p \geq 2$.

Proof:

Accept all through that not all vertices are of a similar request. Give u a chance to be a vertex of G of the littlest request p. Let $N^+(u) = \{u_1, u_2, u_3, u_4, u_5\}$, then we can part $N^+(u)$ into stage cycles regarding rp in one of the accompanying ways: $(u_1)(u_2, u_3, u_4, u_5)$, $(u_1)(u_2)(u_3)(u_4, u_5)$ or $(u_1)(u_2, u_3)(u_4, u_5)$ because of Lemma 3.1 and Theorems 1.1 and 1.2.

In the event that the stage cycles are $(u_1)(u_2, u_3, u_4, u_5)$, at that point because of Lemma 3.1 we should have u is a selfrepeat, thus there is k vertices of request 1 and M (5, k) – k - 1 of request 4. On the off chance that rather the change cyles are $(u_1)(u_2, u_3)(u_4, u_5)$, at that point we could have k vertices of request 1 and M (5, k) –k -1 of request 2 or k + 2 vertices of request $p \geq 2$ and M (5, k) – k - 3 of request 2p.

At last, on the off chance that the stage cycles are $(u_1)(u_2)(u_3)(u_4, u_5)$, at that point if $\phi = r^p$, we would have H to be either a (3, k, −1)- digraph or a (3, k, 1)- digraph. In any case, (3, k, −1)- digraphs don't exist agreeing to Theorem 1.1, in this manner we should have M (3, k) + 1 vertices of request $p \geq 2$ and M (5, k) - M (3, k) - 2 of request 2p.

REFERENCES

1. E.T. Baskoro, Y.M. Cholily and M. Miller, Enumerations of vertex orders of almost Moore digraphs with selfrepeats, *Discrete Mathematics* **308** (2008), 123–128.
2. W.G. Bridges and S. Toueg, On the impossibility of directed Moore graphs, J. Combin. Theory, Series B 29(3) (1980), 339–341.
3. Anita A bildgaard Sillasen, Subdigraphs of almost moore digraphs induced by fixpoints of an automorphism, Electronic Journal of Graph Theory and Applications 3(1) (2015), 1-7
4. Edy Tri Baskoro and Amrullah, Almost Moore digraphs of degree 4 without selfrepeats, *Journal of the Indonesian Mathematical Society (MIHMI)* **8(1)** (2002), 51–60.
5. Edy Tri Baskoro, Mirka Miller, Jozef ˇ Sir´a_n and Martin Sutton, Complete characterization of almost Moore digraphs of degree three, J. Graph Theory 48(2) (2005), 112–126.

6. Anita Abildgaard Sillasen, On k-geodetic digraphs with excess one, *Electronic Journal of Graph Theory and Applications* **2(2)** (2014), 150–159.
7. J. Conde, J. Gimbert, J. Gons´alez, J.M. Miret and R. Moreno, Nonexistence of almost Moore digraphs of diameter three, Electron. J. Combin. 15 (2008), #R87.
8. Miguel Angel Fiol, J. Luis A. Yebra, and Ignacio Alegre de Miquel, Line digraph iterations and the (d; k) digraph problem, IEEE Trans. Computers 33(5) (1984), 400–403.
9. J. Plesn´ık and ˇ S. Zn´am, Strongly geodetic directed graphs, Acta Fac. Rerum Natur. Univ.Comenian., Math. Publ. 23 (1974), 29–34.