Disturbance Observer based Sliding Mode Control of Servo System

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Abstract : Servo systems are nonlinear uncertain systems which are difficult to control by traditional methods. Sliding mode control (SMC) is a special nonlinear control technique which has a quick response and is insensitive to parameters variation and disturbance which make it very suitable for servo system control. In this paper, a new type of sliding mode control with a disturbance observer has been proposed. The chattering has been considerable reduced. The results have been compared with basic SMC. The Lyapunov approach has been used for stability analysis. This method can deal with mismatched disturbance and has better transient state performance with faster response time and lower overshoot along with less chattering effect.

IndexTerms **- Sliding mode control, Disturbance observer, Servo system, Variable structure control.**

I. Introduction

Control problem of non-linear systems subject to external disturbances and model mismatch is a wildly discussed topic. Researchers have proposed a number of solutions to for this problem. Among them the disturbance observer is favourable to estimate the external disturbances and model uncertainties [1].
For the use of a disturbance observer (DOB), an accurate mathematical model of controlled object is not needed, instead an approximate model (nominal model) can also be used in order to estimate the external disturbances that exist in the system. Therefore, DOB is increasingly becoming popular in modern industrial control. The fundamental idea of a DOB is to look for the difference between the actual output and the nominal model output and use it as an equivalent disturbance acting on the nominal model. This equivalent disturbance is then estimated and introduced in the control terminal as a feedback signal to nullify the effects of external disturbances [2].
The effects of external disturbances as well as parameter perturbations can be suppressed by using SMC. But the discontinuous switching of the controller tends to induce high-frequency chattering in mechanical systems. Although the employment of functions in the discontinuous control such as the saturation function method instead of signum function could help in reducing the chattering problem, the prominent advantage of disturbance rejection performance is sacrificed. Such disadvantages severely constrain the applications of SMC.
In order to overcome these limitations, researchers have proposed a technique called active anti-disturbance control (AADC) approach. The idea behind AADC is to directly counteract disturbances by designing feedforward compensation based on measurement of disturbance or its estimations instead of feedback compensation [3].

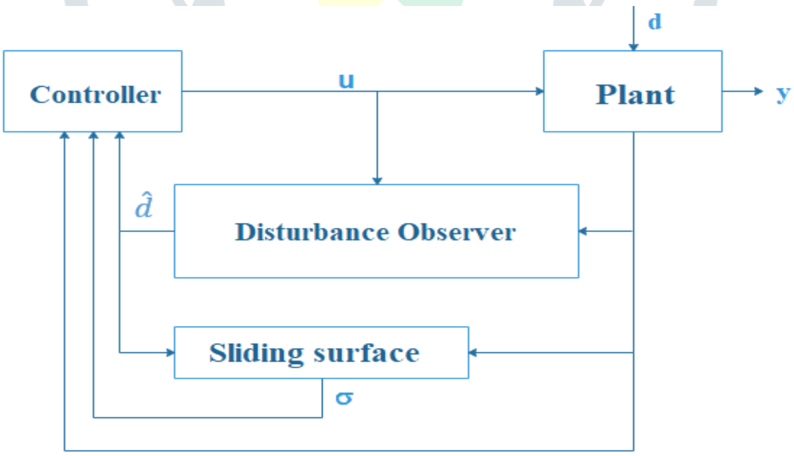

Fig. 1 Block diagram of Disturbance observer based Sliding mode control

II. System Description

The position and speed of electrical servo drive systems are widely used in engineering systems, such as CNC machines, industrial robots, winding machines etc. The main properties necessary for servo systems include good position tracking, no overshoot, no oscillation, quick response and good robustness.
In general, with the implementation of field-oriented control, the mechanical equation of an induction motor drive or a permanent synchronous motor drive can be described as [4].

$$J\ddot{\theta}(t) + B\dot{\theta}(t) + T_d = T_e \quad (1)$$

where θ is the rotor position,
J is the moment of inertia,
B is the damping coefficient,

T_d denotes the external load disturbance, nonlinear friction and unpredicted uncertainties,
T_e represents the electric torque which is defined as

$$T_e = k_t i \tag{2}$$

where k_t is the torque constant and
i is the torque current control input command.
Equation (1) can be written as

$$\ddot{\theta} = -b\dot{\theta} + au - d \tag{3}$$

Where $a = \frac{k_t}{J}$, $b = \frac{B}{J}$, $d = T_d$.
The block diagram of a servo system has been shown in Fig. 2.

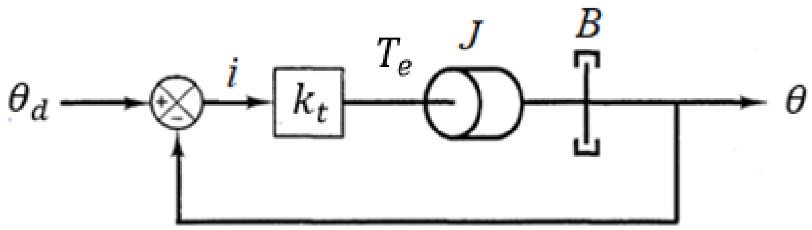

Fig. 2 Block diagram of servo system

Take $x_1 = \theta$, $x_2 = \dot{\theta}$, and $u = i$

Thus, the state-space representation becomes

$$\dot{x_1} = x_2 + T_d$$
$$\dot{x_2} = -\frac{B}{J}x_2 + \frac{k_t}{J}u \tag{4}$$

III. Disturbance Observer based Sliding Mode Control

For demonstration of Disturbance observer based sliding mode controller (DOSMC), the system same as given in section II is considered. The results have been compared with that of a basic SMC design with proportional plus constant rate reaching law.

A. Observer Design

The observer proposed by (Atsuo et al., 1994)[5] as:

$$\dot{\hat{d}} = k_1(\hat{w} - \dot{\theta})$$
$$\dot{\hat{w}} = -\hat{d} + au - k_2(\hat{w} - \dot{\theta}) - b\dot{\theta} \tag{5}$$

Where $\hat{d}$ is the estimation of d, and $\hat{w}$ is the estimation of $\dot{\theta}$, $k_1 > 0$, $k_2 > 0$.
Block diagram for a disturbance observer based SMC applied on a servo plant has been shown in Fig. 3.

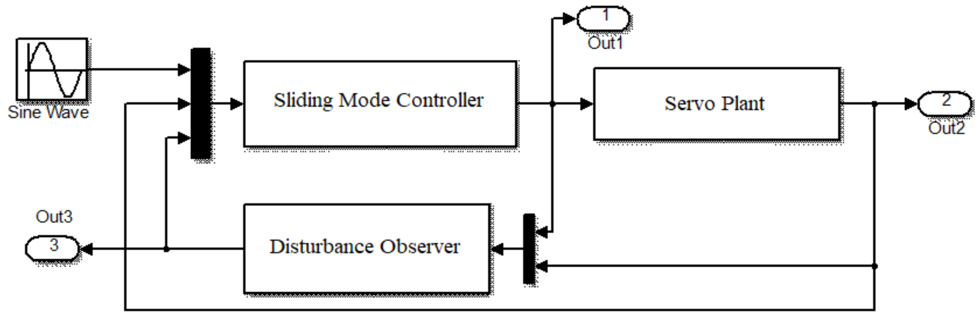

Fig. 3 Block diagram of SMC based Disturbance Observer applied to a Servo system

B. Analysis of stability of observer

Take the Lyapunov function as

$$V_1 = \frac{1}{2k_1}\tilde{d}^2 + \frac{1}{2}\tilde{w}^2 \tag{6}$$

Where
$\tilde{d} = d - \hat{d}$, $\tilde{w} = \dot{\theta} - \hat{w}$
On differentiating equation (6), we have

$$\dot{V_1} = \frac{1}{k_1}\tilde{d}\dot{\tilde{d}} + \tilde{w}\dot{\tilde{w}} = \frac{1}{k_1}\tilde{d}(\dot{d} - \dot{\hat{d}}) + \tilde{w}(\ddot{\theta} - \dot{\hat{w}})$$
$$\dot{V_1} = \frac{1}{k_1}\tilde{d}\dot{\tilde{d}} + \tilde{w}\dot{\tilde{w}} = \frac{1}{k_1}\tilde{d}(\dot{d} - \dot{\hat{d}}) + \tilde{w}(\ddot{\theta} - \dot{\hat{w}})$$

$$\dot{V}_1 = \frac{1}{k_1}\tilde{d}\dot{d} - \frac{1}{k_1}\tilde{d}\dot{\hat{d}} + \tilde{w}(\ddot{\theta} - (-\dot{\hat{d}} + au - k_2(\hat{w} - \dot{\theta}) - b\dot{\theta}))$$
$$= \frac{1}{k_1}\tilde{d}\dot{d} - \frac{1}{k_1}\tilde{d}k_1(\hat{w} - \dot{\theta}) + \tilde{w}(-b\dot{\theta} + au - d - (-\dot{\hat{d}} + au - k_2(\hat{w} - \dot{\theta}) - b\dot{\theta}))$$
$$= \frac{1}{k_1}\tilde{d}\dot{d} - \tilde{d}(\hat{w} - \dot{\theta}) + \tilde{w}(-d + \dot{\hat{d}} + k_2(\hat{w} - \dot{\theta}))$$
$$= \frac{1}{k_1}\tilde{d}\dot{d} + \tilde{d}\tilde{w} + \tilde{w}(-\tilde{d} - k_2\tilde{w})$$
$$= \frac{1}{k_1}\tilde{d}\dot{d} - k_2\tilde{w}^2 \leq 0$$

We take d to be a slowly time varying signal, and d has bounds. When k_1 is relatively large, we obtain $\frac{1}{k_1}\dot{d} \approx 0$. Concurrently, k_2 is also relatively large, and thus,

$$\dot{V}_1 = \frac{1}{k_1}\tilde{d}\dot{d} - k_2\tilde{w}^2 \leq 0 \quad (7)$$

C. Controller Design

Let the desired position input be θ_d, and the error $e = \theta_d - \theta$. The sliding variable is chosen as:

$$s = \dot{e} + ce \quad (8)$$

where c must satisfy the Hurwitz condition c>0.

On differentiating equation (8) and substituting equation (3), we get

$$\dot{s} = \ddot{e} + c\dot{e} = \ddot{\theta}_d - \ddot{\theta} + c\dot{e} = \ddot{\theta}_d + b\dot{\theta} - au + d + c\dot{e} \quad (9)$$

Making use of exponential rate reaching law and equating equation (9) to zero, we get the control input as

$$u = \frac{1}{a}[\ddot{\theta}_d + b\dot{\theta} + c\dot{e} + \hat{d} + ks + \eta sgn(s)] \quad (10)$$

D. Analysis of stability of controller

Taking the Lyapunov function as

$$V_2 = \frac{1}{2}s^2 \quad (11)$$

On differentiating equation (4.13) and substituting the equations (4.11) and (4.12) in it, we get

$$\dot{V}_2 = s\dot{s} = s(\ddot{\theta}_d + b\dot{\theta} - au + d + c\dot{e})$$
$$= s(\ddot{\theta}_d + b\dot{\theta} - \left(\ddot{\theta}_d + b\dot{\theta} + c\dot{e} + \hat{d} + ks + \eta sgn(s)\right) + d + c\dot{e})$$
$$= s\left(d - \hat{d} - ks - \eta sgn(s)\right)$$
$$= \tilde{d}s - ks^2 - \eta|s| \leq 0$$

Lyapunov function for the entire closed system can be given as

$$V = V_1 + V_2 = \frac{1}{2k_1}\tilde{d}^2 + k_2\tilde{w}^2 + \frac{1}{2}s^2 \leq 0 \quad (12)$$

Hence, the stability has been proved.

IV. Simulation and Results

The desired trajectory is taken as $\theta_d = sint$. The parameters of the servo system have been given in Table 1. The initial state is [π/6, 0]. It is assumed that the disturbance is given as $d = 8.5 + 0.5\sin(2\pi t)$. Tracking of desired trajectory by disturbance observer based SMC has been compared with basic SMC. The controller and observer parameters used have been described in Table 2.

Table 1 Parameters of Servo System

Parameters	Symbols	Values
Moment of inertia	J	5.77 x 10^{-2} Nms^2
Damping coefficient	B	8.8 x 10^{-3} Nms/rad
Torque constant	k_t	0.667 Nm/A
External load disturbance	T_d	0.5, 4<t<6 0.5+0.02sin4t, t>6

Table 2 Controller and Observer Parameters for Servo System

Controller	Parameters
SMC	$c = 20, \eta = 8.501, k = 10$
DOSMC	$c = 20, \eta = 8.501, k = 10$ $k_1 = 15000,\ k_2 = 50$

Fig. 4 shows the ideal position signal being tracked. There is only a slight difference in the time taken by both, the basic SMC and DOSMC, to reach the desired trajectory. This can be pointed out in a better way if we look at the plot for tracking error in Fig. 5 and 6. With DOSMC, the error becomes zero at 0.45s while with basic SMC it takes 0.65s. Fig. 7 shows how the observer is successfully able to estimate the disturbance. The control input to the plant has been shown in Fig. 8. It can be seen how the chattering has reduced with the application of disturbance observer.

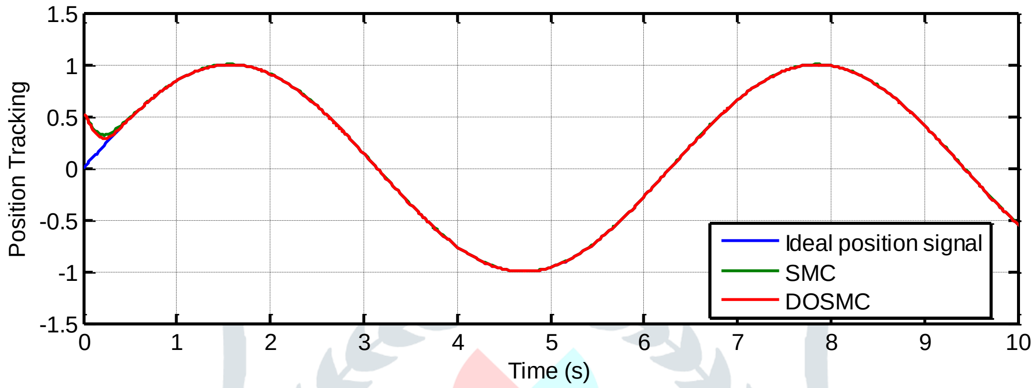

Fig. 4 Position tracking of state x1 vs time

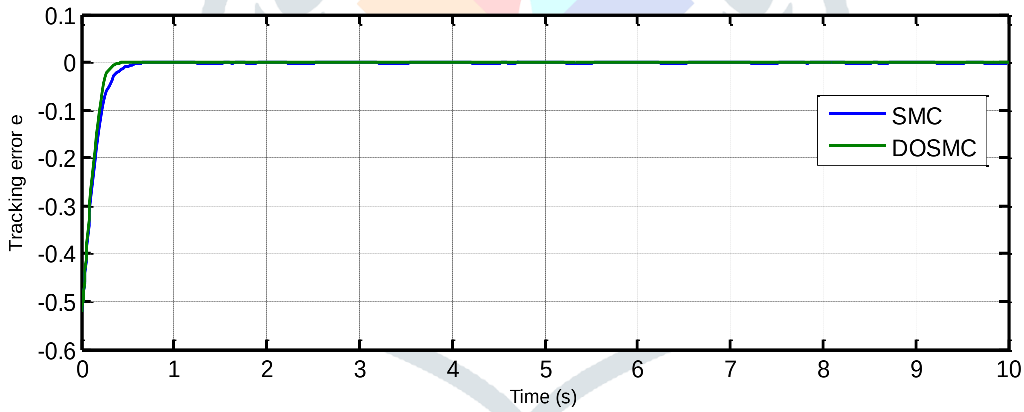

Fig. 5 Tracking error vs time

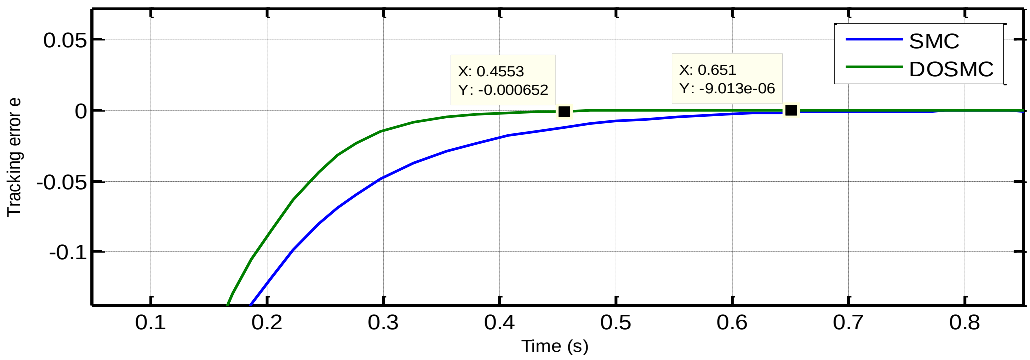

Fig. 6 Tracking error vs time (zoomed in)

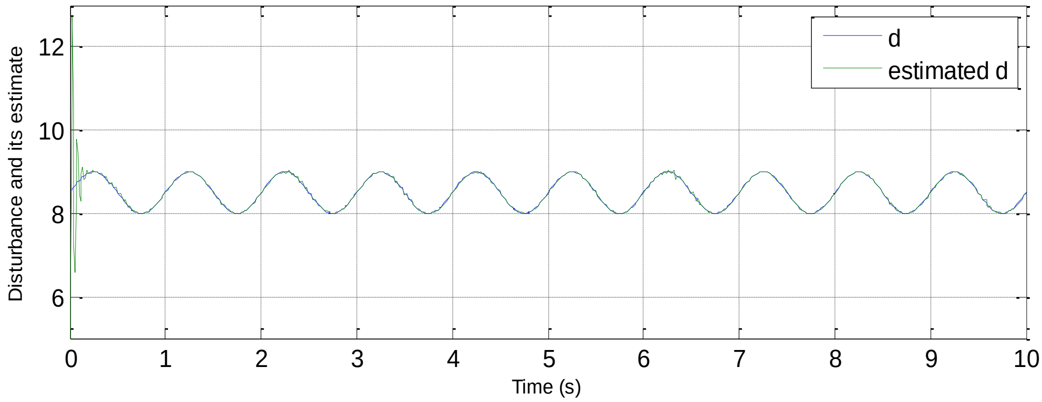

Fig. 7 Disturbance and its estimate vs time

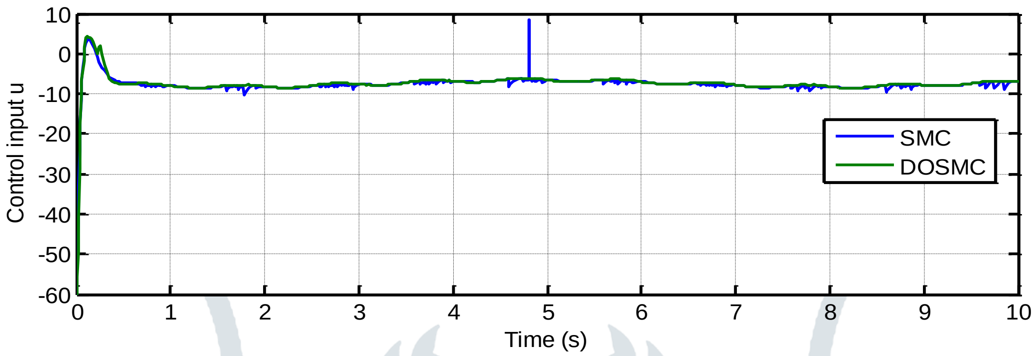

Fig. 8 Control input u vs time

V. Conclusion

Control problem of servo system subject to external disturbances has been solved using a disturbance observer. A disturbance observer based SMC technique gives an accurate estimate of the external disturbance added to the system and demonstrates reduction in chattering. The results have been compared with basic SMC. The Lyapunov stability criterion has been used to verify the stability of the system. This method is better able to deal with mismatched disturbance and has improved transient state performance with faster response time and lower overshoot along with less chattering effect.

References

[1] Lining Tan , Guodong Jin, Changjiang Sun, Zhiguo Xiong, 2018, High-order disturbance observer for nonlinear systems using sliding-mode technology, *Chinese Control And Decision Conference (CCDC)*, https://doi.org/10.1109/CCDC.2018.8407343

[2] M. Aydin, R. Coban, 2017, Second-order sliding mode control design and experimental application to a servo motor, *International Conference Mechanical System and Control Engineering*, pp. 141-144.

[3] S. H. Li, J. Yang, W.-H. Chen, X. S. Chen, 2014, Disturbance Observer Based Control: Methods and Applications, *Boca Raton, FL, USA:CRC Press*

[4] Bartoszewicz, A (ed.), 2005, Robust control, theory and applications, *InTech, 2011. Rijeka, Croatia*

[5] A. Kawamura, H. Itoh, K. Sakamoto, 1994, Chattering reduction of disturbance observer based sliding mode control, *IEEE Trans. Ind. Applications.*, vol. 30, pp. 456-461.

[6] W. Wu, Z. M. Bai, 2010, Sliding mode control based on disturbance observer for servo system, *Proc. 2nd Int. Conf. Comput. Autom. Eng.*, pp. 26-29, 2010-Feb.-26–28.

[7] Liu J K, Er L J. , 2003, Sliding Mode Controller Design for Position and Speed Control of Flight Simulator Servo System with Large Friction, *Systems Engineering and Electronics*,14(3): pp.59-62.

[8] Utkin V I., 1997, Variable structure systems with sliding modes, *IEEE Transactions Automatic control*, 22(2):pp.212-222.

[9] Liu, J.K., Wang, X.H., 2012, Advanced Sliding Mode Control for Mechanical Systems: Design, Analysis and MATLAB Simulation, *Springer, Tsinghua University Press, Berlin, Beijing*

[10] Liu J K,MATLAB Simulation for Sliding Mode Control,2005,10, pp.237-279.