Some properties of gR_1 and pairwise gR_1 Spaces

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Abstract

In this paper, we introduce gR_1 spaces and the bitopological analogue of gR_1 axiom by naming as pairwise gR_1 spaces.
Key words: gR_1 axiom, g-cl, g-ker.

1. *Introduction*

In 1963, N. Levine [5] offered a new notion to the field of general topology by introducing semi-open sets and defined this notion by utilizing the known notion of closure of an open set. By using semi-open sets semi-R_1 introduced by C. Dorsett [1]. In this paper we study and introducing the semi-R_1 separation axiom of topological spaces in terms of g-open sets called gR_1-axiom. Also by replacing open sets by g-open sets in pairwise R_1axioms of Murdeshwar and Naimpally [8], we introduced pairwise gR_1 axiom.

2. *Preliminaries*

A subset A of X is said to be g-closed [6] if cl(A) $\subset$ U whenever A $\subset$ U and U is open in (X, T). Clearly every closed set is g-closed. Complement of g-closed is called g-open. A set U is said to be g-neighbourhood of point x $\in$ X if x $\in$ U and U is g-open [7]. The family of all g-closed sets in a space (X, T) is denoted by GC (X, T). The g closure of a subset A in a space X, denoted by gcl A [6] is defined as the intersection of all g-closed sets that contain A. A space X is said to be a P_g-space [3] if x $\notin$ cl{y} implies that y $\notin$ gcl{x}. A space X is said to be a gR_0-space [4] if x $\notin$ gcl{y} implies that y $\notin$ gcl{x}. A bitopological space (X, T_1, T_2) is said to be pairwise P_g-space [2] if x $\notin$ T_i-cl{y} $\Rightarrow$ y $\notin$ T_j-gcl{x},

where i, j $\in$ {1, 2} and i $\neq$ j. A bitopological space (X, T_1, T_2) is said to be pairwise gR_0-space [4] if x $\notin$ T_i-gcl{y} $\Rightarrow$ y $\notin$ T_j-gcl{x}. A space X is said to be g_1 [9] if for any two distinct points x and y of X, there exists a g-open set U containing x but not y and a g-open set V containing y but not x. A bitopological space (X, T_1, T_2) is said to be pairwise g_1 [9] if for each pair of distinct points x, y of X, there is a T_i -g-open set U containing x but not y and a T_j-g-open V containing y but not x. A bitopological space X is pairwise R_0 [8] if for each G $\in$ T_i, x $\in$ G implies T_j-cl ({x}) $\subset$ G.

3. *gR_1 Spaces*

Definition 3.1: A space (X, T) is said to be gR_1 if for every pair of distinct points x, y of X, with gcl{x} $\neq$ gcl{y} there exists a g-open set U and a g-open set V such that x $\in$ U, y $\in$ V and U $\cap$ V =ϕ.

Example 3.2: Let X = {a, b, c}, T = {ϕ, {a}, {b, c}, X}.

GC (X, T) = {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, X}.
Clearly (X, T) is a gR_1 space. But it is not R_1.

Theorem 3.3: Every g_2-space is gR_1-space.

Proof: By definition, (X, T) is said to be g_2 if for each pair of distinct points x, y of X, there is a g-open set U and a g-open V such that x $\in$ U and y $\in$ V and U $\cap$ V =ϕ. Therefore y $\notin$ gcl{x} and x $\notin$ gcl{y}. Hence (X, T) is gR_1.

Theorem 3.4: Every g_1 and gR_1-space is g_2.

Proof: Let (X, T) be g_1 and gR_1-space. Let x, y be two distinct points of X. Since (X, T) is g_1 therefore gcl{x} $\neq$ gcl{y}. Since (X, T) be gR_1, there exists a g-open set U and a g-open set V such that x $\in$ U, y $\in$ V and U $\cap$ V = ϕ. Hence (X, T) is g_2.

Theorem 3.5: Every gR_1-space is gR_0-space.

Proof: Let (X, T) be gR_1-space. Let G be any g-open set and $x \in G$. For each $y \in X - G$, $gcl\{x\} \neq gcl\{y\}$. Therefore there exists a g-open set U_y and a g-open set V_y such that $x \in U_y$, $y \in V_y$ and $U_y \cap V_y = \phi$. If $A = \{V_y : y \in X - G\}$, then $X - G \subseteq A$ and $x \notin A$. g-openness of A implies $gcl\{x\} \subseteq X - A \subseteq G$. Hence (X, T) is gR_0.

Theorem 3.6: A space (X, T) is gR_1 if and only if for every pair of distinct points x, y of X such that $gcl\{x\} \neq gcl\{y\}$, there exists a g-open set U and a g-open V such that $gcl\{x\} \subseteq V$, $gcl\{x\} \subseteq U$ and $U \cap V = \phi$.

Proof: Let (X, T) be gR_1-space. Let x, y be two distinct points of X such that $gcl\{x\} \neq gcl\{y\}$, then there exists a g-open set U and a g-open set V such that $x \in V$, $y \in U$ and $U \cap V = \phi$. Since a gR_1-space is gR_0, therefore $x \in V$ implies $gcl\{x\} \subseteq V$ and $y \in U$ implies $gcl\{y\} \subseteq U$. Hence the result follows. The converse is obvious.

4. *Pairwise gR_1Spaces*

Definition 4.1: A space (X, T_1, T_2) is said to be pairwise gR_1 if for every pair of distinct points x, y of X, with T_i-$gcl\{x\} \neq T_j$-$gcl\{y\}$ there exists a T_j-g-open set U and a T_i -g-open set V such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Example 4.2: Let X = {a, b, c}, $T_1 = \{\phi, \{a\}, \{b, c\}, X\}$, $T_2 = \{\phi, \{b\}, \{a, c\}, X\}$.

$GC(X, T_1) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$,
$GC(X, T_2) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$,
Clearly (X, T_1, T_2) is a pairwise gR_1 space. But it is not pairwise R_1.

Theorem 4.3: Every pairwise g_2-space is pairwise gR_1-space.

Proof: By definition, (X, T_1, T_2) is said to be pairwise g_2 if for each pair of distinct points x, y of X, there is a T_i -g-open set U and a T_j-g-open V such that $x \in U$ and $y \in V$ and $U \cap V = \phi$. Therefore $y \notin T_i$-$gcl\{x\}$ and $x \notin T_j$-$gcl\{y\}$. Hence (X, T_1, T_2) is pairwise gR_1.

Theorem 4.4: Every pairwise g_1 and pairwise gR_1-space is pairwise g_2.

Proof: Let (X, T_1, T_2) be pairwise g_1 and pairwise gR_1. Let x, y be two distinct points of X. Since (X, T_1, T_2) is pairwise g_1 therefore T_i-$gcl\{x\} \neq T_j$-$gcl\{y\}$ [Sharma]. Since (X, T_1, T_2) be pairwise gR_1, there exists a T_j-g-open set U and a T_i -g-open set V such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Hence (X, T_1, T_2) is pairwise g_2.

Theorem 4.5: Every pairwise gR_1-space is pairwise gR_0-space.

Proof: Let (X, T_1, T_2) be pairwise gR_1-space. Let G be any T_i-g-open set and $x \in G$. For each $y \in X - G$, T_j-$gcl\{x\} \neq T_i$-$gcl\{y\}$. Therefore there exists a T_i-g-open set U_y and a T_j -g-open set V_y such that $x \in U_y$, $y \in V_y$ and $U_y \cap V_y = \phi$. If $A = \{ V_y : y \in X - G\}$, then $X - G \subseteq A$ and $x \notin A$. T_j-g-openness of A implies T_j-$gcl\{x\} \subseteq X - A \subseteq G$. Hence (X, T_1, T_2) is pairwise gR_0.

Theorem 4.6: A space (X, T_1, T_2) is pairwise gR_1 if and only if for every pair of distinct points x, y of X such that T_i-$gcl\{x\} \neq T_j$-$gcl\{y\}$, there exists a T_i-g-open set U and a T_j-g-open V such that T_i-$gcl\{x\} \subseteq V$, T_j-$gcl\{x\} \subseteq U$ and $U \cap V = \phi$.

Proof: Let (X, T_1, T_2) be pairwise gR_1-space. Let x, y be two distinct points of X such that T_i-$gcl\{x\} \neq T_j$-$gcl\{y\}$, then there exists a T_i-g-open set U and a T_j-g-open set V such that $x \in V$, $y \in U$ and $U \cap V = \phi$. Since a pairwise gR_1-space is pairwise gR_0, therefore $x \in V$ implies T_i-$gcl\{x\} \subseteq V$ and $y \in U$ implies T_j-$gcl\{y\} \subseteq U$. Hence the result follows. The converse is obvious.

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