# APPLICATIONS OF DIOPHANTINE EQUATIONS IN CHEMICAL EQUATIONS

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**Abstract:** Linear Diophantine equations is used in all the fields especially in chemistry.It is used to solve the chemical equations.Some examples are illustrated in this paper in a detailed manner.

**Keywords** - Diophantine equations, Chemical equations, input, output, reactants, products.

## 1. INTRODUCTION

Number theory is a broad subject with many strong connections with other branches of mathematics. Number theory is the purest branch of pure mathematics and from the attendant suspicion that it can have few substantive applications to real world problems. The importance of number theory derives from its central position in mathematics; its concepts and problems have been instrumental in the creation of large parts of mathematics.

## 2. DIOPHANTINE EQUATIONS

The term Diophantine equation to any equation in one or more unknowns that is to be solved in the integers. The simplest type of Diophantine equation is the linear Diophantine equation in two unknowns:

$$ax+by=c,$$

where a,b,c are given integers.

## 3. CHEMICAL EQUATIONS

We can reduce all chemical compounds to a combination of chemical elements in a special way. We can also reduce all integers to a product of prime factors in a particular way. Both the reductions work like building blocks which help to construct more complicated objects.

## 4. BALANCING CHEMICAL EQUATIONS

A chemical equation is nothing but a chemical reaction. When the left side shows the input, the right side shows the output from the inputs.

Consider a chemical equation in the form

$$uX_{x1}Y_{y1}Z_{z1}\ldots+vX_{x2}Y_{y2}Z_{z2}\ldots+wX_{x3}Y_{y3}Z_{z3}\ldots+\ldots\rightarrow u'X_{x'1}Y_{y'1}Z_{z'1}\ldots+v'X_{x'2}Y_{y'2}Z_{z'2}\ldots+w'X_{x'3}Y_{y'3}Z_{z'3}\ldots+\ldots \quad (1)$$

where X,Y,Z are the elements in the reaction, $x_1,y_1,z_1$ ,… $x'_1,y'_1,z'_1$ ,… are positive integers or zero, and u,v,w,…, u',v',w',… are the unknown coefficients of the reactants and products.

From equation 1, we have

$$\left.\begin{aligned} ux_1+vx_2+wx_3+\ldots &= u'x'_1+v'x'_2+w'x'_3+\ldots \\ uy_1+vy_2+wy_3+\ldots &= u'y'_1+v'y'_2+w'y'_3+\ldots \\ uz_1+vz_2+wz_3+\ldots &= u'z'_1+v'z'_2+w'z'_3+\ldots \end{aligned}\right\} \quad (2)$$

Equation 2 represent a elementary problem of Diophantine equation to find all integer solutions [u,v,w,…u',v',w',…].

Example 1:

Consider the chemical equation

$$uK_2Cr_2O_7+vFeSO_4+wH_2SO_4\rightarrow u'H_2O+v'Cr_2(SO_4)_3+w'Fe_2(SO_4)_3+t'K_2SO_4 \quad (3)$$

From equation 3, we have

$$\left.\begin{array}{ll} 2u=2v' & \text{for Cr} \\ 7u+4v+4w= u'+12v'+12w'+4t' & \text{for O} \\ v=2w' & \text{for Fe} \\ v+w=3v'+3w'+t' & \text{for S} \\ 2w=2u' & \text{for H} \\ 2u=2z' & \text{for K} \end{array}\right\} \quad (4)$$

The system of equations 4 reduced to

$$6u=v$$

This is a linear Diophantine equation in two unknowns with a solution [u,v]=[1,6].
Hence [u,v,w,u',v',w',t'] = [1,6,7,7,1,3,1].
Equation 3 becomes

$$K_2Cr_2O_7+6FeSO_4+7H_2SO_4\rightarrow 7H_2O+Cr_2(SO_4)_3+3Fe_2(SO_4)_3+K_2SO_4$$

Example 2:

Consider the chemical equation

$$uKMnO_4+vH_2SO_4+wHNO_2\rightarrow u'MnSO_4+v'Mn(NO_3)_2+w'H_2O+t'KNO_3 \quad (5)$$

From equation 5, we have

$$\left.\begin{array}{ll} u=t' & \text{for K} \\ u=u'+v' & \text{for Mn} \\ 4u+4v+4w=4u'+6v'+w'+3t' & \text{for O} \\ 2v+w=2w' & \text{for H} \\ v=u' & \text{for S} \\ w=2v'+t' & \text{for N} \end{array}\right\} \quad (6)$$

The system of equations 6 reduced to

$$10v - w = 0$$

This is a linear Diophantine equation in two unknowns with a solution [v,w]=[1,10].
Hence [u,v,w,u',v',w',t']=[4,1,10,1,3,6,4].
Equation 5 becomes

$$4KMnO_4+H_2SO_4+10HNO_2\rightarrow MnSO_4+3Mn(NO_3)_2+6H_2O+4KNO_3$$

Example 3:

Consider the chemical equation

$$uMnO_2+vCl_2+wKOH\rightarrow u'KMnO_4+v'KCl+w'H_2O \quad (7)$$

From equation 7, we have

$$\left.\begin{array}{ll} u=u' & \text{for Mn} \\ 2u+w=4u'+w' & \text{for O} \\ 2v=v' & \text{for Cl} \\ w=u'+v' & \text{for K} \\ w=2w' & \text{for H} \end{array}\right\} \quad (8)$$

The system of equations 8 reduced to

$$2v - 3u = 0$$

This is a linear Diophantine equation in two unknowns with a solution [u,v]=[2,3].
Hence [u,v,w,u',v',w'] = [2,3,8,2,6,4,].
Equation 7 becomes

$$2MnO_2+3Cl_2+8KOH\rightarrow 2KMnO_4+6KCl+4H_2O$$

Example 4:

Consider the chemical equation

$$uCu+vHNO3 \rightarrow u'Cu(NO_3)_2+v'NO+w'H_2O \quad (9)$$

From equation 9, we have

$$\left.\begin{array}{ll} u=u' & \text{for Cu} \\ 3v=6u'+v'+w' & \text{for O} \\ v=2u'+v' & \text{for N} \end{array}\right\} \quad (10)$$

$$v=2w' \quad \text{for H}$$

The system of equations 10 reduced to

$$3v' - 2u = 0$$

This is a linear Diophantine equation in two unknowns with a solution [u,v']=[3,2].

Hence [u,v,u',v',w'] = [3,8,3,2,4].

Equation 9 becomes

$$3Cu+8HNO3 \rightarrow 3Cu(NO_3)_2+2NO+4H_2O$$

## 5. CONCLUSION

The role of Linear Diophantine equations plays a vital role in chemistry as it is fully based on equations. Because most of the chemical equations will be ended in Linear Diophantine equations.

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