Three Dimensional Geometry

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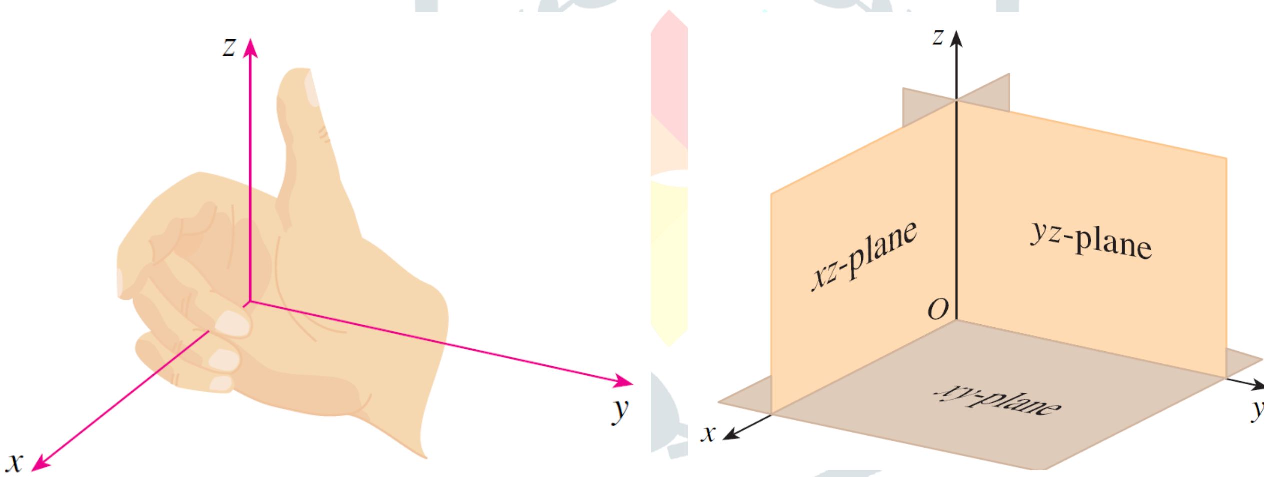
ABSTRACT: -

We have studied co-ordinate geometry in two dimensions in which the position of a point is determined by the perpendicular distance from the point to two intersecting perpendicular lines, known as the co-ordinate axes. In actual life, we do not have to deal with points lying in a plane only. For example, suppose, if we have to locate the position of the lowest tip of a ceiling fan hanging from the ceiling of a room. Here we will not only require the perpendicular distances of the point to be located from two perpendicular walls of the room, but also the height of the lowest point from the floor of the room. Therefore, we require not two but three numbers representing the perpendicular distances of the points from three mutually perpendicular planes, namely the floor of the room and the two adjacent walls of the room. The three numbers representing the three distances are called the coordinates of the point with reference to the three co-ordinate planes. Thus, to locate a point in space, we require three dimensions. The geometry which defines the position of a point in space by three numbers x, y and z is called three dimensional geometries, which we shall be studying in this paper. Three-dimensional space (also: 3-space) is a geometric setting in which three values (called parameters) are required to determine the position of an element (i.e., point). This is the informal meaning of the term dimension. An object that has height, width and depth, like any object in the real world. For example: your body is three-dimensional. It is also known as "3D". In this topic three dimensional geometry of space (3-D space) is explained in details. This is a very important topic in mathematics. We are looking at the equations of graphs in 3-D space as well as vector valued functions and how we do calculus with them. In the 3-D Coordinate system. In this topic I have discussed about the concepts and notation for the three dimensional coordinate system, equations of lines and develop the various forms for the equation of lines in three dimensional space. In the equations of planes, the concept of equation of a plane is also discussed in detail with 3D dimensional figures. I have discussed in vector functions, the

concept of vector functions, Types of vector (particular unit vector), position vector of a Point, distance of a point P (x, y, z) from origin and are explained with the help of diagrams.

INTRODUCTION: -

Def^n:- CO-ORDINATE AXES AND CO-ORDINATE PLANES - Let XOX', YOY' and ZOZ' be three mutually perpendicular lines intersecting at O. The point O is called the origin and the lines XOX', YOY' and ZOZ' are called co-ordinate axes. These axes determine three mutually perpendicular planes viz., XOY, YOZ and ZOX called the coordinate planes. XOX' is called x-axis, YOY' is called y-axis and ZOZ' is called z-axis. It may be noted that the distances measured along OX, OY, OZ are taken as positive while distances measured along OX', OY', OZ' are considered as negative. The plane XOY is called xy-plane, the plane YOZ is called yz-plane and the plane ZOX is called zx-plane. These planes divide the whole space into eight parts called octants.



Def^n - CO-ORDINATES OF A POINT IN SPACE - Let OX, OY, OZ be the co-ordinate axes and let P be any point in space. Through P, draw planes parallel to the co-ordinate planes meeting x-axis at A, y-axis at B and a-axis at C respectively. Complete the parallelepiped whose coterminous edges are OA, OB and OC. Let the distances OA, OB, OC be x, y, z: These are known as co-ordinates of the point P. Thus to every point in space, we get an ordered triplet of numbers associated with it. The co-ordinates of P are written as (x, y, z), OA= x is the x co-ordinate, OB = y is the y co-ordinate and OC = z is the z co-ordinate of P.

Def^n - BASE VECTORS AND POSITION VECTORS:-The three unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ in the direction of co-ordinate axes OX, OY, OZ respectively are called the Cartesian base vectors. Let P be any point in space with co-ordinates (x, y, z) and the position vector $\vec{r}$**.** Then

$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

Def^n - CO-ORDINATES OF A POINT IN TERMS OF DIRECTION COSINES :- If the direction cosines of the line OP are l,m, n and OP = r, where O is the origin,then the co-ordinates of P are (l r, m r, n r)

Def^n - RELATION BETWEEN DIRECTION COSINES OF A LINE :- If l, m, n are the direction cosines of any line, then $l^2 + m^2 + n^2 = 1$.

Def^n - DIRECTION COSINES OF A VECTOR

Let P (x, y, z) be a point in space whose position vector is $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ **.**If $\vec{r}$ makes angles α ,β ,γ with the positive direction of co-ordinate axes x, y, z respectively, then the angles α ,β ,γ are called the direction angles of $\vec{r}$ and cosines of direction angles i.e., cos α, cos β, cos γ are called direction cosines. These are usually denoted by l, m, n

i.e. l = cos α, m = cosβ, n = cosγ.

$$P_1 P_2 = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$$

----References----

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