# Study of $C_k$-Convex Function

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## 1. Abstract:-

We define $C_k$-Convex Function of a C- set in a linear Space and established some important theorems.

**2. Introduction:-** In this paper we use the notions of convex set and convex function in a real or complex linear space as given in Doneford and Schwarz [1] Rayden [2] and Rudin [3] in the analogy to the above notions. we define
$C_k$- Convex function and established some results in connection with this definition.

## 3.Definition.

A real Valued function f: R$\longrightarrow R$ is a $C_k$-Convex function ($k \geq 0$) if

$$K + f(x+a) - f(x) - \frac{a}{b}\,[f(x) - f(x-b)] \geq 0$$

For all $x$ in R and all positive reals a,b.

**Theorem (3.1)** : If f is $C_k$ -Convex then f is $C_m$ –Convex for all m$\geq k$.

Proof:- Since f is $C_k$ –Convex,

$K + f(x+a) - f(x) - \frac{a}{b}\,[f(x) - f(x-b)] \geq 0,$

For all real $x$ and and all positive numbers a,b.

Hence m$\geq k$ implies

$m + f(x+a) - f(x) - \frac{a}{b}\,[f(x) - f(x-b)] \geq 0,$ Thus f is $C_m$ –Convex.

**Theorem(3.2) :** If f and g are $C_k$ –Convex and $C_m$ –Convex respectively and if $\alpha, \beta > 0$ $then\ \alpha f + \beta g$

is $C_{\alpha k+\beta m}$-Convex.

In particular, the set of all $C_k$ –Convex functions is convex.

**Proof:** Since f is $C_k$ –Convex,

$K + f(x+a) - f(x) - \frac{a}{b}\,[f(x) - f(x-b)] \geq 0,$

For all real $x$ in R and all positive reals a,b.

Since $\alpha > 0,$

$\alpha K + (\alpha f)(x+a) - (\alpha f)\,(x) - \frac{a}{b}\,[(\alpha f)(x) - (\alpha f)(x-b)] \geq 0$ --------(3.21)

For all $x$ in R and all positive reals a,b. Also g is $C_m$ –Convex,

Hence

$m + g(x+a) - g(x) - \frac{a}{b}\,[g(x) - g(x-b)] \geq 0$

Where a , b are positive reals and all $x \in R$.

Hence $\beta > 0$ implies

$\beta m + (\beta g)(x+a) - (\beta g)(x) - \frac{a}{b}\,[(\beta g)(x) - (\beta g)(x-b)] \geq 0$ ………(3.22)

With x, a, b as before .

Adding the inequalities (3.21) and (3.22),

$$\alpha k + \beta m + (\alpha f + \beta g)(x+a) - (\alpha f + \beta g)(x) - \frac{a}{b}\,[(\alpha f + \beta g)(x) - (\alpha f + \beta g)(x-b)] \geq 0$$

For all x in R and all posiive reals a, b.

But this shows that $\alpha f + \beta g$ is $C_{\alpha k + \beta m}$-Convex.

As a Particulars case let g be $C_k$ –Convex i.e. m = k

Let $\alpha + \beta = 1, \quad then$

$\alpha k + \beta m = \alpha k + \beta k = (\alpha + \beta)k = k$

Thus $\alpha f + \beta g$ is also $C_k$ –Convex. Hence in particular , the set of all $C_k$ –Convex function is convex.

**Theorem (2.3) :** If f is $C_k$ –Convex then a function g is given by

$$g(x) \equiv f(x-z)$$

is also $C_k$ –Convex for all z in R.

**Proof:** Let z be a fixed element in R and x be any element of R.

Since f is $C_k$ –Convex ,

$$k + f(x+a) - f(x) - \frac{a}{b}\,[\,f(x) - f(x-b)] \geq 0$$

For all positive reals a,b.

Replacing x by x - z in the above inequality.

$$k + f(x-z+a) - f(x-z) - \frac{a}{b}\,[\,f(x-z) - f(x-z-b)] \geq 0$$

or, $k + f(x+a-z) - f(x-z) - \frac{a}{b}\,[\,f(x-z) - f(x-b-z)] \geq 0$

or, $k + g(x+a) - g(x) - \frac{a}{b}\,[\,g(x) - g(x-b)] \geq 0$

For all $x$ in R and all positive reals a, b.

Hence g is also $C_k$ –Convex for all z in R.

## Reference :

[1]. Dunford, N. and Schwartz, J. “Linear Operators Part –I” Inter Science Publishers, Inc., New York,1967.

[2]. Royden,H.L. “Real Analysis,” Macmillan Company, NewYork, 1964.

[3]. Rudin, W. “Functional Analysis ,” Mc. Graw Hill Book Company,Inc.New York,1973.