An FM /FE$_k$/C Queueing Model With k-Phases Using DSW Algorithm

S.Shanmugasundaram* and K. Thamilselvi**

*Assistant Professor, Department of Mathematics, Government Arts College, Salem-7, TamilNadu, India.

**Assistant Statistical Investigator, District Rural Development Agency, Karur-639007, TamilNadu , India.

ABSTRACT

An FM/FE$_k$/C queueing model whose arrivals are Fuzzy Markovian and services are Fuzzy Erlangian , multi server with k-phases. The membership functions and performance measures of the queueing model are defined by applying DSW(Dong,Shah &Wong) algorithm, based on α-cut principle in Fuzzy environment. The numerical examples are given to test the feasibility of the model.

KEYWORDS : Trapezoidal Fuzzy numbers – α-cut – DSW algorithm – k phases.

1. INTRODUCTION

Queueing theory is originated in telephony with work of Erlang.A.K, [2]. In the recent decades many articles are published on queueing theory. Any queueing model is characterized by situations where both arrivals and departures take place simultaneously. In particular, the inter arrival times and service times are restricted to follow specific probability distributions. An Effective method has been developed to analyse the queueing model. The Fuzzy queues are much more accurate than the normal crisp queues. The parameters Fuzzy arrival and Fuzzy erlang service rate are best described linguistic terms very high, high, low, very low and moderate. Fuzzy logic was initiated in 1965 by Zadeh[12]. Later many authors like Buckley J.J[1], Li R.J and Lee E.S[3] , Negi D.S and Lee E.S[5] are improved the Fuzzy logic. Timothy Rose [11] explained Fuzzy logic and its applications to engineering. Srinivasan.R,[7] dicussed DSW algorithm for the detail description of his Fuzzy queueing model. Shanmugasundaram.S and Venkadesh.B,[8] explained about Fuzzy multi server queueing model. Mohammed shapique.A,[4] analysed Fuzzy queue with erlang service model using DSW algorithm for single server. Sujatha. N, Murthy Akella. V.S.N and Deekshitulu G.V.S.R [10] studied Fuzzy queueing model using α-cuts with erlang service and also executed its performance measures.

2. PRELIMINARIES

2.1 Fuzzy Set:

Let X be a non empty set. A Fuzzy set F in X is characterized by its membership function

$\mu_F : X \to [0,1]$ and $\mu_F(x)$ is interpreted as the degree of membership of element x, in fuzzy set F for each $x \in X$. It is clear that F is completely determined by the set of tuples $F = \{ \mu_F(x) / x \in X\}$.

2.2 α- Cuts:

If a Fuzzy set is defined on for any $\alpha \in [0,1]$ the cuts is represented by the following crisp set.

Strong α-Cuts:

$A_F^* = \{ x \in X / \mu_A(x) > \alpha , \alpha \in [0,1] \}$

Weak α- Cuts:

$A_F = \{ x \in X / \mu_A(x) \geq \alpha , \alpha \in [0,1] \}$

2.3 Trapezoidal Fuzzy number:

A trapezoidal fuzzy number A* is a 4-tuple, its membership function is

$$\mu_{A^*}(z) = \begin{cases} L(z) & : z_1 \leq z \leq z_2 \\ 1 & : z_2 \leq z \leq z_3 \\ R(z) & : z_3 \leq z \leq z_4 \end{cases}$$

Where $z_1 \leq z_2 \leq z_3 \leq z_4$ and $L(z_1) = R(z_4) = 0$.An approximate method of extension is propagating fuzziness for continuous valued mapping determined the membership functions for the output variables.

3. INTERVAL ANALYSIS

Let I_1 and I_2 be two intervals defined by ordered pairs of real numbers with lower and upper bounds.

$I_1 = [a_1,a_2]$; $I_2 = [a_3,a_4]$, Define a general arithmetic property with the symbol *, where * = [+,-,×,÷] symbolocally. The operation $I_1 * I_2 = [a_1,a_2], * [a_3,a_4]$, represents another interval.

The interval calculation depends on the magnitudes and signs of the element a_1,a_2 , a_3,a_4.

$I_1+I_2 = [a_1+ a_3, a_2+ a_4]$, $I_1-I_2 = [a_1 - a_4, a_2 - a_3]$,

$I_1 \cdot I_2 = [\min (a_1 a_3, a_1 a_4 , a_2 a_3 , a_2 a_4),\max(a_1 a_3, a_1 a_4 , a_2 a_3 , a_2 a_4)]$

$I_1 \div I_2 = [a_1, a_2] \cdot [\frac{1}{a_4} , \frac{1}{a_3}]$ provided $a_4 , a_3 \neq 0$

$$\alpha[a_1 , a_2] = \begin{cases} [\alpha a_1, \alpha a_2] , \alpha > 0 \\ [\alpha a_2, \alpha a_1] , \alpha < 0 \end{cases}$$

4. DSW ALGORITHM

DSW (Dong, Shah and Wong) algorithm is one of the approximate methods to make use of intervals at various α – cut levels in defining membership functions. The DSW algorithm streamlines the manipulation of the extension principle for continuous valued Fuzzy variables, such a Fuzzy numbers defined on the real line. It avoids the irregularity in the output membership function due to application of the discrimination etching on the Fuzzy variable domain it can prevent the spreading of resulting functional expression by conventional interval analysis method. Any membership function which is continuous can be represented by continuous curve of α-cut N term from α = 0 to α =1. Suppose we have single input mapping given by y = f(x) that is to be extended for membership function for selected α – cut level.

The DSW algorithm consists of the following steps:

1. Choose the value of the α – cut in [0,1]
2. Find the intervals in the input membership functions that correspond to this α.

3. Using standard binary interval operations, calculate the interval for the output membership function for the selected α – cut level.
4. Continue the steps 1 -3 for various values of α – cut representation of the solution.

5. DESCRIPTION OF THE MODEL:

In this paper,we discussed about a multiple server queueing model with infinite capacity. Here both arrival and service rate are Fuzzy in nature. Arrival rate follows Poisson distribution and service rate follows Erlang distribution with k phases under First Come First Served discipline.

ie) (FM /FE$_k$ / C) : (FCFS/ ∞/∞/)

Here C means the number of service channels and a chain of ‘k’ similar stages gaining an average service time 1/sμ individually. Each customer to be served enters the system in first phase before proceeding to the second phase up to kth phase. The assumption is that a new service does not start until a customer completes all k-phases.

Let the number of arrivals and services per unit time be β and γ respectively.

1. Expected number of customers in the queue is

$$L_q = \frac{(k+1)\beta^2}{2kC\gamma(C\gamma-\beta)}$$

2. Expected waiting time of customer in the queue is

$$W_q = \frac{L_q}{\beta}$$

3. Expected time of a customer spends in the system is

$$W_s = W_q + \frac{1}{C\gamma}$$

4. Expected number of customers in the system is

$$L_s = \beta W_s$$

6. NUMERICAL EXAMPLE

Consider (FM/ FE $_k$ /C) queue which consists of 4 phases and 3 servers (k=4, C=3), both arrival and service rate are trapezoidal Fuzzy numbers represented by $\tilde{\beta} = [11, 12, 13, 14]$ $\tilde{\gamma} = [7, 8, 9, 10]$. The confidence interval for α is [11 + α , 14 – α] and [7 + α, 10 – α].Consider x_1 = [11 + α , 14 – α] , y_1 = [7 + α, 10 – α] and x_2 = [11 + α , 7 + α] , y_2 = [14 - α, 10 – α].

$L_q = \frac{(k+1)x^2}{2kcy(cy-x)}$, $W_q = \frac{L_q}{x}$, Ws = Wq + $\frac{1}{cy}$, Ls = xW$_s$

Table 1: The α – cuts of L_q , W_q , W_s & L_s at α values

A	L_q	W_q	W_s	L_s
0	[0.13267, 0.83333]	[0.01206, 0.05952]	[3.34539, 2.39287]	[36.7993 , 33.5]
0.1	[0.13939, 0.76612]	[0.01255, 0.05511]	[3.31255, 2.42178]	[36.76939, 33.66278]
0.2	[0.14652, 0.70646]	[0.01308, 0.05119]	[3.27974, 2.45119]	[36.73318, 33.8264]
0.3	[0.15407, 0.65322]	[0.01363, 0.04768]	[3.24696, 2.48101]	[36.69073, 33.98989]
0.4	[0.16208, 0.60548]	[0.01421, 0.04452]	[3.21421, 2.51118]	[36.64208, 34.15215]
0.5	[0.17060, 0.5625]	[0.01483, 0.04166]	[3.18150, 2.54166]	[36.58726, 34.3125]
0.6	[0.17965, 0.52363]	[0.01548, 0.03907]	[3.14882, 2.57241]	[36.52632, 34.4702]
0.7	[0.18929, 0.48836]	[0.01617, 0.03671]	[3.11617, 2.60338]	[36.45929, 34.62503]
0.8	[0.19956, 0.45625]	[0.01691, 0.03456]	[3.08357, 2.63456]	[36.38622, 34.77625]
0.9	[0.21051, 0.42694]	[0.01769, 0.03259]	[3.05102, 2.66592]	[36.30718, 34.92360]
1	[0.22222, 0.40009]	[0.01851, 0.03077]	[3.01851, 2.69744]	[36.22222, 35.06676]

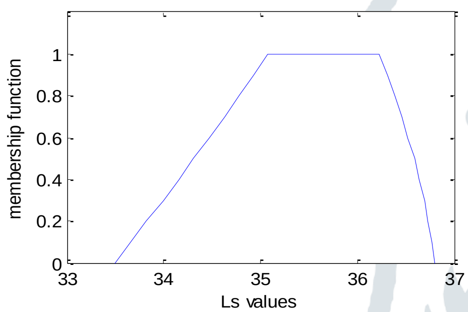

Fig1: Membership function of the queue length in system Ls

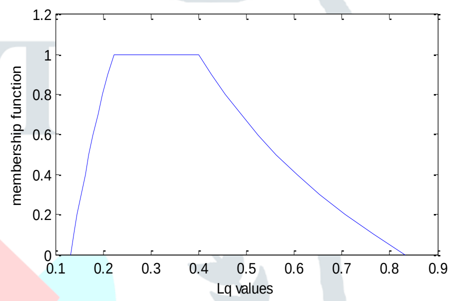

Fig2: Membership function of the expected length Lq

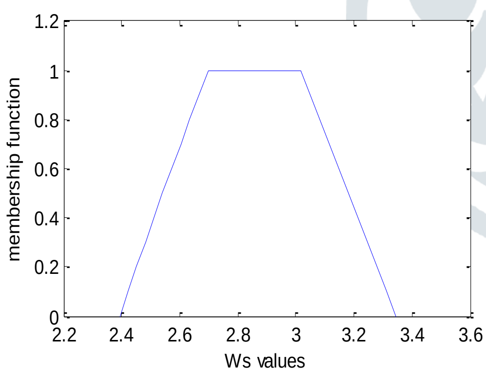

Fig3: Membership function of the waiting time Ws

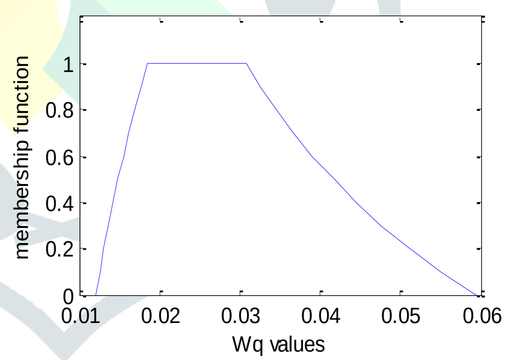

Fig4: Membership function of the waiting time Ws

Table 2: The α – cuts of L_q , W_q , W_s & L_s at α values

A	L_q	W_q	W_s	L_s
0	[0.36011, 0.25520]	[0.03273, 0.01822]	[2.36607, 3.35156]	[26.02678, 46.92187]
0.1	[0.35444, 0.25733]	[0.03193, 0.01851]	[2.39859, 3.31851]	[26.62444, 46.12733]
0.2	[0.34900, 0.25951]	[0.03116, 0.01880]	[2.43116, 3.28547]	[27.22900, 45.33951]
0.3	[0.34378, 0.26176]	[0.03042, 0.01910]	[2.46375, 3.25244]	[27.84045, 44.55842]
0.4	[0.33877, 0.26407]	[0.02971, 0.01941]	[2.49638, 3.21941]	[28.45877, 43.78407]
0.5	[0.33396, 0.26644]	[0.02904, 0.01973]	[2.52904, 3.18640]	[29.08396, 43.01644]
0.6	[0.32933, 0.26889]	[0.02839, 0.02006]	[2.56172, 3.15339]	[29.71600, 42.25555]
0.7	[0.32488, 0.27141]	[0.02776, 0.02040]	[2.59443, 3.12040]	[30.35488, 41.50141]
0.8	[0.32060, 0.27400]	[0.02716, 0.02075]	[2.62716, 3.08742]	[31.00060, 40.75400]

0.9	[0.3164, 0.27667]	[0.02659, 0.02112]	[2.65992, 3.05445]	[31.65314, 40.01334]
1	[0.3125, 0.27943]	[0.02604, 0.02149]	[2.69270, 3.02149]	[32.3125, 39.27943]

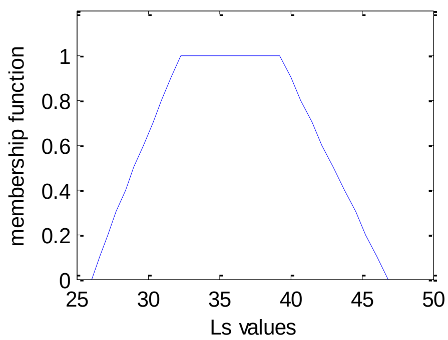

Fig1: Membership function of the queue length in system Ls

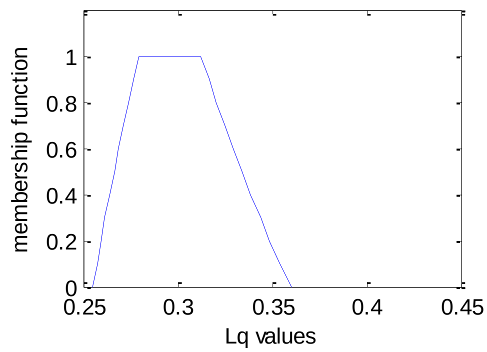

Fig2: Membership function of the expected length Lq

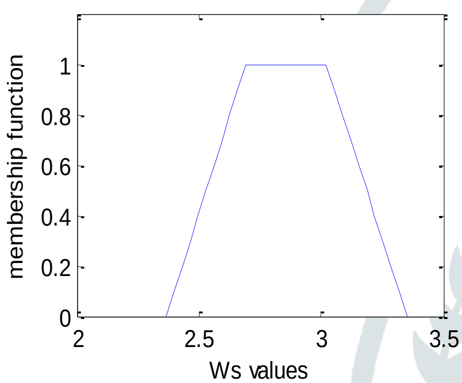

Fig3: Membership function of the waiting time Ws

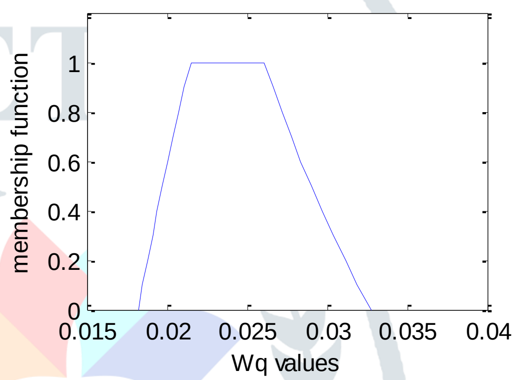

Fig4: Membership function of the waiting time Wq

With the help of MATLAB software we perform α-cuts of arrival rate, service rate and fuzzy expected number of jobs in queue at eleven distinct levels of α : 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for Fuzzy expected number of jobs in queue at different possibility α levels are presented in table1 and 2. Similarly the performance measure such as Ls, Lq ,Ws and Wq also derived in table1 and 2. The α– cut represents the possibility that these four performance measure will lie in the associated range. In particular, α = 0 the range, the performance measures could appear and for α =1 the performance measures are in the table1 while these four performance measures are Fuzzy, the most likely value of the expected queue length Lq falls between 0.22222 and 0.40009 (α=1) and its value is impossible to fall outside the range of 0.13267 and 0.83333 (α=0) also the expected length of the system Ls falls between 36.22222 and 35.06676 (α=1) and won't fall outside the range of [36.7993, 33.5](α=0).Similarly from table 2 , the most likely value of the expected queue length Lq falls between 0.3125 and 0.27943 (α=1) and its value is impossible to fall outside the range of 0.36011 and 0.25520 (α=0) also the expected length of the system Ls falls between 32.3125 and 39.27934 (α=1) and won't fall outside the range of [26.02678, 46.92187] (α=0). The above data will be very suitable for designing a queueing system.

7. CONCLUSION

In this paper we apply DSW algorithm, the performance measures of L_s , L_q , W_s & W_q in intervals $x_1 = [11 + \alpha , 14 - \alpha]$, $y_1 = [7 + \alpha, 10 - \alpha]$ and $x_2 = [11 + \alpha , 7 + \alpha]$, $y_2 = [14 - \alpha, 10 - \alpha]$ are analyzed. The numerical example shows efficiency of DSW algorithm in Trapezoidal Fuzzy numbers.

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