A Note on G(γ_{mtss}) of Certain Graphs.

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***Abstract*:** A total dominating set D of graph G = (V, E) is a total strong split dominating set if the induced sub graph < V−D > is totally disconnected with at least two vertices. The total strong split domination number γ_{tss}(G) is the minimum cardinality of a total strong split dominating set. In this paper we define the modified γ_{tss} −graph G(γ_{mtss}) = (V(γ_{mtss}), E(γ_{mtss})) of G to be the graph whose vertices V(γ_{mtss}) corresponds injectively with the γ_{tss} −sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in G(γ_{mtss}) if there exists a vertex v ∈ D_1 and w ∈ D_2 such that $D_1 = D_2 - \{w\} \cup \{v\}$ and $D_2 = D_1 - \{v\} \cup \{w\}$. Thus two γ_{tss} −sets are said to be adjacent if they differ by one vertex. We also determine G(γ_{mtss}) of certain graphs.

***Keywords* - Domination number, total strong split domination number, γ_{tss} − graph of a graph, γ_{mtss} − graph of a graph.**

I. INTRODUCTION

The graphs considered here are finite, undirected, without loops, multiple edges. For all graph theoretic terminology not defined here, the reader is referred to [2]. A set of vertices D in a graph G is a dominating set, if every vertex in V−D is adjacent to some vertex in D. The domination number γ(G) is the minimum cardinality of a dominating set. A total dominating set D of a connected graph G is a total split dominating set if the induced sub graph < V−D > is disconnected. The total split domination number γ_{ts}(G) is the minimum cardinality of a total split dominating set. This concept was introduced by B. Janakiram, Soner and Chaluvaraju in [3]. Strong split domination was introduced by V. R. Kulli and B. Janakiram in [4]. A dominating set D of a graph G = (V, E) is a strong split dominating set if the induced sub graph < V – D > is totally disconnected with at least two vertices. The strong split domination number γ_{ss} (G) is the minimum cardinality of a strong split dominating set. A total dominating set D of a connected graph G is a total strong split dominating set if the induced sub graph < V−D > is totally disconnected with at least two vertices. The total strong split domination number γ_{tss}(G) is the minimum cardinality of a total strong split dominating set. This concept was introduced by T. Sheeba Helen and T.Nicholas in [5]. Gerd H. Fricke et al. [1] introduced γ −graph of a graph. Consider the family of all γ-sets of a graph G and define the γ −graph G(γ) = (V(γ), E(γ)) of G to be the graph whose vertices V(γ) correspond 1−1 with the γ −sets of a graph G, and two γ −sets, say D_1 and D_2, form an edge in E(γ) if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$. With this definition, two γ −sets are said to be adjacent if they differ by one vertex, and the two vertices defining this difference are adjacent in G. T. Sheeba Helen and T.Nicholas in [6] introduced the concept γ_{tss} − graph of a graph G and defined the graph G(γ_{tss}) = (V(γ_{tss}), E(γ_{tss})) of G to be the graph whose vertices V(γ_{tss}) corresponds injectively with the γ_{tss} −sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in G(γ_{tss}) if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$. With this definition, two γ_{tss} −sets are said to be adjacent if they differ by one vertex, and the two vertices defining this difference are adjacent in G.

In this paper we define the modified γ_{tss} −graph G(γ_{mtss}) = (V(γ_{mtss}), E(γ_{mtss})) of G to be the graph whose vertices V(γ_{mtss}) corresponds injectively with the γ_{tss} −sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in G(γ_{mtss}) if there exists a vertex v ∈ D_1 and w ∈ D_2 such that $D_1 = D_2 - \{w\} \cup \{v\}$ and

$D_2 = D_1 - \{v\} \cup \{w\}$. Thus two γ_{tss} −sets are said to be adjacent if they differ by one vertex. We also determine $G(\gamma_{mtss})$ of certain graphs.

Gerd H. Fricke et al. [8] introduced γ −graph of a graph. The concept of γ −graph inspired the following concept.

Definition 1.1 Consider the family of all γ −sets of a graph G and define the modified γ −graph $G(\gamma_m) = (V(\gamma_m), E(\gamma_m))$ of G to be the graph whose vertices $V(\gamma_m)$ corresponds injectively with the γ −sets of a graph G and two γ −sets D_1 and D_2 form an edge in $G(\gamma_m)$ if there exists a vertex $v \in D_1$ and $w \in D_2$ such that $D_1 = D_2 - \{w\} \cup \{v\}$ and $D_2 = D_1 - \{v\} \cup \{w\}$.

We have introduced γ_{tss} −graph of the graph G.

Definition 1.2 Consider the family of all γ_{tss} − sets of a graph G and define the graph $G(\gamma_{tss}) = (V(\gamma_{tss}), E(\gamma_{tss}))$ of G to be the graph whose vertices $V(\gamma_{tss})$ corresponds injectively with the γ_{tss}-sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in $G(\gamma_{tss})$ if there exists a vertex $v \in D_1$ and $w \in D_2$ such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$.

In this section we define $G(\gamma_{mtss})$ and we determine $G(\gamma_{mtss})$ of some graphs.

Definition 1.3. Consider the family of all γ_{tss} −sets of a graph G and define the modified γ_{tss} −graph $G(\gamma_{mtss}) = (V(\gamma_{mtss}), E(\gamma_{mtss}))$ of G to be the graph whose vertices $V(\gamma_{mtss})$ corresponds injectively with the γ_{tss} −sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in $G(\gamma_{mtss})$ if there exists a vertex $v \in D_1$ and $w \in D_2$ such that $D_1 = D_2 - \{w\} \cup \{v\}$ and $D_2 = D_1 - \{v\} \cup \{w\}$. Thus two γ_{tss} −sets are said to be adjacent if they differ by one vertex.

Example 1.4.

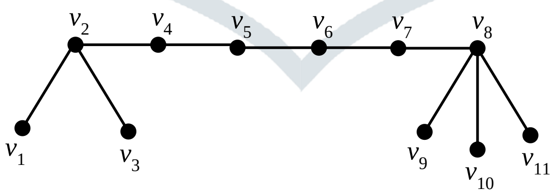

Figure 1.1

For the given graph in Figure 1.1 the total strong split dominating sets are

$D_1 = \{v_2, v_4, v_5, v_7, v_8\}$.

$D_2 = \{v_2, v_4, v_6, v_7, v_8\}$.

$v = v_5$ and $w = v_6$

Then $D_1 - \{v\} \cup \{w\} = \{v_2, v_4, v_5, v_7, v_8\} - \{v_5\} \cup \{v_6\}$

$= \{v_2, v_4, v_6, v_7, v_8\} = D_2$

$D_2 - \{w\} \cup \{v\} = \{v_2, v_4, v_6, v_7, v_8\} - \{v_6\} \cup \{v_5\} = \{v_2, v_4, v_5, v_7, v_8\} = D_1$

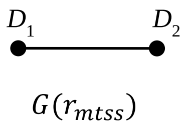

Figure 1.2

Proposition 1.5. $C_{3k}(\gamma_{mtss}) \cong \overline{K_3}$, for k ≥ 2.

Proof: Let $\{v_1, v_2, \ldots, v_{3k}\}$ be the vertex set of C_{3k}, for k ≥ 2. Let D be the minimal total strong split domination set of C_{3k}. $D_1 = \{v_1, v_2, v_4, v_5, \ldots, v_{3k-2}, v_{3k-1}\}$, $D_2 = \{v_2, v_3, v_5, \ldots, v_{3k-2}, v_{3k}\}$, $D_3 = \{v_1, v_3, v_4, v_6, \ldots, v_{3k-2}, v_{3k}\}$ are the γ_{tss}–sets of C_{3k}. Since each C_{3k}, for k ≥ 2 has 3 disjoint γ_{tss}–sets $C_{3k}(\gamma_{mtss}) \cong \overline{K_3}$. ■

Proposition 1.6. $K_{1,n}(\gamma_{mtss}) \cong K_n$.

Proof: Let D be the minimal total strong split domination set of $K_{1,n}$. Let v, $u_1, u_2, u_3, \ldots, u_n$ be the vertices of $K_{1,n}$. $D_i = \{v, v_i\}$, i = 1, 2, . . . , n is an element of $V(\gamma_{mtss})$ and each pair (D_i, D_j), $(1 \leq i, j \leq n)$ form an edge in $K_{1,n}(\gamma_{mtss})$. Hence $K_{1,n}(\gamma_{mtss}) \cong K_n$.

■

Proposition 1.7. For $3 \leq m \leq n$, $K_{m,n}(\gamma_{mtss}) \cong K_n$.

Proof: Let $X = \{u_1, u_2, u_3, \ldots, u_m\}$, $Y = \{v_1, v_2, v_3, \ldots, v_n\}$ be the two partitions of the complete bipartite graph $K_{m,n}$. Let D be the minimal total strong split domination set of $K_{m,n}$.

Suppose m = n = 2 then <V−D> results in an isolated vertex, which violates the definition of total strong split domination.

Suppose $m \leq n$, $m \geq 3$. Then $D_1 = \{v_1, u_1, u_2, u_3, \ldots, u_m\}$, $D_2 = \{v_2, u_1, u_2, u_3, \ldots, u_m\}$, . . . , $D_n = \{v_n, u_1, u_2, u_3, \ldots, u_m\}$ be the γ_{tss}–sets of $K_{m,n}$. Each γ_{tss}–set of $K_{m,n}$ differ by one vertex. Each pair (D_i, D_j), $(1 \leq i, j \leq n)$ form an edge in $K_{m,n}(\gamma_{mtss})$.

Hence $K_{m,n}(\gamma_{mtss}) \cong K_n$. ■

Proposition 1.8. $P_2 \times P_3(\gamma_{mtss}) \cong C_4$

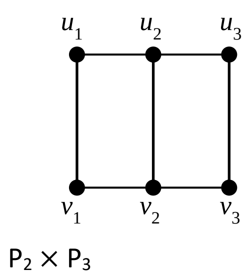

Figure 1.3

Proof: Let $\{u_1, u_2, u_3, v_1, v_2, v_3\}$ be the vertex set of the grid $P_2 \times P_3$. Then $D_1 = \{u_1, u_2, v_2, v_3\}$, $D_2 = \{u_2, u_3, v_1, v_2\}$, $D_3 = \{u_1, u_2, u_3, v_2\}$, $D_4 = \{u_2, v_1, v_2, v_3\}$ are the γ_{tss-}sets of $P_2 \times P_3$.

Here D_1 is adjacent to D_3, D_4. D_2 is adjacent to D_3, D_4. D_3 is adjacent to D_1, D_2. D_4 is adjacent to D_1, D_2. Order of $P_2 \times P_2(\gamma_{mtss})$ is 4 and each vertices D_i have degree 2. Thus $P_2 \times P_3(\gamma_{mtss})$ is isomorphic to C_4.

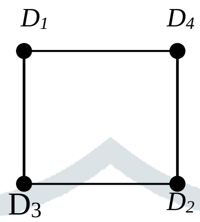

Figure 1.4 ■

Proposition1.9. $P_2 \times P_4$ (γ_{mtss}) is isomorphic to the graph with 14 vertices of which 12 vertices are of degree 4 and 2 vertices are of degree 3.

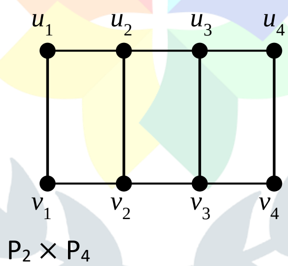

Figure 1.5

Proof: Let $\{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$ be the vertex set of the grid $P_2 \times P_4$.

Then $D_1 = \{u_2, u_3, v_1, v_2, v_3, v_4\}$, $D_2 = \{u_1, u_2, u_3, u_4, v_2, v_3\}$, $D_3 = \{u_1, u_2, u_3, u_4, v_1, v_3\}$,

$D_4 = \{u_1, u_2, u_3, v_2, v_3, v_4\}$, $D_5 = \{u_2, u_3, u_4, v_1, v_3, v_4\}$, $D_6 = \{u_1, u_2, u_3, v_1, v_2, v_4\}$, $D_7 = \{u_1, u_2, u_4, v_1, v_3, v_4\}$, $D_8 = \{u_1, u_3, v_1, v_2, v_3, v_4\}$, $D_9 = \{u_2, u_4, v_1, v_2, v_3, v_4\}$, $D_{10} = \{u_1, u_2, u_3, u_4, v_2, v_4\}$, $D_{11} = \{u_2, u_3, u_4, v_1, v_2, v_3\}$, $D_{12} = \{u_1, u_3, u_4, v_2, v_3, v_4\}$, $D_{13} = \{u_1, u_2, u_4, v_1, v_2, v_3\}$,

$D_{14} = \{u_1, u_3, u_4, v_1, v_2, v_4\}$ are the γ_{tss-}sets of $P_2 \times P_4$. Here D_1 is adjacent to D_4, D_5, D_6, D_{14}. D_2 is adjacent to D_3, D_{10}, D_{13}. D_3 is adjacent to D_2, D_8, D_{10}, D_{12}. D_4 is adjacent to D_1, D_5, D_{11}, D_{14}. D_5 is adjacent to D_1, D_4, D_7, D_{13}. D_6 is adjacent to D_1, D_7, D_8, D_9. D_7 is adjacent to D_5, D_6, D_9, D_{13}. D_8 is adjacent to D_3, D_6, D_9, D_{12}. D_9 is adjacent to D_6, D_7, D_8, D_{12}. D_{10} is adjacent to D_2, D_3, D_{11}, D_{14}.D_{11} is adjacent to D_4, D_{10}, D_{12}, D_{14}. D_{12} is adjacent to D_3, D_8, D_9, D_{11}. D_{13} is adjacent to D_2, D_5, D_7. D_{14} is adjacent to D_1, D_4, D_{10}, D_{11}. Order of $P_2 \times P_4(\gamma_{mtss})$ is 14 of which 12 vertices are of degree 4 and 2 vertices are of degree 3.

Hence $P_2 \times P_4$ (γ_{mtss}) is isomorphic to the graph with 14 vertices of which 12 vertices are of degree 4 and 2 vertices are of degree 3. Thus we get the following graph.

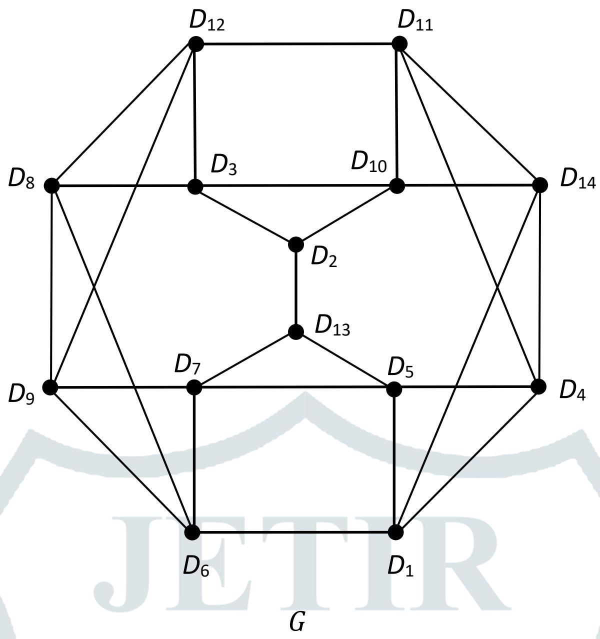

Figure 1.6 ■

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