DOMINATING, STRONG DOMINATING AND SPLIT STRONG DOMINATING SETS TOWARDS HIGHEST DEGREE OF AN INTERVAL GRAPH USING AN ALGORITHM

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Abstract:

Among the various applications of the theory of dominating sets, strong and split strong dominating sets the most often discussed in a communication networks. Their sincere efforts are observed in establishing the relevance of the subject in practicality in coherence with reality. In this paper we tried to present dominating, strong dominating and split strong dominating sets towards highest degree of an interval graph using an algorithm.

Key words:

Interval family, interval graph, degree of vertices, dominating set, strong dominating set, split strong dominating sets.

Introduction:

We have defined a graph as a set and a certain relation on that set. It is often convenient to draw a "picture" of the graph. This may be done in many ways usually one draws an interval graph corresponding to Interval family I for each vertex and connects vertex u and vertex v with a directed arrow whenever uv is an edge. If both uv and vu are edges then some times a single line joints u and v without arrows.

Preliminaries:

Let G be a graph, with vertex set V and edge set E. The open neighborhood set of a vertex $v \in V$ is $nbd(v) = \{u \in V / uv \in E\}$ .The closed neighborhood set of a vertex $v \in V$ is $nbd[v] = nbd(v) \cup \{v\}$ .A vertex in a graph G dominates itself and its neighbours. A set $D \subseteq V$ is called dominating set if every vertex in V/D is adjacent to some vertex in D. The domination studied in [1-2]. The domination number γ of G is the minimum cardinality of a dominating set. The domination number is well-studied parameter. We can see this from the bibliography [3] on domination. In [4], Sampath kumar and Pushpa Latha have introduced the concept of strong domination in graphs. Strong domination has been studied [5-6]. Kulli.V.R. et all [7] introduced the concept of split and non-split domination in graphs. A dominating set D is called split dominating set if the induced sub graph $<V-D>$ is disconnected. The split domination number of γ_s of G is the minimum cardinality of a split dominating set. Let $G=(V,E)$ be a graph and $u,v \in V$ .Then u strongly dominates v if

(i) $uv \in E$

(ii) $\deg v \le \deg u$.

A set $D_{st} \subseteq V$ is a strong dominating set of G if every vertex in $V-D_{st}$ is strongly dominated by at least one vertex in D_{st}. The strong domination number $\gamma_{st}(G)$ of G is the minimum cardinality of a strong dominating set. Define $NI(i)=j$, if $b_i < a_j$ and there do not exist an interval k such that $b_i < a_k < a_j$. If there is no such j, then define $NI(i)=null$. nbd^+ (i) is the set of all adjacent vertices of i which are greater than i. nbd^- (i) is the set of all adjacent vertices of i which are less than i. nbd^{-1}(i) is a nearest vertex to i ,which belongs to nbd^-(i). d^+(i) is the number of adjacent vertices which are greater than i. d^- (i) is the number of adjacent vertices which are less than i.

Main Algorithms:

1. To find a minimum dominating set of an interval graph using an algorithm:

Input: Interval family $I=\{I_1, I_2, ..., I_n\}$.

Output: Minimum Dominating set of an interval graph of a given interval family.

Step 1: Take i=1, D = ϕ.

Step 2: S_1= nbd [i].

Step3: LHDI= Largest interval of the highest degree intervals in S_1.

Step4: D= D $\cup$ {LHDI}.

Step5: Find NI (LHDI).

Step6: If i= NI (LHDI) exists.

then goto step 2.

Else

Step7: Write Minimum Dominating Set = D.

Step8: End.

2. To find Strong Dominating Set of an interval graph using an algorithm:

Input: Interval family $I = \{I_1, I_2, ..., I_n\}$.

Output: Strong Dominating set of an interval graph of a given interval family.

Step 1: Take i= 1, $D_{st} = \phi$.

Step 2: S_1 = nbd [i].

Step 3: S_2 =The set of vertices in S_1, which are adjacent to all other vertices in S_1.

Step 4: LHDI = Largest interval of the highest degree interval in S_2.

Step 5: $D_{st} = D_{st} \cup$ {LHDI}.

Step 6: Find NI (LHDI).

Step 7: If i=NI (LHDI) exists.

then go to step 2

Else

Step 8: Write Strong Dominating set = D_{st}.

Step 9: End

3. To find Split Strong Dominating Set of an interval graph using an algorithm:

Input: Interval family $I = \{I_1, I_2, ..., I_n\}$.

Output: Split Strong Dominating set of an interval graph of a given interval family.

Step 1: Take i= 1, $SD_{st} = \phi$, Count = 0.

Step 2: S_1 = nbd [i].

Step 3: S_2 = The set of vertices in S_1, which are adjacent to all other vertices in S_1.

Step 4: LHDI = Largest interval of the highest degree intervals in S_2.

Step 5: $SD_{st} = SD_{st} \cup$ {LHDI}

Step 6: If $d(nbd^{-1}(LHDI)) = 1$ then

Count = Count + 1

Step 7: If d(LHDI) = 1 then

Count = Count + 1

Step 8: If Count = 0 then

Step8.1: If there exists an edge (u, v) $\in$ E(G) such that u$\in$nbd$^-$(LHDI) & v$\in$nbd$^+$(LHDI).

Step8.2: x = Largest vertex of {u, v}.

Step8.3: Count = Count +1.

Step8.4: $SD_{st} = SD_{st} \cup \{x\}$ and go to Step10.

else .

Step9: go to Step10.

Step 10: Find NI (LHDI).

Step 11: If i=NI (LHDI) exists

then go to step 2

else

Step 12: Write Split Strong Dominating set = SD_{st}.

Step 11: End.

Main Theorems:

Theorem 1: Let G be an interval graph corresponding to an interval family $I = \{I_1, I_2, ..., I_n\}$. If i and j are any two intervals in I such that $i \in D_{st}$, where D_{st} is a minimum strong dominating set of the given interval graph G, $j \neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and at least one interval $k \neq i$ to the right of j that intersects j then $\gamma_{st}(G) < \gamma_{sst}(G)$.

Proof: Let G be an interval graph corresponding to an interval family $I = \{I_1, I_2, ..., I_n\}$. Let i and j be any two intervals in I such that $i \in D_{st}$, where D_{st} is a minimum strong dominating set of the given interval graph G, $j \neq 1$ and j is contained in i and Suppose there is at least one interval to the left of j that intersects j and at least one interval $k \neq i$ to the right of j that intersects j. Then it is obviously we know that j is adjacent to k in the induced sub graph $< V - D_{st} >$. Then there will be a connection in $< V - D_{st} >$. Since there is at least one interval to the left of j that intersects interval j, there will be a connection in $< V - D_{st} >$ to it's left. In this connection we introduce another interval 'h', which is to the right of j and i and also interest i and j to D_{st} for disconnection in the induced subgraph $< V - D_{st} >$. We also formulated Split strong dominating set as follows

$$SD_{st} = D_{st} \cup \{h\} \Rightarrow |SD_{st}| = |D_{st} \cup \{h\}|.$$

Since D_{st}, {h} are disjoint $\Rightarrow |SD_{st}| = |D_{st}| + |\{h\}|$

or

$$\Rightarrow \quad |D_{st}| + |\{h\}| = |SD_{st}|.$$

$$\Rightarrow \gamma_{st}(G) + |\{h\}| = \gamma_{sst}(G).$$

$$\Rightarrow \gamma_{st}(G) < \gamma_{sst}(G).$$

Illustration:

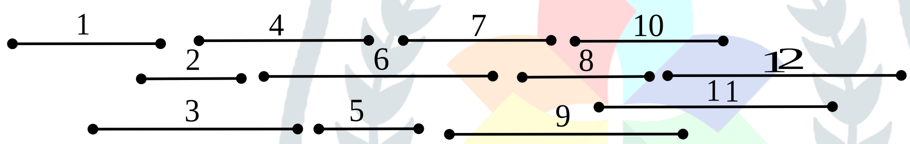

Fig: Interval Family

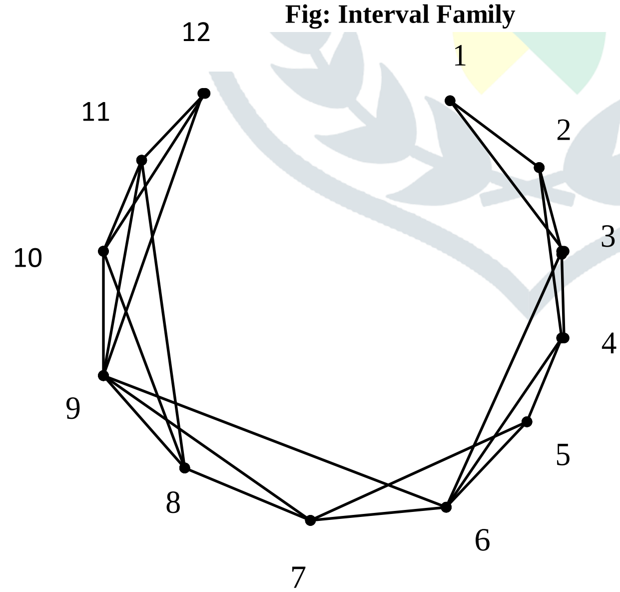

Fig: Interval Graph

As follows an algorithm with illustration for neighbours as given interval family I.

We construct an interval graph G from an interval family I = {1, 2,……,12}as follows

Neighbourhood of vertices:

nbd [1] = {1,2,3}	nbd [2] = {1,2,3,4}	nbd [3] = {1,2,3,4,6}
nbd [4] = {2,3,4,5,6}	nbd [5] = {4,5,6,7}	nbd [6] = {3,4,5,6,7,9}
nbd [7] = {5,6,7,8,9}	nbd [8] = {7,8,9,10,11}	nbd [9] = {6,7,8,9,10,11,12}
nbd [10] = {8,9,10,11,12}	nbd[11] = {8,9,10,11,12}	nbd[12] = {9,10,11,12}

Degree of vertices:

d(1) = 2 d(2) = 3 d(3) = 4 d(4) = 4 d(5) = 3 d(6) = 5

d(7) = 4 d(8) = 5 d(9) = 6 d(10) = 4 d(11) = 4 d(12) = 3

Nonintersecting vertex of vertices:

NI (1) = 4 NI (2) = 5 NI (3) = 5 NI (4) = 7

NI (5) = 8 NI (6) = 8 NI (7) = 10 NI (8) = 12

NI (9) = null NI (10) = null NI (11) = null NI (12) = null

Procedure for finding a minimum dominating set(D) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 12}.

Step 1**:** Take i = 1, D = ϕ

Step 2**:** S_1 = nbd[1] = {1, 2, 3}

Step 3**:** LHDI = 3

Step 4**:** D = D∪{LHDI} = ϕ∪{3} = {3}

Step 5**:** Find NI(LHDI) = NI(3) = 5

Step 6**:** if i = NI(LHDI) = 5 exists

then go to Step2

Step 2**:** S_1 = nbd[5] = {4, 5, 6, 7}

Step 3**:** LHDI = 6

Step 4**:** D = D∪{LHDI} = {3}∪{6} = {3, 6}

Step 5**:** Find NI(LHDI) = NI(6) = 8

Step 6**:** if i = NI(LHDI) = 8 exists

then go to Step2

Step 2**:** S_1 = nbd[8] = {7, 8, 9, 10, 11, 12}

Step 3**:** LHDI = 9

Step 4**:** D = D∪{LHDI} = {3, 6}∪{9} = {3, 6, 9}

Step 5**:** Find NI(LHDI) = NI(9) = null

Step 6**:** if i = NI(LHDI) = null

else

Step7: Minimum Dominating Set = D = {3, 6, 9}

Step8: End.

Output: Minimum Dominating Set = D = {3, 6, 9}

Procedure for finding a strong dominating set(D_{st}) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 12}.

Step 1**:** Take i = 1, D_{st} = ϕ

Step 2**:** S_1 = nbd[1] = {1, 2, 3}

Step3: S_2 = {1, 2, 3}

Step4**:** LHDI = 3

Step5**:** D_{st} = D_{st} ∪ {LHDI} = ϕ ∪ {3} = {3}

Step6**:** Find NI(LHDI) = NI(3) = 5

Step 7**:** if i = NI(LHDI) = 5 exists

then go to Step2

Step 2**:** S_1 = nbd[5] = {4, 5, 6, 7}

Step3: S_2 = {5, 6}

Step4**:** LHDI = 6

Step5**:** $D_{st} = D_{st} \cup \{LHDI\} = \{3\} \cup \{6\} = \{3, 6\}$

Step6**:** Find NI(LHDI) = NI(6) = 8

Step 7**:** if i = NI(LHDI) = 8 exists

then go to Step2

Step 2**:** S_1 = nbd[8] = {7, 8, 9, 10, 11, 12}

Step3: S_2 = {8, 9}

Step4**:** LHDI = 9

Step5**:** $D_{st} = D_{st} \cup \{LHDI\} = \{3, 6\} \cup \{9\} = \{3, 6, 9\}$

Step6**:** Find NI(LHDI) = NI(9) = null

Step 7**:** if i = NI(LHDI) = null

else

Step8: Write Strong Dominating Set = D_{st} = {3, 6, 9}

Step9: End.

Procedure for finding a split strong dominating set (SD_{st}) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 12}.

Step 1: Take i= 1, $SD_{st} = \phi$, Count = 0.

Step 2: S_1 = nbd [1] = {1, 2, 3}

Step 3: S_2 = {1, 2, 3}

Step 4: LHDI = 3

Step 5: $SD_{st} = SD_{st} \cup \{LHDI\} = \phi \cup \{3\} = \{3\}$

Step 6: $d(nbd^{-1}(LHDI)) = d(2) = 2 \neq 1$

Step 7: $d(LHDI) = d(3) = 4 \neq 1$

Step 8: If Count = 0 then

Step8.1: there exists an edge (2, 4) $\in$ E(G) such that 2$\in$nbd$^-$(3) & 4$\in$nbd$^+$(3).

Step8.2: x = Largest vertex of {2, 4} = 4

Step8.3: Count = Count +1 = 0+1 = 1

Step8.4: $SD_{st} = SD_{st} \cup \{x\} = \{3\} \cup \{4\} = \{3, 4\}$

goto Step10. .

Step 10: Find NI (LHDI) = NI(3) = 5

Step 11: If i=NI (LHDI) = 5 exists

then go to step 2

Step 2: S_1 = nbd [5] = {4, 5, 6, 7}

Step 3: S_2 = {5, 6}

Step 4: LHDI = 6

Step 5: $SD_{st} = SD_{st} \cup$ {LHDI} = $\{3,4\} \cup \{6\} = \{3, 4, 6\}$

Step 6: If d(nbd^{-1}(LHDI)) = d(5) = 3 $\neq$ 1

Step 7: If d(LHDI) = d(6) = 5 $\neq$ 1

Step 8: If Count = 1$\neq$0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(6) = 8

Step 11: If i=NI (LHDI) = 8 exists

then go to step 2

Step 2: S_1 = nbd [8] = {7, 8, 9, 10, 11}

Step 3: S_2 = {8, 9}

Step 4: LHDI = 9

Step 5: $SD_{st} = SD_{st} \cup$ {LHDI} = $\{3,4, 6\} \cup \{9\} = \{3, 4, 6,9\}$

Step 6: If $d(nbd^{-1}(LHDI)) = d(8) = 5 \neq 1$

Step 7: If $d(LHDI) = d(9) = 6 \neq 1$

Step 8: If Count = $1 \neq 0$

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(9) = null

Step 11: If i=NI (LHDI) = null

else

Step 12: Write Split Strong Dominating set = SD_{st}= {3, 4, 6, 9}.

Step 11: End.

Output: Split Strong Dominating set = SD_{st}= {3, 4, 6, 9}.

Note: $D_{st} = \{3,6,9\}$, $SD_{st} = \{3,4,6,9\}$

$|D_{st}| < |SD_{st}|$

$\therefore \gamma_{st}(G) < \gamma_{sst}(G)$

Theorem 2: Let D_{st} be a strong dominating set of the given interval graph G corresponding to an interval family $I = \{I_1, I_2, ..., I_n\}$. If i and j are any two intervals in I such that j is contained in i and if there is no other interval $k \neq i$ that intersects j then the strong dominating set D_{st} is also a split strong dominating set of an interval graph G.

Proof: Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family and G is an interval graph corresponding to I. Let i and j be any two intervals in I such that j is contained i. If there is no interval $k \neq i$ that intersect j. Then clearly i lies in the strong dominating set D_{st}. Further in induced sub graph $< V - D_{st} >$ the vertex j is not adjacent to any other vertex and then j becomes as an isolated vertex in induced sub graph $< V - D_{st} >$. There is a disconnection in $< V - D_{st} >$.Hence the strong dominating set, which we considered is split strong dominating set. Hence we follows an algorithm to find strong dominating set and split strong dominating set of an interval graph with an illustration.

Illustration:

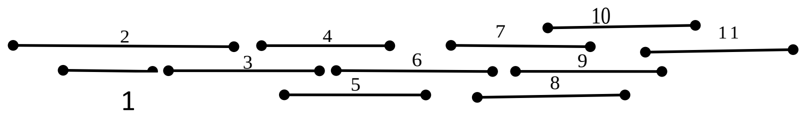

Fig: Interval Family

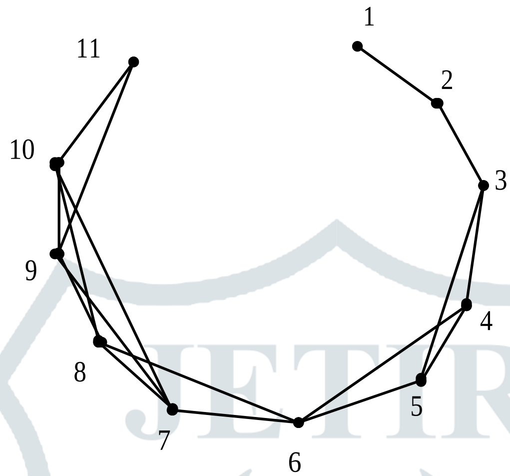

Fig: Interval Graph

As follows an algorithm with illustration for neighbours as given interval family I.

We construct an interval graph G from an interval family I = {1, 2,…….. ,11}as follows

Neighbourhood of vertices:

nbd[1] = {1,2} nbd[2] = {1, 2, 3} nbd[3] = {2,3,4,5}

nbd[4] = {3,4,5,6} nbd[5] = {3,4,5,6} nbd[6] = {4,5,6,7,8}

nbd[7] = {6,7,8,9,10} nbd[8] ={6,7,8,9,10} nbd[9] = {7,8,9,10,11}

nbd[10] ={7,8,9,10,11} nbd[11] = {9,10,11}

Degree of vertices:

d(1) = 1 d(2) = 2 d(3) = 3 d(4) = 3 d(5) = 3 d(6) = 4

d(7) = 4 d(8) = 4 d(9) = 4 d(10) = 4 d(11) = 2

Nonintersecting vertex of vertices:

NI (1) = 3 NI (2) = 4 NI (3) = 6 NI (4) =7 NI (5) =7 NI (6) = 9

NI (7) = 11 NI (8) = 11 NI (9) = null NI (10) = null NI (11) = null

Procedure for finding a minimum dominating set(D) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 11}.

Step 1**:** Take i = 1, D = ϕ

Step 2**:** S_1 = nbd[1] = {1, 2}

Step 3**:** LHDI = 2

Step 4**:** D = D∪{LHDI} = ϕ∪{2} = {2}

Step 5**:** Find NI(LHDI) = NI(2) = 4

Step 6**:** if i = NI(LHDI) = 4 exists

then go to Step2

Step 2**:** S_1 = nbd[4] = {3,4, 5, 6}

Step 3**:** LHDI = 6

Step 4**:** D = D∪{LHDI} = {2}∪{6} = {2, 6}

Step 5**:** Find NI(LHDI) = NI(6) = 9

Step 6**:** if i = NI(LHDI) = 9 exists

then go to Step2

Step 2**:** S_1 = nbd[9] = {7, 8, 9, 10, 11}

Step 3**:** LHDI = 10

Step 4**:** D = D∪{LHDI} = {2, 6}∪{10} = {2, 6, 10}

Step 5**:** Find NI(LHDI) = NI(10) = null

Step 6**:** if i = NI(LHDI) = null

else

Step7: Minimum Dominating Set = D = {2, 6, 10}

Step8: End.

Output: Minimum Dominating Set = D = {2, 6, 10}

Procedure for finding a strong dominating set(D_{st}) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 11}.

Step 1**:** Take i = 1, $D_{st} = \phi$

Step 2**:** S_1 = nbd[1] = {1, 2}

Step3: S_2 = {1, 2}

Step4**:** LHDI = 2

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = $\phi \cup$ {2} = {2}

Step6**:** Find NI(LHDI) = NI(2) = 4

Step 7**:** if i = NI(LHDI) = 4 exists

then go to Step2

Step 2**:** S_1 = nbd[4] = {3,4, 5, 6}

Step3: S_2 = {4, 5}

Step4**:** LHDI = 5

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = {2} $\cup$ {5} = {2, 5}

Step6**:** Find NI(LHDI) = NI(5) = 7

Step 7**:** if i = NI(LHDI) = 7 exists

then go to Step2

Step 2**:** S_1 = nbd[7] = {6,7, 8, 9, 10}

Step3: S_2 = {7,8}

Step4**:** LHDI = 8

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = {2,5} $\cup$ {8} = {2, 5, 8}

Step6**:** Find NI(LHDI) = NI(8) = 11

Step 7**:** if i = NI(LHDI) = 11 exists

then go to Step2

Step 2**:** S_1 = nbd[11] = { 9, 10, 11}

Step3: S_2 = {9, 10, 11}

Step4**:** LHDI = 10

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = {2,5, 8} $\cup$ {10} = {2, 5, 8, 10}

Step6**:** Find NI(LHDI) = NI(10) = null

Step 7**:** if i = NI(LHDI) = null

else

Step8: Write Strong Dominating Set = D_{st} = {2, 5, 8, 10}

Step9: End.

Output: Strong Dominating Set = D_{st} = {2, 5, 8, 10}

Procedure for finding a split strong dominating set(SD_{st}) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 11}.

Step 1: Take i= 1, $SD_{st} = \phi$, Count = 0.

Step 2: S_1 = nbd [1] = {1, 2}

Step 3: S_2 = {1, 2}

Step 4: LHDI = 2

Step 5: $SD_{st} = SD_{st} \cup$ {LHDI} = $\phi \cup$ {2} = {2}

Step 6: If $d(nbd^{-1}(LHDI))$ = d(1) = 1

Count = Count + 1 = 0 + 1 = 1

Step 7: d(LHDI) = d(2) = 2 $\neq$ 1

Step 8: If Count = 0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(2) = 4

Step 11: If i=NI (LHDI) = 4 exists

then go to step 2

Step 2: S_1 = nbd [4] = {3,4, 5, 6}

Step 3: S_2 = {4,5}

Step 4: LHDI = 5

Step 5: SD_{st}= SD_{st} $\cup$ {LHDI} = {2} $\cup$ {5} ={2, 5}

Step 6: d(nbd^{-1}(LHDI)) = d(4) = 3 $\neq$1

Step 7: d(LHDI) = d(5) = 3 $\neq$ 1

Step 8: If Count = 1$\neq$0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(5) = 7

Step 11: If i=NI (LHDI) = 7 exists

then go to step 2

Step 2: S_1 = nbd [7] = {6, 7, 8, 9, 10}

Step 3: S_2 = {7, 8}

Step 4: LHDI = 8

Step 5: SD_{st}= SD_{st} $\cup$ {LHDI} = {2, 5} $\cup$ {8} ={2, 5, 8}

Step 6: d(nbd^{-1}(LHDI)) = d(7) = 4 $\neq$1

Step 7: d(LHDI) = d(8) = 4 $\neq$ 1

Step 8: If Count = 1$\neq$0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(8) = 11

Step 11: If i=NI (LHDI) =11 exists

then go to step 2

Step 2: S_1 = nbd 11] = { 9, 10,11}

Step 3: S_2 = {9, 10, 11}

Step 4: LHDI = 10

Step 5: $SD_{st}= SD_{st} \cup$ {LHDI} = {2, 5,8} $\cup$ {10} ={2, 5, 8, 10}

Step 6: d(nbd^{-1}(LHDI)) = d(9) = 4 $\neq$1

Step 7: d(LHDI) = d(10) = 4 $\neq$ 1

Step 8: If Count = 1$\neq$0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(10) = null

Step 11: If i=NI (LHDI) = null

else

Step 12: Write Split Strong Dominating set = SD_{st}= {2, 5, 8, 10}.

Step 11: End.

Output : Split Strong Dominating set = SD_{st}= {2, 5, 8, 10}.

Note: D_{st} = {2,5,8,10}, SD_{st} = {2,5,8,10}

Hence $D_{st} = SD_{st}$

Theorem 3: Let $I = \{I_1, I_2, ..., I_n\}$ be an interval family and D_{st} is a strong dominating set of the given interval graph G. If i, j, k are any three consecutive intervals such that $i < j < k$ and if $j \in D_{st}$, and i intersects j, j intersect k and i does not intersect k then $D_{st} = SD_{st}$.

Proof: Suppose $I = \{I_1, I_2, ..., I_n\}$ be an interval family . If i, j, k be three consecutive intervals such that $i < j < k$ and i intersect j, j intersect k, but i does not intersect k. Suppose $j \in D_{st}$, where D_{st} is a strong dominating set. Then i and k are not adjacent in the induced subgraph $< V - D_{st} >$. There exists a a disconnection between i and k. That is, there is no $m \in I$, $m < k$ such that m intersects k. If possible suppose that such an m exists, then since $m < k$ we must have $m < i < j < k$ ($\because m < k$). Now m intersects k implies i and j also intersect. Then there is a path between i and k and are adjacent. This is a contradiction to hypothesis. So such a m does not exists. Hence we get disconnection. Hence D_{st} is also a split strong dominating set of the given interval graph G .As usual as follows an algorithm to find a strong dominating set and split strong dominating set of an interval graph G .

Illustration:

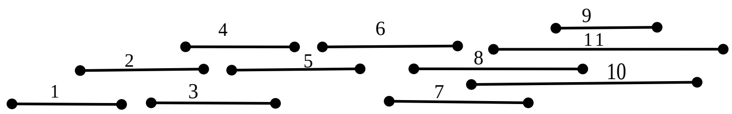

Fig: Interval Family

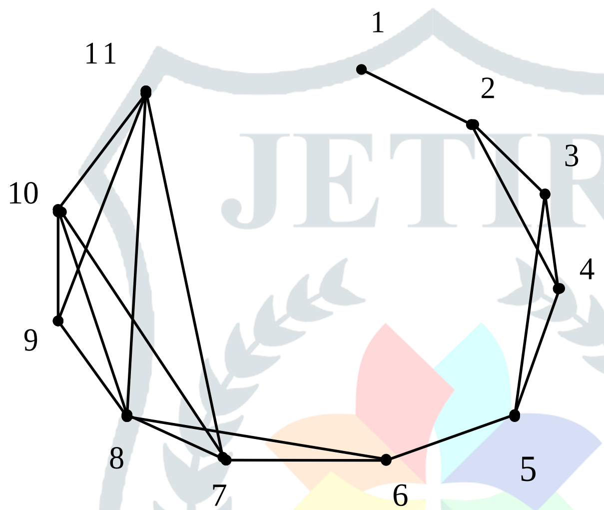

Fig: Interval Graph

We construct an inter [illegible] ,11} as follows.

Degree of vertices:

nbd [1] = {1,2} nbd [2] = {1,2,3,4} nbd [3] = {2,3,4,5}

nbd [4] = {2,3,4,5} nbd [5] = {3,4,5,6} nbd [6] = {5,6,7,8}

nbd [7] = {6,7,8,10,11} nbd[8]={6,7,8,9,10,11} nbd[9]={8,9,10,11}

nbd[10]={7,8,9,10,11} nbd[11] ={7,8,9,10,11}

Degree of vertices:

d(1) = 1 d(2) = 3 d(3) = 3 d(4) = 3 d(5) = 3 d(6) = 3

d(7) = 4 d(8) = 5 d(9) = 3 d(10) = 4 d(11) = 4

Nonintersecting vertex of vertices:

NI (1) = 3 NI (2) = 5 NI (3) = 6 NI (4) =6 NI (5) =7 NI (6) = 9

NI (7) = 9 NI (8) = null NI (9) = null NI (10) = null NI (11) = null

Procedure for finding a minimum dominating set(D) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 11}.

Step 1**:** Take i = 1, D = ϕ

Step 2**:** S_1 = nbd[1] = {1, 2}

Step 3**:** LHDI = 2

Step 4**:** D = D$\cup${LHDI} = $\phi\cup${2} = {2}

Step 5**:** Find NI(LHDI) = NI(2) = 5

Step 6**:** if i = NI(LHDI) = 5 exists

then go to Step2

Step 2**:** S_1 = nbd[5] = {3, 4, 5, 6}

Step 3**:** LHDI = 6

Step 4**:** D = D$\cup${LHDI} = {2}$\cup${6} = {2, 6}

Step 5**:** Find NI(LHDI) = NI(6) = 9

Step 6**:** if i = NI(LHDI) = 9 exists

then go to Step2

Step 2**:** S_1 = nbd[9] = {7, 8, 9, 10, 11}

Step 3**:** LHDI = 8

Step 4**:** D = D$\cup${LHDI} = {2, 6}$\cup${8} = {2, 6, 8}

Step 5**:** Find NI(LHDI) = NI(8) = null

Step 6**:** if i = NI(LHDI) = null

else

Step7: Minimum Dominating Set = D = {2, 6, 8}

Step8: End.

Output: Minimum Dominating Set = D = {2, 6, 8}

Procedure for finding a strong dominating set(D_{st}) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 11}.

Step 1**:** Take i = 1, $D_{st} = \phi$

Step 2**:** S_1 = nbd[1] = {1, 2}

Step3: S_2 = {1, 2}

Step4**:** LHDI = 2

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = $\phi \cup$ {2} = {2}

Step6**:** Find NI(LHDI) = NI(2) = 5

Step 7**:** if i = NI(LHDI) = 5 exists

then go to Step2

Step 2**:** S_1 = nbd[5] = {3,4, 5, 6}

Step3: S_2 = {5}

Step4**:** LHDI = 5

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = {2} $\cup$ {5} = {2, 5}

Step6**:** Find NI(LHDI) = NI(5) = 7

Step 7**:** if i = NI(LHDI) = 7 exists

then go to Step2

Step 2**:** S_1 = nbd[7] = {6,7, 8, 10,11}

Step3: S_2 = {7, 8}

Step4**:** LHDI = 8

Step5**:** $D_{st} = D_{st} \cup$ {LHDI} = {2, 5} $\cup$ {8} = {2, 5, 8}

Step6**:** Find NI(LHDI) = NI(8) = null

Step 7**:** if i = NI(LHDI) = null

else

Step8: Write Strong Dominating Set = D_{st} = {2, 5, 8}

Step9: End.

Output: Strong Dominating Set = D_{st} = {2, 5, 8}

Procedure for finding a split strong dominating set(SD_{st}) of an interval graph using an algorithm:

Input: Interval family I = {1, 2, 3,………, 11}.

Step 1: Take i= 1, $SD_{st} = \phi$, Count = 0.

Step 2: S_1 = nbd [1] = {1, 2}

Step 3: S_2 = {1, 2}

Step 4: LHDI = 2

Step 5: $SD_{st} = SD_{st} \cup$ {LHDI} = $\phi \cup$ {2} = {2}

Step 6: If $d(nbd^{-1}(LHDI))$ = d(1) = 1

Count = Count + 1 = 0 + 1 = 1

Step 7: d(LHDI) = d(2) = 3 $\neq$ 1

Step 8: If Count = 1 $\neq$ 0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(2) = 5

Step 11: If i=NI (LHDI) = 5 exists

then go to step 2

Step 2: S_1 = nbd [5] = {3,4, 5, 6}

Step 3: S_2 = {5}

Step 4: LHDI = 5

Step 5: $SD_{st} = SD_{st} \cup$ {LHDI} = {2} $\cup$ {5} = {2, 5}

Step 6: $d(nbd^{-1}(LHDI))$ = d(4) = 3 $\neq$1

Step 7: d(LHDI) = d(5) = 3 $\neq$ 1

Step 8: If Count = 1$\neq$0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(5) = 7

Step 11: If i=NI (LHDI) = 7 exists

then go to step 2

Step 2: S_1 = nbd [7] = {6, 7, 8, 10, 11}

Step 3: S_2 = {7, 8}

Step 4: LHDI = 8

Step 5: SD_{st}= $SD_{st} \cup$ {LHDI} = {2, 5} $\cup$ {8} ={2, 5, 8}

Step 6: d(nbd^{-1}(LHDI)) = d(7) = 4 $\neq$1

Step 7: d(LHDI) = d(8) = 5 $\neq$ 1

Step 8: If Count = 1$\neq$0

else

Step 9: go to Step 10

Step 10: Find NI (LHDI) = NI(8) = null

Step 11: If i=NI (LHDI) = null

else

Step 12: Write Split Strong Dominating set = SD_{st} = {2, 5, 8}.

Step 11: End.

Output: Split Strong Dominating set =SD_{st} = {2, 5, 8}.

D_{st} = {2, 5, 8} and SD_{st} = {2, 5, 8}

$\therefore D_{st} = SD_{st}$

Therefore an illustration is verified.

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