SOME EXTENSION FORMULAE INCLUDING A FUNDAMENTAL OF GENERALIZED HYPER GEOMETRIC WORK

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Abstract:

In this paper, the author will explain some expansion formula of the basic analogue of generalized hypergeometric function (I-function) in the relationship with the applications of q-Leibnitz rule for the Weyl type q-derivatives of a product of two functions.

Expansion formulae involving a basic analogue of H-function, Meijer's G-function and Mac'Robert's E-function have been derived as special cases of the results.

Key Words: q-Leibnitz rule, Weyl type q-derivatives of product of two function.

Introduction:

Clarifying certain extension of the essential simple of I-work in association with the uses of q-Leibnitz regulation aimed at the Weyl q-subsidiaries of a result of two capacities. Extension methods including a fundamental simple of H-work, Meijer's G-capacity, MacRobert's E-work have been inferred as exceptional instances of the outcomes.

Our interpretation of genuine issues to scientific articulations depends on analytics, which thusly depends on the separation and joining tasks of subjective request, kind of contradiction fragmentary analytics that are additionally a characteristic speculation of analytics, numerical past is similarly extended. Now assuming noteworthy job, for example, material science, rheology, quantitative science, electro-science, dispersing hypothesis, dissemination, transport hypothesis, likelihood, flexibility, control hypothesis, building arithmetic and numerous others. Partial analytics like numerous other scientific controls and thoughts has its source in the mission of specialists for to grow its applications to new fields. This opportunity of request opens new measurements and numerous issues of connected sciences can be handled in progressively effective manner by methods for partial math.

The motivation is to build availability of various components of q-partial math, speculation of essential hyper geometric capacities to this present reality issues of designing, science and financial matters.This uncovers a concise past, uses of fundamental hyper geometric capacities ,the speculations in light of various numerical orders.

It is committed to infer extension methods of a fundamental simple of I-work characterized [1] as far as Gamma work as pursues: $I^{m,n}_{q,A_{i},B_{i};R}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n}(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,m}(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right]=$

$$\frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^{m}\Gamma_q(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma_q(1-a_j+\alpha_j s)}{\sum_{i=1}^{R}\left\{\prod_{j=m+1}^{B_i}\Gamma_q(1-b_{ji}+\beta_{ji}s)\prod_{j=1}^{n}\Gamma_q(a_{ji}-\alpha_{ji}s)\right\}\Gamma_q(s)\Gamma_q(1-s)\sin\pi s}\pi z^s ds$$

(4.1.1)

Saxena et.al.[2] likewise characterized as far as Mellin-Barne's vital as pursues:

$$I^{m,n}_{A_i,B_i;R}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,A_i}\\(b_j,\beta_j)_{1,n};(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\omega}\int_L\frac{\prod_{j=1}^{m}G(q^{b_j-\beta_j s})\prod_{j=1}^{n}G\left(q^{1-a_j+\alpha_j s}\right)\pi z^s ds}{\sum_{i=1}^{R}\{\prod_{j=m+1}^{B_i}G\left(q^{1-b_{ji}+\beta_{ji}s}\right)\prod_{j=n+1}^{A_i}G\left(q^{a_{ji}-\alpha_{ji}s}\right)\}G(q^{1-s})\sin\pi s} \qquad (4.1.2)$$

where $z\neq 0, 0<|q|<1, \omega=\sqrt{-1}$ and

$0\le m\le B_i,\ 0\le n\le A_i,\ i=1,2,3,\dots,R$; R is finite and

$G(q^\alpha)=\prod_{n=0}^{\infty}(1-q^{\alpha+n})^{-1}=\frac{1}{(q^\alpha;q)_\infty}$

Also, $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}$ are real and positive and $a_j,b_j,a_{ji},b_{ji}$are complex numbers.

L runs from $-i\infty\ to+i\infty$ so that altogether the extremes of $G(q^{b_j-\beta_j s})$; $1\le j\le m$, are to the precise and $G\left(q^{1-a_j+\alpha_j s}\right)$; $1\le j\le n$ are to its left and are at least some $\varepsilon>0$.

According to [7, 8] ,

$$H^{m,n}_{q,A,B}\left[z;q\left|\begin{matrix}(a_j,\alpha_j)_{1,A}\\(b_j,\beta_j)_{1,B}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L\frac{\prod_{j=1}^{m}G(q^{b_j-\beta_j s})\prod_{j=1}^{n}G(q^{1-a_j+\alpha_j s})\pi z^s ds}{\prod_{j=m+1}^{B}G(q^{1-b_j+\beta_j s})\prod_{j=n+1}^{A}G\left(q^{a_j-\alpha_j s}\right)G(q^{1-s})\sin\pi s}$$

Where $G(q^\alpha)=\{\prod_{n=0}^{\infty}(1-q^{\alpha+n})\}^{-1}=\frac{1}{(q^\alpha;q)_\infty}$

The above meaning is summarized as :

$$H^{m,n}_{A,B}\left[z;q\left|\begin{matrix}(a,1)_{1,p}\\(b,1)_{1,q}\end{matrix}\right.\right]=G^{m,n}_{A,B}\left[z;q\left|\begin{matrix}a_1,a_2,\dots,a_p\\b_1,b_2,\dots,b_q\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_i\frac{\prod_{j=1}^{m}G(q^{b_j-s})\prod_{j=1}^{n}G(q^{1-a_j+s})\pi z^s ds}{\prod_{j=m+1}^{q}G(q^{1-b_j+s})\prod_{j=n+1}^{p}G\left(q^{a_j-s}\right)G(q^{1-s})\sin\pi s}$$

Riemann-Liouville partial integrals: As characterized in [3, 4],

$$(I^\alpha_{a+}f)x=\frac{1}{\Gamma(\alpha)}\int_a^x(x-t)^{\alpha-1}f(t)\,dt,\ (x>a, a\epsilon R). \qquad (4.1.3)$$

in this way, when all is said in done the Riemann-Liouville partial integrals of discretionary request for a capacity f(t), is a characteristic outcome of the notable recipe (Cauchy-Dirichlet's) that decreases the estimation of the n-overlay crude of a capacity f (t) to a solitary fundamental of convolution type.

As of late, Yadav et. al. [5] presents another q-augmentation of the Leibniz rule for the subordinates of a result of two fundamental capacities as far as a limited q-arrangement including Weyl type q-subsidiaries of the capacities in the accompanying way.

$${}_zD^\alpha_{\infty,q}[U(z)V(z)]=\sum_{r=0}^{\infty}\frac{(-1)^r q^{r(r+1)/2}(q^{-\alpha};q)_r}{(q;q)_r}\ {}_zD^{\alpha-r}_{\infty,q}[U(z)]\ {}_zD^\alpha_{\infty,q}[V(zq^{\alpha-r})]$$

(4.1.4)

$${}_zD^{\alpha}_{\infty,q}[f(z)] = \frac{q^{r(r+1)/2}}{\Gamma_q(-\alpha)}\int_z^{\infty}(t-z)^{-q-1} f(tq^{1+\alpha})d_q(t) \quad (4.1.5) \quad \underset{z,\ \infty,q}{\overset{\alpha}{D}}[f(z)] = \frac{q^{\frac{-\alpha(\alpha+1)}{2}}}{\Gamma_q(-\alpha)}\int_z^{\infty}(t-z)_{-q-1} f(tq^{1+\alpha})d_q(t)$$

(5)

In specific,

$$\underset{z,\ \infty,q}{\overset{\alpha}{D}}\left[z^{-p}\right] = \frac{\Gamma_q(p+\alpha)q^{-\alpha p+\frac{\alpha(1-\alpha)}{2}} z^{-p-\alpha}(1-q)}{\Gamma_q(p)} \quad (6)$$

Primary Results:

In this segment, we will build up specific outcomes related with essential simple of I-work by doling out appropriate qualities to the capacities z, and α in the q-Leibnitz rule. The fundamental outcomes to be set up are as per the following:

$$I^{m+1,n}_{A_i+1,B_i+1;R}\left[P(zq^{\mu})^k;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n}\ (a_{ji},\alpha_{ji})_{n+1,A_i}(\lambda,k)\\(\mu+\lambda,k)(b_j,\beta_j)_{1,m}\ (b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right] = \sum_{r=0}^{\mu}\frac{(-1)^r q^{\frac{r(r+1)}{2}+\lambda r}(q^{-\mu};q)_r(q^{\lambda};q)_{\mu-r}}{(q;q)_r}$$
$$\times I^{m+1,n}_{A_i+1,B_i+1;R}\left[P(zq^{\mu})^k;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,A_i}(0,k)\\(r,k)(b_j,\beta_j)_{1,m}\ (b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right] \quad (7)$$

Where $0 \le m \le B$, $0 \le n \le A$, $Re[slog(z) - logsin\pi s] < 0, k \ge 0$, $P \in C$. Proof: On putting,

$U(z) = z^{-\lambda}$ and $V(z) = I^{m,n}_{A_i,B_i;R}\left[Pz^k;q\left|\begin{matrix}(a_j,\alpha_j)\ (a_{ji},\alpha_{ji})\\(b_j,\beta_j)\ (b_{ji},\beta_{ji})\end{matrix}\right.\right]$ (8)

$$\underset{z,\ \infty,q}{\overset{\mu}{D}}\left\{z^{-\lambda}I^{m,n}_{A_i,B_i;R}\left[Pz^k;q\left|\begin{matrix}(a_j,\alpha_j)\ (a_{ji},\alpha_{ji})\\(b_j,\beta_j)\ (b_{ji},\beta_{ji})\end{matrix}\right.\right]\right\} = \sum_{r=0}^{\mu}\frac{(-1)^r q^{\frac{r(r+1)}{2}}(q^{-\mu};q)_r}{(q;q)_r}\underset{z,\ \infty,q}{\overset{\mu-r}{D}}\left\{z^{-\lambda}\right\}$$
$$\times \underset{z,\ \infty,q}{\overset{\alpha}{D}}\left\{I^{m,n}_{A_i,B_i;R}\left[P(zq^{\mu-r})^k;q\left|\begin{matrix}(a_j,\alpha_j)\ (a_{ji},\alpha_{ji})\\(b_j,\beta_j)\ (b_{ji},\beta_{ji})\end{matrix}\right.\right]\right\} \quad (9)$$

Now, in view of (1), the L.H.S. of (9) reduces to

$$\underset{z,\ \infty,q}{\overset{\mu}{D}}\left\{z^{-\lambda}I^{m,n}_{A_i,B_i;R}\left[Pz^k;q\left|\begin{matrix}(a_j,\alpha_j)\ (a_{ji},\alpha_{ji})\\(b_j,\beta_j)\ (b_{ji},\beta_{ji})\end{matrix}\right.\right]\right\} =$$

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma_q(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma_q(1-a_j+\alpha_j s)\,\pi P^s}{\sum_{i=1}^{R}\left\{\prod_{j=m+1}^{B_i}\Gamma_q(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{A_i}\Gamma_q(a_{ji}-\alpha_{ji}s)\right\}\Gamma_q(s)\Gamma_q(q^{1-s})\sin\pi s}\underset{z,\ \infty,q}{\overset{\mu}{D}}\left\{z^{-(\lambda-ks)}\right\}ds \quad (10)$$

On utilizing fragmentary q-subsidiary recipe in the above condition, we get following fascinating changes for the work after specific rearrangements.

$$\underset{z,\,\infty,q}{\overset{\mu}{D}}\left\{z^{-\lambda} I^{m,n}_{A_i,B_i;R}\left[Pz^k;q\left|\begin{matrix}(a_j,\alpha_j)\,(a_{ji},\alpha_{ji})\\(b_j,\beta_j)\,(b_{ji},\beta_{ji})\end{matrix}\right.\right]\right\}=\frac{z^{-\lambda-\mu}q^{\mu\lambda+\mu(1-\mu)/2}}{(1-q)^{\mu}}$$

$$\times I^{m+1,n}_{A_i+1,B_i+1;R}\left[P(zq^{\mu})^k;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,A_i}(\lambda,k)\\(\mu+\lambda,k)(b_j,\beta_j)_{1,m}\,(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right] \quad (11)$$

Now taking $\lambda = 0$ and replacing μ by r and then z by $zq^{\mu-r}$ respectively in(11) to get the result

$$\underset{z,\,\infty,q}{\overset{r}{D}}\left\{z^{-\lambda} I^{m,n}_{A_i,B_i;R}\left[P(zq^{\mu-r})^k;q\left|\begin{matrix}(a_j,\alpha_j)\,(a_{ji},\alpha_{ji})\\(b_j,\beta_j)\,(b_{ji},\beta_{ji})\end{matrix}\right.\right]\right\}=\frac{z^{-r}q^{\frac{r(r+1)}{2}-r\mu}}{(1-q)^{r}}$$

$$\times I^{m+1,n}_{A_i+1,B_i+1;R}\left[P(zq^{\mu})^k;q\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,A_i}(0,k)\\(r,k)(b_j,\beta_j)_{1,m}\,(b_{ji},\beta_{ji})_{m+1,B_i}\end{matrix}\right.\right] \quad (12)$$

In view of (6), we can easily get the following result

$$\underset{z,\,\infty,q}{\overset{\mu-r}{D}}\left\{z^{-\lambda}\right\}=\frac{\Gamma_q(P+\mu-r)q^{(\mu-r)(1-\mu+r-2P)/2}z^{-P-\mu+r}}{(1-q)^{\mu}} \quad (13)$$

On substituting the estimations of the articulations engaged with (9), from (11), (12) and (13) we touch base at the correct hand side of the given hypothesis (7).

Unique cases:

In this segment, we will think about some unique instances of the fundamental outcome and derive certain extension formulae for essential simple of H, G and E-works as pursues:

(I) It is intriguing to see that for taking R = 1, Ai = A, Bi = B, we get outstanding consequence of Saxena et.al. [6]

$$H^{m+1,n}_{A+1,B+1}\left[P(zq^{\mu})^k;q\left|\begin{matrix}(a,\alpha)\,(\lambda,k)\\(\mu+\lambda,k)\,(b,\beta)\end{matrix}\right.\right]=\sum_{r=0}^{\mu}\frac{(-1)^r q^{\frac{r(r+1)}{2}+\lambda r}(q^{-\mu};q)_r(q^{\lambda};q)_{\mu-r}}{(q;q)_r}\times H^{m+1,n}_{A+1,B+1}\left[P(zq^{\mu})^k;q\left|\begin{matrix}(a,\alpha)\,(0,k)\\(r,k)(b,\beta)\end{matrix}\right.\right] \quad (14)$$

(ii) If we set $\alpha = \beta = 1,\ and k = 1$, in equation (14), we get the following interesting expansion formula for q-analogue of Meijer's G-function as,

$$G^{m+1,n}_{A+1,B+1}\left[P\,zq^{\mu};q\left|\begin{matrix}a_1,a_2,...,a_p,\lambda\\\mu+\lambda,b_1,b_2,...,b_q\end{matrix}\right.\right]=\sum_{r=0}^{\mu}\frac{(-1)^r q^{\frac{r(r+1)}{2}+\lambda r}(q^{-\mu};q)_r(q^{\lambda};q)_{\mu-r}}{(q;q)_r}\times G^{m+1,n}_{A+1,B+1}\left[P\,zq^{\mu};q\left|\begin{matrix}a_1,a_2,...,a_A,0\\r,b_1,b_2,...,b_B\end{matrix}\right.\right] \quad (15)$$

(iii) Finally for n = 0, and m = B,

$$E_q\left[B+1;b_j,\mu+\lambda,A+1,A_j,\lambda P\,zq^{\mu}\right]=\sum_{r=0}^{\mu}\frac{(-1)^r q^{\frac{r(r+1)}{2}+\lambda r}(q^{-\mu};q)_r(q^{\lambda};q)_{\mu-r}}{(q;q)_r}\times E_q\left[B+1;b_j,r,A+1,a_j,0,P\,zq^{\mu}\right]$$

CONCLUSION

On investigating the likelihood for determination of certain developments of fundamental simple I-work. The outcomes subsequently inferred and liable to discover definite uses in the hypothesis of hyper geometric capacities. At last, finish up the comment with the outcomes, administrators demonstrated in this paper seem, by all accounts, to be new and liable to have helpful applications to a wide scope of issues of arithmetic, insights and physical sciences.

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