

ANTI-MAGIC LABELINGS FOR ARMED CROWN AND DRAGON GRAPHS.

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ABSTRACT:

A Graph with "q" edges is called Anti-magic, if its edges can be labeled with 1,2,...,q without repetition such that the sum of the labels of the edges incident to each vertex are distinct. In this paper, we discuss about the anti-magic labelings for Dragon and armed crown graphs.

KEYWORDS:

Anti-magic Labeling, Dragon Graph, Armed Crown Graphs.

INTRODUCTION:

A Graph is a collection of nodes and lines that we call vertices and edges respectively. A Graph can be labeled or unlabeled. A Graph Labeling is an assignment of integers to the vertices and edges. Here in this paper we are interested in labeling a special types of graphs. In many labeled graphs, the labels are used for identification only. Labeling can be used not only to identify vertices and edges, but also to signify some additional properties depending on the particular labeling.

It may seem strange to term a graph as having an "Anti-magic" labeling, but the term comes from the connections to magic labelings and magic squares. Magic squares can trace their origin back to ancient china somewhere around the 7th century BC. A magic square is an arrangement of numbers into a square such that the sum of each row, column and diagonal are equal. The term "Anti-magic" then comes from being the opposite of magic or arranging numbers in such a way that no two sums are equal.

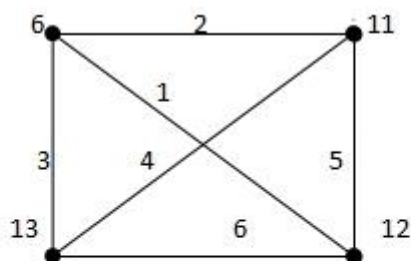
In 1990, Hartsfield and Ringel introduced the concept of anti-magic labeling, which is an assignment of distinct values to different objects in a graph in such a way that when taking certain sums of the labels, the sums will all be different. They had already conjectured that every graph except for K_2 has an anti-magic edge labeling. Then Bodendiek and Walther proved that from some natural number "n" any connected graph other than K_2 will have a weak anti-magic edge labeling if you allow the labels to be natural numbers with an upper bound of "n". A weak anti-magic labeling is similar to an anti-magic labeling except one does not require bound of "n".

In this paper we discuss about the anti-magic labeling of Armed Crown and Dragon graphs.

Definitions:

Anti-magic labeling:

A Graph with q edges is called Anti-magic, if its edges can be labeled with $1, 2, \dots, q$ without repetition such that the sums of the labels of the edges incident to each vertex are distinct.



Antimagic labeling for K_4

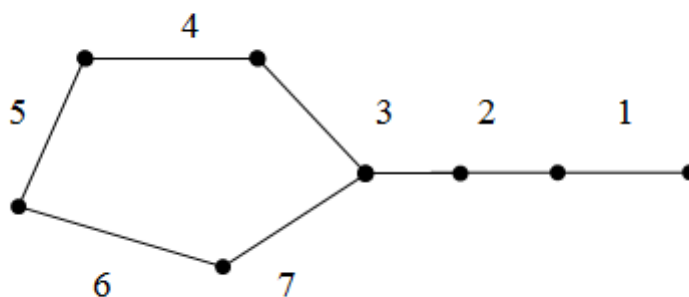
Armed Crown Graph:

The Armed Crown graph is obtained by joining three pendant edge to each vertex of the cycle C_n .

Dragon Graph:

The Graph obtained by joining a cycle Graph with a Bridge is called Dragon Graph.

Dragon Graph is otherwise known as Tadpole Graph.



Dragon Graph ($T_{5,3}$)

Main Results:

Theorem 1:

The Armed Crown Graph of cycle on “ n ” vertices is Anti-magic.

Proof:

Let v_1, v_2, \dots, v_n be the vertices of “n” cycles. Let e_1, e_2, \dots, e_n be the edges of “n” cycles.

Let us define the labeling function as $f: E(G) \rightarrow \{1, 2, \dots, 2n\}$ where “n” is the number of Edges.

For (i=1 to q-1)

$u_i v_i$ u_i of degree 1, v_i of degree 1

$$f(e_i) = i$$

$u_i v_i$ u_i of degree 2, v_i of degree 1

$$f(e_i) = i$$

$u_i v_i$ u_i of degree 2, v_i of degree 1

$$f(e_i) = i$$

$u_i v_i$ u_i of degree 3, v_i of degree 3

$$f(e_{i+1}) = i + 1$$

$u_i v_i$ u_i of degree n, v_i of degree n

$$f(e_{i+n}) = i + n$$

For (i=1 to n)

v_i of degree 3

if $i=1$

$$f(v_i) = f(e_1) + f(e_2) + f(e_{2n}) \text{ for all } i.$$

If $i \neq 1$

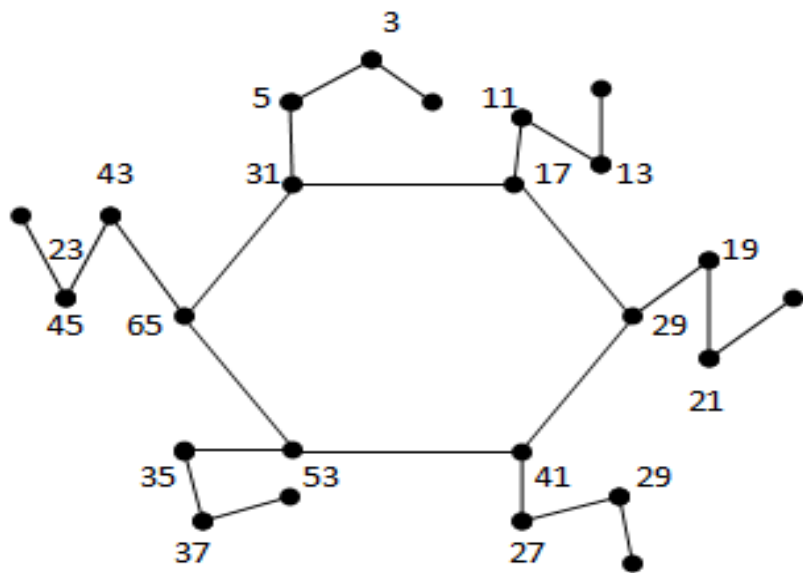
$$f(v_i) = f(e_{2n}) + f(e_{2n-1}) + f(e_{2n-2}) \text{ for all } i$$

v_i of degree 1

$$f(v_i) = f(e_i)$$

$$f(v_{i+1}) = f(e_{i+1})$$

It is verified that, edges can be labeled with $1, 2, \dots, q$ without repetition such that the sums of the labels of the edges incident to each vertex are distinct.



Antimagic Labeling for Armed Crown Graph

Theorem:2

The Dragon Graph of cycle $T_{i,3}$ vertices is Anti-magic.

Proof:

Let v_1, v_2, \dots, v_n be the vertices of “n” cycles. Let e_1, e_2, \dots, e_n be the edges of “n” cycles.

Let us define the labeling function as $f : E(G) \rightarrow \{1, 2, \dots, 2n\}$ where “n” is the number of Edges.

For (i=1 to q-1) $u_i v_i$ u_i of degree 1, v_i of degree 1

$$f(e_i) = i$$

$u_i v_i$ u_i of degree 1, v_i of degree 1

$$f(e_i) = i$$

$u_i v_i$ u_i of degree 1, v_i of degree 3

$$f(e_i) = i$$

$u_i v_i$ u_i of degree 2, v_i of degree 2

$$f(e_{i+1}) = i + 1$$

$u_i v_i$ u_i of degree n, v_i of degree n

$$f(e_{i+n}) = i + n$$

For (i=1 to n)

v_i is of degree 3

If $i=1$

$$f(v_i) = f(e_1) + f(e_2) + f(e_{2n}) \text{ for all } i.$$

If $i \neq 1$

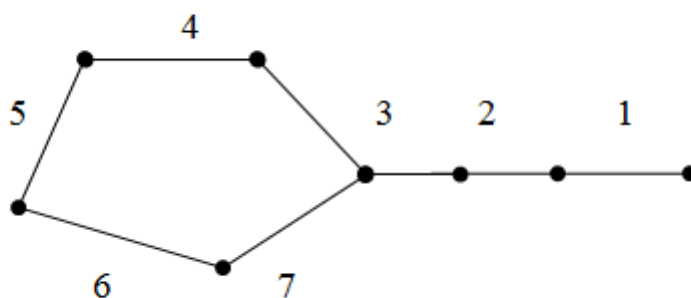
$$f(v_i) = f(e_{2n}) + f(e_{2n-1}) \text{ for all } i$$

v_i of degree 1

$$f(v_i) = f(e_i)$$

$$f(v_{i+1}) = f(e_{i+1})$$

It is verified that, edges can be labeled with $1, 2, \dots, q$ without repetition such that the sums of the labels of the edges incident to each vertex are distinct.



Antimagic Labeling for Dragon Graph

Conclusion:

In this paper, we have presented anti-magic labeling for Dragon and Armed Crown graphs.

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