A Mathematical Study of Stenosis in the presence of Pressure Gradient, Flow Rate and Shear Stress by using Magnetic Effect

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Abstract: In this paper, a precise model is planned and examined to study the effects of bell-shaped stenosis. Blood is supposed to be homogeneous incompressible and non-Newtonian fluid. Radial velocity has been taken very small. Various different parameters have been taken to found the variation of pressure gradient, flow rate and shear stress.

Index Terms: Power-law fluid model, shear stress, pressure drop, stenosis height.

I. INTRODUCTION

To become stenosis in human arteries is a common occurrence and hemodynamic factors have significance in the formation and proliferation of cardiovascular diseases. It is such type of disease which becomes a reason for thickening, hardening, and loss of elasticity of the arterial walls, leading to impaired blood circulation. Above reasons are responsible for the build-up of calcium deposition in the lumen of the artery causing stenosis. Aging, hypertension, diabetes are the main source for the becoming of stenosis.

D.S. Sankar and Ahmad Izani Md. Ismail [1] have noticed and identified that the wall shear stress, pressure drop, resistance and plug core radius in the two-fluid Casson model are significantly much lower than those of the Herschel-Bulkley model. Sahu et al. [2] observed that the resistive impedance augmented with increasing blood tube radius for a constant value of stenosis height. It may be said that resistive impedance and wall shear stress augmented for a particular value of stenosis height. Musad et al. [3] indicate that wall shear stress augmented with the increases of height stenosis and while pressure gradient so, wall shear stress augmented with the augment of length stenosis, other parameters have been taken constantly. The effect of magnetism on different fluid parameters like flow rate, blood velocity, wall shear stress, blood flow resistance and acceleration in the presence of multiple stenoses has been given by Gaurav Varshney, V.K. Katiyar, Sushil Kumar [4]. All the blood flow uniqueness are examined to be affected by the influence of applied magnetism as well as the existence of multiple stenoses. Laskar et al. [5] concluded that the blood flow is generally affected by the non-Newtonian rheology, change in the blood flow pattern and augment in the shear stress at the wall, in comparisons with those in a Newtonian model. Saktipada Nanda and Ratan Kumar Bose [6] have examined from the theoretical investigation that the blood flow resistance at the stenosed areas of the artery augmented marginally with the augment of the average flow velocity. A precise model for examining the non-Newtonian blood flow through a stenosed arterial area has been developed by Pankaj Mathur and Surekha Jain, [7]. It has been concluded that the pressure drop as well as shear stress increases as the size of stenosis augmented for a given non-Newtonian precise model of the blood. Alimohamadi et al. [8] show that the changing in temperature and pressure reduction is significantly related to the value of magnetic field intensity. A precise study of pressure drop and shear stress of non newtonian blood has done by jain et al. [9]. Noreen Sher Akbar [10] has investigated that resistive impedance in a diverging tapering emerges to be smaller rather than those in converging tapering as the blood flow rate is higher in the previous than that in the later, as anticipated, as well as impedance resistance, have its maximum worths in the symmetric stenosis. The precise analysis of heat, as well as mass transfer on the blood flow through a tapered stenosed pipe with the suspension of nanoparticles, has been given by Sapna R Shah, Rohit Kumar and Anamika [11]. It concluded that the velocity profile decreases with an augment in the Grashof number and local Grashof number whereas the temperature profile and concentration profile decreases with an augment in the Brownian motion parameter and Thermophoresis parameter. Blood flow behavior was studied by Sharma and Raghav [12]. This study says that Hemodynamic play the main role to build an arterial single or multiple stenoses in the body, which indicate to the abnormalities of the cardio system. Blood functioning, especially in the diseased arteries can be identified by mathematical simulation. A detailed study of modeling of uncharacteristic blood flow through arteries in the existence of slip condition has done by Raghav and Sharma [13]. Further, they examined the modeling of abnormal blood flow through arteries in the occurrence of stenosis, slip condition and magnetic field [14].

II. DEVELOPMENT OF THE MODEL

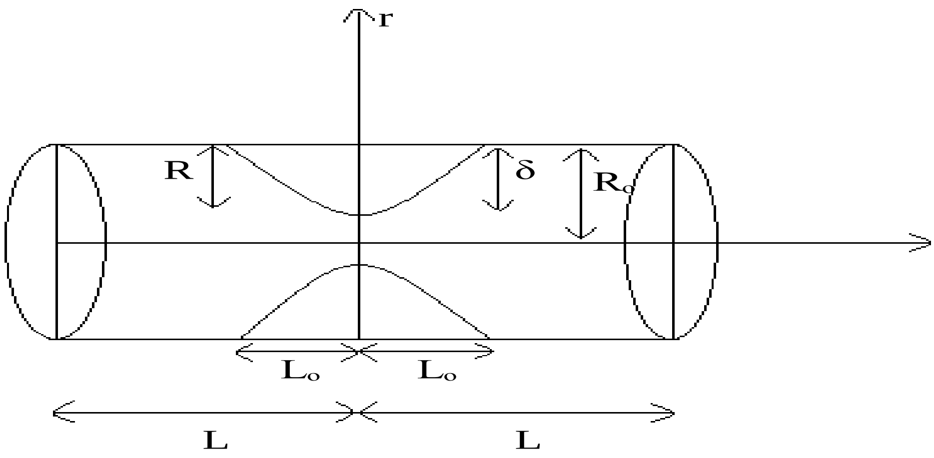


Figure 1: Geometry of bell-shaped stenosis in the artery

The radius of the artery depends upon the geometry of the stenosis and can be written as follows

$$R_o = R\left\{1 - \frac{\sigma}{R} e^{-\alpha z^2}\right\} \quad 1(a)$$

Where $\alpha = \frac{M^2\beta^2}{{R_o}^2}$

And δ is the height of the stenosis assumed to be much smaller in comparison to the unobstructed radius R of the artery $(\delta \ll R_o)$. R_o is the radius of the artery in the stenosed region at the axial distance z. M is a parametric constant, β is the relative length of the constriction, defined as the ratio of the radius to the half-length of the stenosis, i.e.,

$$\beta_o = \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}} \quad 1(b)$$

Where β_o is the magnetic effect

Now let us consider the laminar and steady flow of the fluid. When the inertial and entrance effects are neglected, the one-dimensional flow equation is given by

$$0 = \frac{-dp}{dz} + \frac{\mu}{r}\frac{d}{dr}\left\{r\left(\frac{-dw}{dr}\right)^n\right\} + \beta_o \quad (2)$$

Where w is the axial velocity and p is the fluid pressure. The boundary conditions associated with equation (2) are given as follows:

$$\frac{w}{dr} = 0 \text{ at } r = 0 \quad 3(a)$$

$$w = 0 \text{ at } r = R_o \quad 3(b)$$

III. ANALYTICAL SOLUTION OF THE PROBLEM

Following Shukla et. al. (1979) and solving equation (2) and using equation (3) we get

$$w = \frac{n}{n+1}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \quad (4)$$

Also in case of no stenosis $\delta = 0$

$$w' = \frac{n}{n+1}\left(\frac{P}{2\mu}\right)^{\frac{1}{n}}\left({R_o}^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right) \quad (5)$$

From the above two results we have

$$\bar{w} = \frac{w}{w'} = \frac{R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}}{{R_o}^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}} \quad (6)$$

The constant flux Q is given by

$$Q = \int_0^R 2\pi r w dr$$

$$Q = \pi \int_0^R r^2 \left(\frac{-dw}{dr}\right) dr \quad (7)$$

From equation (2)

$$\left(\frac{-dw}{dr}\right) = \left\{\frac{r}{2}\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_O\sqrt{\mu\sigma}}\right]\right\}^{\frac{1}{n}} \tag{8}$$

Using equation (8) in (7)

$$Q = \pi\int_0^R r^2\left(\frac{-dw}{dr}\right)dr$$

$$Q = \frac{\pi n R^{\frac{3n+1}{n}}}{(3n+1)2^n}\left[\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_O\sqrt{\mu\sigma}}\right]^{\frac{1}{n}} \tag{9}$$

$$\frac{dp}{dz} = \mu\left\{\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right\}^n + \frac{H_a}{R_O}\sqrt{\frac{\mu}{\sigma}} \tag{10}$$

$$dp = \left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\}dz$$

Integrating equation (9) along with the condition $p = p_o$ at $z = -L$, and $p = p_L$ at $z = L$

$$\int_{p_0}^{p_L} dp = \left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\}\int_{-L}^{L} dz$$

$$p_L - P_0 = \left\{\mu\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_o}\sqrt{\frac{\mu}{\sigma}}\right\}2L \tag{11}$$

The shearing stress at the wall is given by

$$\tau = \mu(r)\left(\frac{-dw}{dr}\right)^n$$

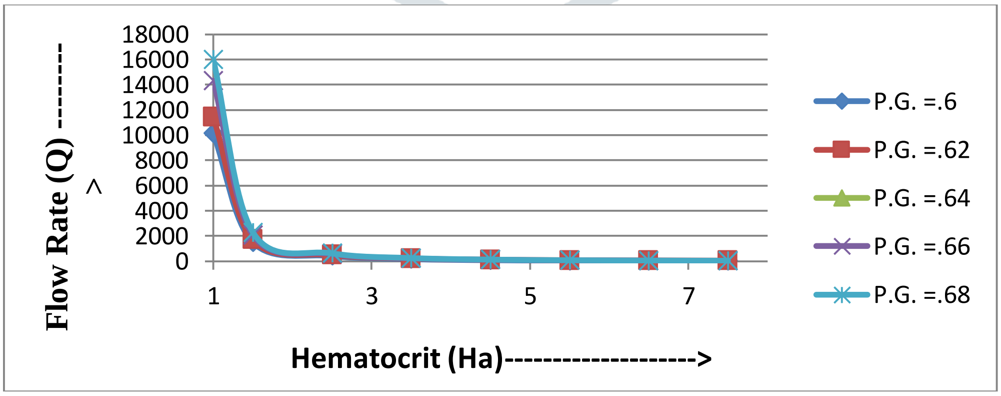
$$\tau = \mu\left[\frac{r}{2}\left(\frac{1}{\mu}\frac{dp}{dz} - \frac{H_a}{R_O\sqrt{\mu\sigma}}\right)\right]$$

Using equation (8)

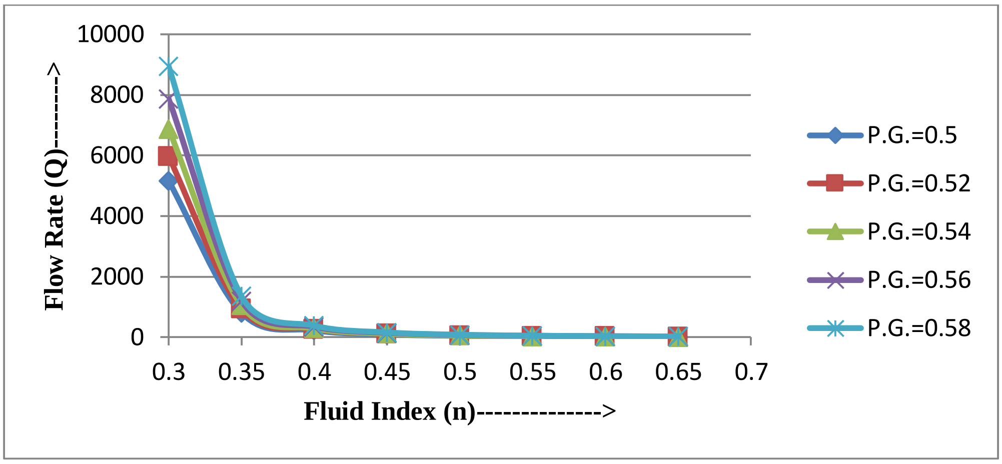
$$\tau = \frac{r\mu}{2}\left[\frac{Q(3n+1)2^n}{\pi n R^{\frac{3n+1}{n}}}\right]^n + \frac{H_a}{R_O}\sqrt{\frac{\mu}{\sigma}}\left(\frac{r}{2} - 1\right) \tag{12}$$

IV. RESULTS AND DISCUSSION

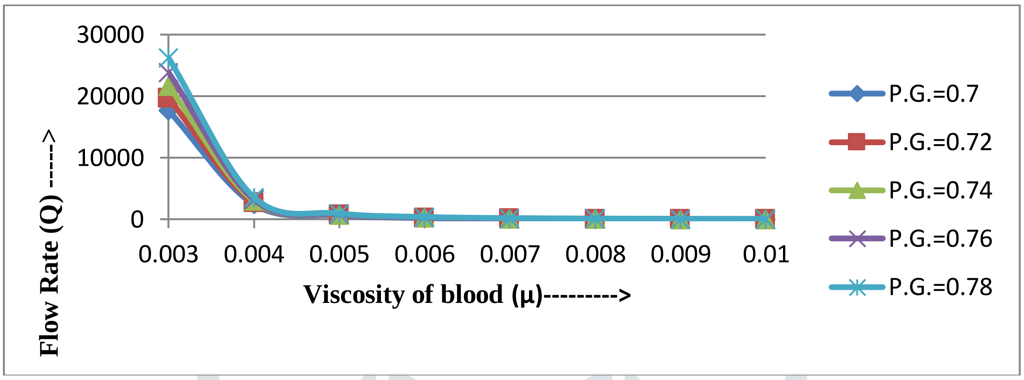
The expression for flow rate given by equation (9) is plotted in the graph (1) to (3). Graph (1) shows that the flow rate decreases with increasing pressure gradient and Hematocrit. Similarly, graph (2) and (3) shows that the flow rate decreases with increasing pressure gradient and fluid index as well as the viscosity of blood. The expression for pressure gradient given by equation (10) is plotted in the graph (4) to (6). Graph (4) shows that the pressure gradient increases with increasing flow rate and fluid index. Similarly, graph (5) and (6) shows that the pressure gradient also increases with increasing flow rate and Hematocrit as well as the viscosity of blood. The expression for shearing stress given by equation (12) is plotted in the graph (7) to (9). The graph (7) shows that the shearing stress increases with increasing the flow rate and fluid index. Similarly, graph (8) and (9) shows that the shearing stress also increases with increasing flow rate and Hematocrit as well as the viscosity of blood.



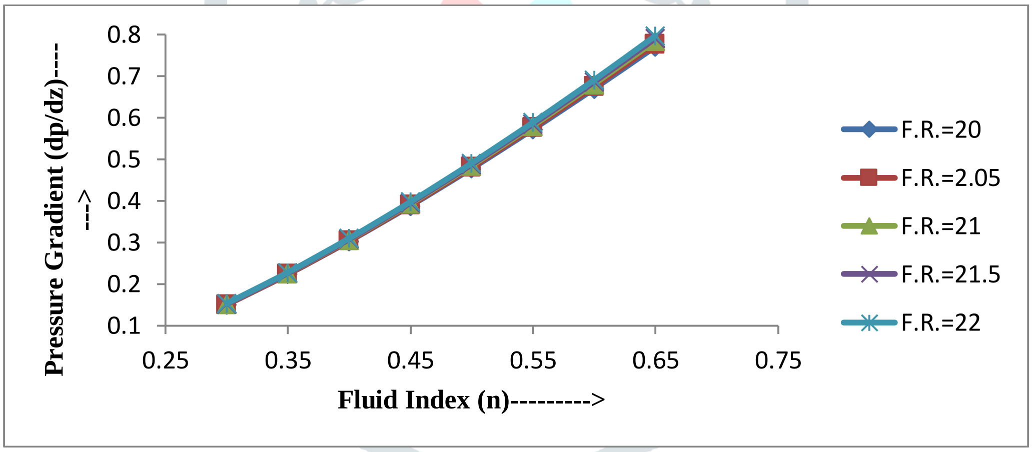
Graph 1: Variation of flow rate with different Hematocrit (Ha) and pressure gradient



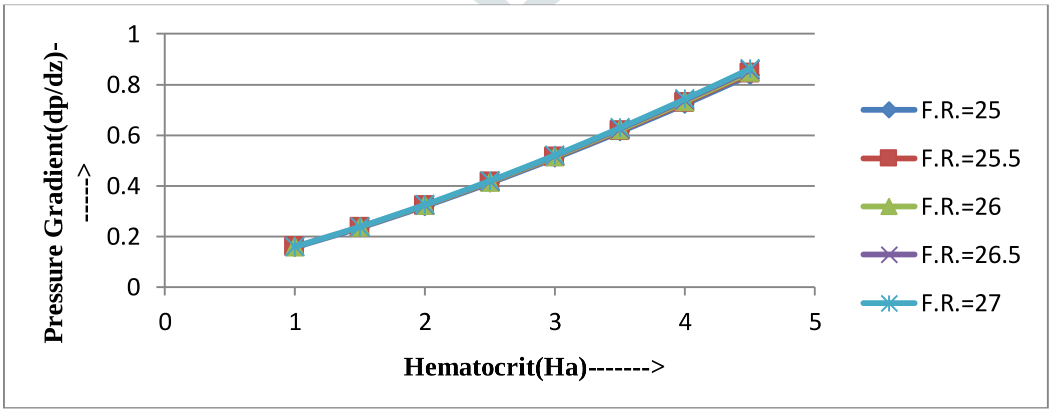
Graph 2: Variations of Flow Rate with different Fluid Index (n) and Pressure Gradient



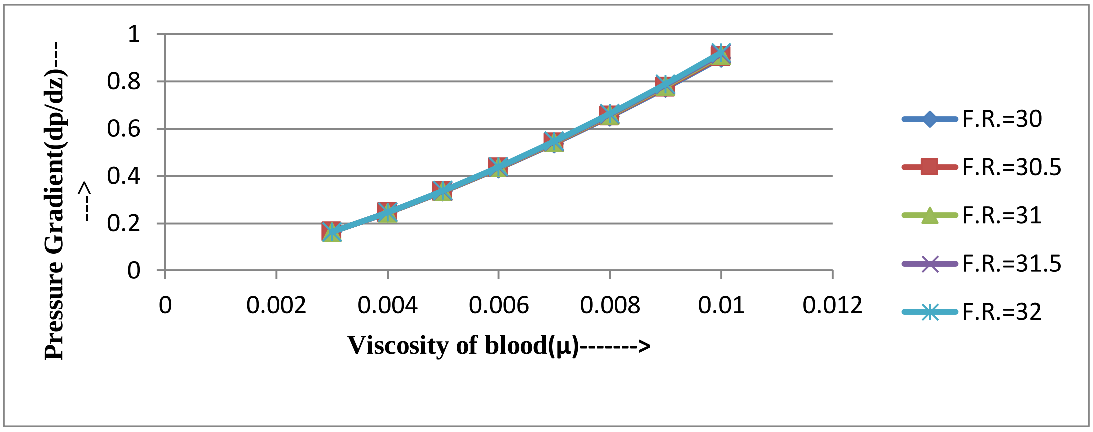
Graph 3: Variations of Flow Rate in the terms of different Viscosity of Blood (μ) and Pressure Gradient



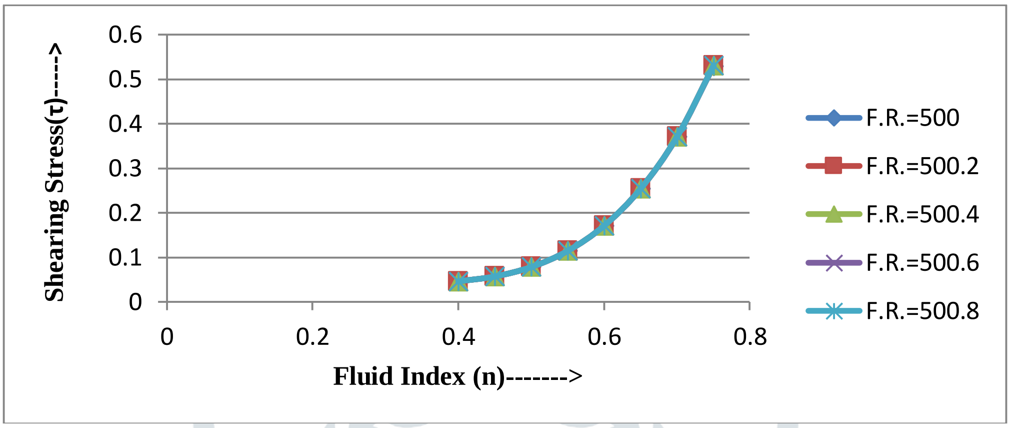
Graph 4: Variations of Pressure gradient with different Fluid Index (n) and Flow Rate



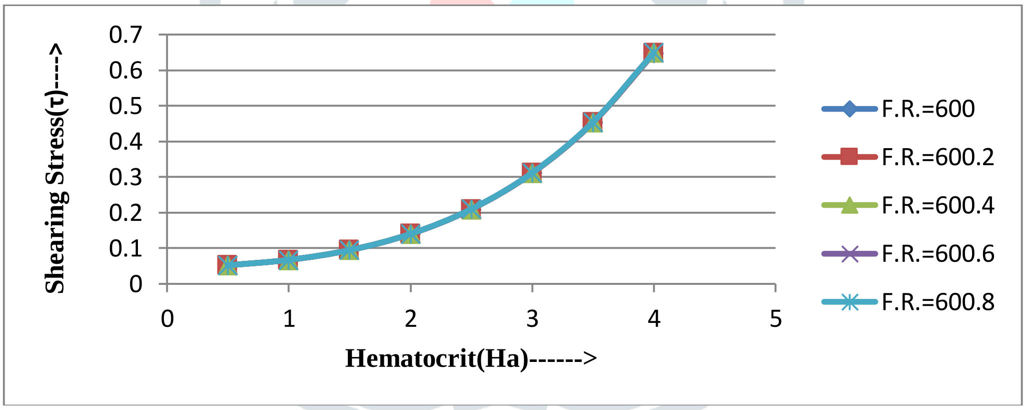
Graph 5: Variations of Pressure Gradient with different Hematocrit(Ha) and Flow Rate



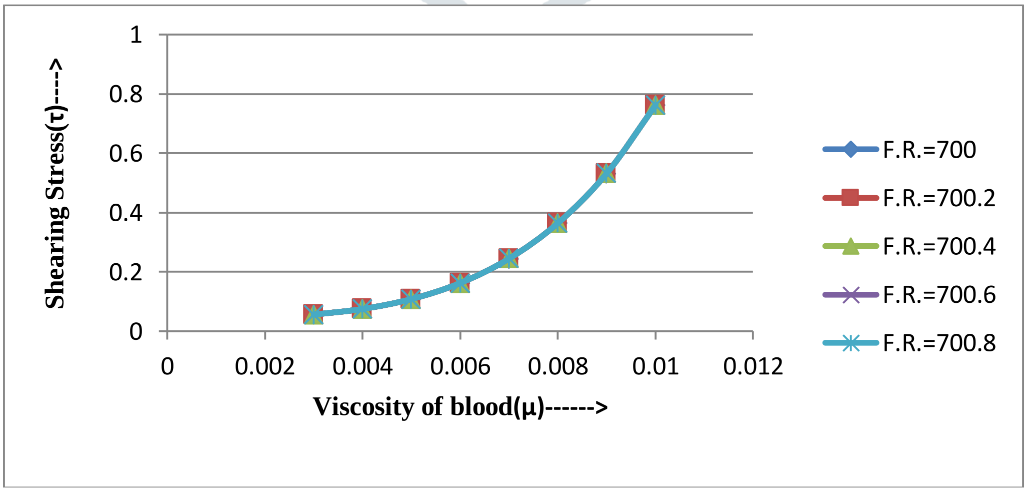
Graph 6: Variations of the Pressure Gradient in terms of Viscosity of Blood (μ) and Flow Rate



Graph 7: Variations of Shearing Stress (τ) with different Fluid Index (n) and Flow Rate



Graph 8: Variations of Shearing Stress with different Hematocrit(Ha) and Flow Rate



Graph 9: Variations of Shearing Stress in terms of Viscosity of Blood (μ) and Flow Rate

V. CONCLUSION

In this paper, a deep study has been done to find the results. Variation of flow rate, pressure gradient, and shearing stress has been analyzed in the presence of other parameters. Graph 1 to 3 reveals that variation of flow rate is decreasing in the presence of pressure gradient, fluid index and hematocrit value, graph 4 to 6 reveals that variation of pressure gradient is increasing in the presence of fluid index, flow rate, hematocrit and viscosity of blood whereas graph 7 to 9 reveals that variation of shearing stress increases in the presence of fluid index, flow rate, hematocrit and viscosity of blood.

VI. ACKNOWLEDGMENT

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