Fuzzy labeling on wheel graph

M.K .Pandurangan[1*], T.Bharathi[2], S.Antony Vinoth[3]

[1]*Associate Professor, Pachaiyappa's College, Chenai-600 030*
[2]*z Assistant Professor, Loyola College (Autonomous), Chenai-600 034*
[3]*Research Scholar, Loyola College (Autonomous), Chenai-600 034*

Abstract: In this paper the concept of fuzzy labeling for wheel graph has been introduced. The generalized formula for *n*- dimensional wheel graph of the vertices and edges has been discussed. A graph G = (σ, μ) is said to be a fuzzy labeling graph, if σ: *V* → [0, 1] and μ: *V*×*V* → [0, 1] is bijective such that the membership value of edges and vertices are distinct and μ (*u*, *v*) < σ (*u*) Λ σ (*v*) for all *u*, *v* ϵ *V*.

Keywords: Fuzzy path, Fuzzy graph, Fuzzy labeling graph, Wheel graph.

1. Introduction

In 1965, Zadeh introduced concept of uncertainty by fuzzy set [5]. A fuzzy set is defined mathematically by assigning each possible individual in the universe of discourse a value, representing its grade of membership, which corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set.

The fuzzy graph was introduced by Rosen field using fuzzy relation where represents the relationship between the objects by previously indicating the level of the relationship between the objects of the function sets [3]. Fuzzy graphs have many more applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision [6].

2. Preliminaries

The basic concept of fuzzy graph and fuzzy labelling graph were discussed. Let U and V be two sets. Then ρ is said to be a fuzzy relation from *U* into *V* if ρ is a fuzzy set of *U*×*V*. A fuzzy graph $G = (V, \sigma, \mu$) of a nonempty set V together with a pair of function σ: $V \rightarrow [0,1]$ and μ: $E \rightarrow [0,1]$,it satisfy $\mu(v, u) \leq \sigma(v) \wedge \sigma(u)$ for all $v, u \in V$. A path *P* in a fuzzy graph is a sequence of distinct nodes $v_1, v_2 \ldots v_n$ such that $\mu(v_1, v_{i+1})$ >0; 1≤ *i* ≤ *n*; here *n* ≥ 1 is called the length of the path *P*. The consecutive pairs (v_1, v_{i+1}) are called the edge of the path.

A path P is called a cycle if $v_1 = v_n$ and *n* ≥ 3. The strength of a path *P* is defined to be the weight of the weakest arc of the path. An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of the connectedness of its end nodes when it is deleted. A graph $G = (V, \sigma, \mu)$ is said to be a fuzzy labelling graph, if σ: $V \rightarrow [0,1]$ and μ:$V \times V \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ [1] [2].

An elegant definition of a metric in a fuzzy graph has been given by Rosenfeld [3]. If ρ is the path consisting of the vertices $x_1, x_2 \ldots x_n$ in a fuzzy graph $G = (\tau, \gamma)$, the μ- length fuzzy graph is define by $l(\rho)$ where$l(\rho) = \sum_{i=1}^{n} \mu(x_{i-1}, x_i)^{-1}$ For two vertices *x*, *y* in *G*, the μ- length of all paths joining *X* and *Y*. The μ- distance $\delta(u, v)$ is the smallest μ -length of any $u - v$ path and δ is metric. Suppose $G = (\sigma, \mu)$is a fuzzy graph with *V* as the set of vertices. The eccentricity $e(v)$ of a vertex $v \in V$ is defined to be the maximum of all the μ- distance $\delta(v, u)$ for all *u* in *V*. The radius of a connected fuzzy graph is the minimum of all eccentricities of the vertices of the fuzzy graph. An eccentric node *v*, is a node v^* such that $e(v) = \delta(v, v^*)$. A node *v* is called a diametrical node if $e(v) = diam(G)$ [3] [6]. If *G** is a cycle then the fuzzy labeling cycle G_ω has exactly only one weakest arc [1].

Definition2.1: A cycle graph *G* is said to be a fuzzy labeling cycle graph if it has fuzzy labelling [1].

Proposition2.1: Let G_ω be a fuzzy labeling cycle such that *G** is a cycle, then it has $(n - 1)$ bridges.

Proposition2.2: Let $G = (\sigma_\omega, \mu_\omega)$ be a fuzzy graph such that G* is a cycle. Then a node is a fuzzy cut node of G if and only if it is a common node of two fuzzy bridges [7].

Proposition2.3: If G* is a cycle with fuzzy labeling then it has $(n - 2)$ cut nodes [1].

3. Methodology

3.1 Fuzzy graphs

A fuzzy graph $G = (V, \sigma, \mu)$ is a triple consisting of a nonempty set V together with a pair of functions $\sigma : V \longrightarrow [0, 1]$ and $\mu : E \longrightarrow [0, 1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$.

3.2 Fuzzy labeling

A graph $G = (\sigma, \mu)$ is said to be a fuzzy labeling graph, if σ: $V \longrightarrow [0, 1]$ and μ: $V \times V \longrightarrow [0, 1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Proposition3.1: If *G** is a cycle with fuzzy labeling then, the graph has exactly two end nodes.

Proposition3.2: If G_ω is a fuzzy labeling cycle graph, then every bridge is strong and vice versa.

Proposition3.3: If G_ω is a connected fuzzy labeling graph then there exists a strong path between any pair of nodes.

Proposition3.4: Every fuzzy labeling graph has atleast one weakest arc. Proposition: For any fuzzy labeling graph G_ω , $\delta(G_\omega)$ is a fuzzy end node of G_ω such that the number of arcs incident on, $\delta(G_\omega)$ is at least two.

Proposition3.5: Every fuzzy labeling graph has at least one end nodes.[1].

4. Result and discussion

4.1 Fuzzy labelling of Wheel graph

Wheel graph is obtained from a cycle C_n by adding a new vertex and edges joining it to all the vertices of the cycle [7].A wheel graph is said to be a fuzzy wheel graph in which all the vertices and edges has membership function and said to satisfy

$\gamma(v_i, v_j) \leq \tau(v_i) \wedge \tau(v_j)$ for all $v_i, v_j \in V$

Fuzzy wheel graph is denoted by FW_n.

Theorem1: Every n – dimensional wheel graph admits fuzzy labelling of a graph.

Proof: Let W_n be wheel graph, Define a fuzzy wheel graph $FW_n = (V, \tau, \gamma)$, a non-empty set V together with a pair function $\tau : V \rightarrow [0,1]$ and $\gamma : E \rightarrow [0,1]$ for all $v_k, v_l \in V$,it satisfy

$$\gamma(v_k, v_l) \leq \tau(v_k) \wedge \tau(v_l) \rightarrow (1.1)$$

A fuzzy labeling of a graph $FW_n = (V, \tau, \gamma)$ of a nonempty set V together with a pair of functions $\tau : V \rightarrow [0,1]$ and $\gamma : E \rightarrow [0,1]$ it satisfy

$\gamma(v_k, v_l) < \tau(v_k) \wedge \tau(v_l) \rightarrow (1.2)$ for all $v_k, v_l \in V$

Wheel graph is obtained from a cycle C_n by adding a new vertex and edges joining it to all the vertices of the cycle

Let us consider the n- dimension of wheel graph, $n \geq 3$, is obtained from a cycle C_n by adding a new vertex and edges joining it to all the vertices of the cycle

Define the membership function of the vertices of a wheel graph is

$\tau(v_i) = \frac{2n+i}{10^{n-2}}$ where i =1,2,3….n

Case (i)

Let us consider any the consecutive path of W_n which contains all the vertices and maximum number of edges of the graph of a graph W_n,

$v_1, v_2, v_2 \ldots\ldots v_i \in V$ and $e_1, e_2, e_3 \ldots\ldots e_{i-1} \in V$

The path of W_n is

$v_1 e_1 v_2 e_2 \ldots\ldots v_i e_{i-1} \in P_n$

Then define every pair of the vertices of the edges is

$\gamma(e_i): v_i \times v_{i+1} \rightarrow \frac{i}{10^{n-2}}$ where i =1,2,3….$2n$

Case (ii)

Let us consider star graph S_n of W_n,

In star graph, the edges which adjacent with v_n expect e_{i-1} edge. Define

$\gamma(e_{n+i}): v_{n+1} \times v_i \rightarrow \frac{n+i}{10^{n-2}}$ where i =1,2,3….$2n$

Case (iii)

Consider the edge which lies in path of the cycle does not belong in the path P_n (i,e) only one edge of W_n.

$e_i: v_{n-1} \times v_i \rightarrow \frac{i}{10^{n-2}}$, where i =$2n$.

Here every membership function of vertices $\tau(v_1), \tau(v_2), \tau(v_3) \ldots \tau(v_n)$ and $\gamma(v_k, v_l)$ are distinct, and it satisfies the equation 1.1 and 1.2

Hence n - wheel graph admits Fuzzy labelling of graph.

5. Algorithm of fuzzy labeling wheel graph

Step:1 Fix n, (i,e) the dimension of a wheel graph

Step:2 Fix the membership value to maximum number edges of any path of W_n by

$\gamma(e_i): v_i \times v_{i+1} \rightarrow \frac{i}{10^{n-2}}$,where i =1,2,3….$2n$.

Step:3 Fix the membership value of remaining edges which does not belong to path P_n by

$\gamma(e_{n+i}): v_{n+1} \times v_i \rightarrow \frac{n+i}{10^{n-2}}$,where i =1,2,3….$2n$.

Step:4 Fix the membership value of remaining edges which does not belong to path P_n and star graph by

$\gamma(e_i): v_{n-1} \times v_i \rightarrow \frac{i}{10^{n-2}}$,where i =$2n$.

Step:5 After giving membership value to all edges, than fix membership function to vertices by

$\tau(v_i) = \frac{2n+i}{10^{n-2}}$ where i =1,2,3….n.

Step:6 By step 1,step 2,step 3 must satisfies $\gamma(v_k, v_l) < \tau(v_k) \wedge \tau(v_l)$ for all $v_k, v_l \in V$ otherwise repeat step2.

Verification:

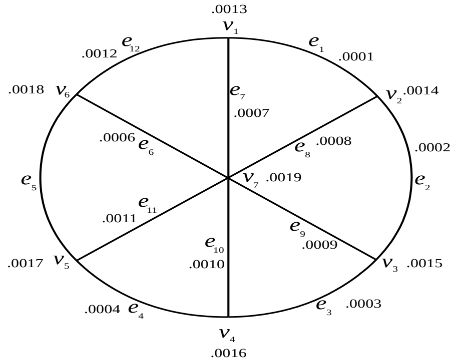

Figure 5.1: Fuzzy labelling of wheel graph

Step:1 Fix $n = 6$

Step:2 In any consecutive path,

$i = 1$ then $\gamma(e_1): v_1 \times v_2 \to \frac{1}{10^{6-2}}$ which implies membership of $\gamma(e_1)$is 0.0001

$i = 2$ then $\gamma(e_2) = 0.0002$

$i = 3$ then $\gamma(e_3) = 0.0003$

$i = 4$ then $\gamma(e_4) = 0.0004$

$i = 5$ then $\gamma(e_5) = 0.0005$

$i = 6$ then $\gamma(e_6) = 0.0006$

Step:3 Fix the membership value of edges (i,e) v_7 adjacent with $v_1, v_2, v_3, v_4, v_5, v_6$

$i = 7$ then $e_7: v_7 \times v_1 \to \frac{6+1}{10^{6-2}}$ (i,e) $\gamma(e_7) = 0.0007$

$i = 8$ then $\gamma(e_8) = 0.0008$

$i = 9$ then $\gamma(e_9) = 0.0009$

$i = 10$ then $\gamma(e_{10}) = 0.0010$

$i = 11$ then $\gamma(e_{11}) = 0.0011$

Step:4 Fix membership value of edge which does not belong to cyle of the path

$i = 1$ $\gamma(e_{12}): v_5 \times v_1 \to \frac{12}{10^4} = 0.0012.$

Step:5 There are 7 vertices of wheel graph FW_6 then fix membership function of the vertices are $v_1, v_2, v_3, v_4, v_5, v_6, v_7$

$\tau(v_1) = \frac{2(6)+1}{10^{6-2}} = 0.0013$

$\tau(v_2) = 0.0014$

$\tau(v_3) = 0.0015$

$\tau(v_4) = 0.0016$

$\tau(v_5) = 0.0017$

$\tau(v_6) = 0.0018$

$\tau(v_7) = 0.0019$

6. Conclusion

In In this paper, the concept of fuzzy labeling has been derived for wheel graphs. Fuzzy labeling for cycle related graph have been discussed. We further extend study on cycle free graph.

References

[1] A Nagoor Gani D. Rajalaxmi (a) Subahashini properties of Fuzzy Labeling Graph, Ap-plied Mathematical Sciences, Vol. 6, 2012, no. 70, 3461 – 3466

[2] A Nagoor Gani D. Rajalaxmi (a) Subahashini Metric in Fuzzy Labeling Graph Interna-tional journal of Fuzzy Mathematical Ar-chive Vol. 5, No. 2, 2014, 113-122.

[3] A.Rosenfeld, Fuzzy Graph, In: L.A. Za-deh, K.S. Fu and M.Shimura, Editors, Fuzzy Sets And their Applications to cogni-tive and decision Process, Academic press, New York (1975) 77-95.

[4] L.A.Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.

[5] M.S.Sunitha, A.Vijayakumar, A characterization of fuzzy trees, Information sciences 113(1999) 293-300.

[6] P.Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letters, 6(1987) 297-302.

[7] T.Wang and D.M.Li, Gracefulness of special graph classes, J.Whuan Univ. Natur. Sci. Ed, 58, no.5 (2012) 437-440.