PASCAL TRIANGLE IN BINOMIAL EXPANSION

Prof.SunithaJanga

Assistant Professor, T. John Institute of Management and Science, Bangalore, India,

ABSTRACT:

This paper aims to understand the concepts of binomial theorem and pascal triangle.This paper also focusing on a right utilization of binomial coefficients using pascaltriangle instead of combinatorial notations in binomial articulations which means the direct application in finding binomial coefficients.This paper also facilitates better idea on expanding the binomials by the raised exponents in difffent ways.This article also provides clear picture on what magic involved in pascal triangle in giving idea on fobonacci series,counting numbers etc…

Keywords:*Articulations, Binomial theorem ,Binomial coefficients, Combinatorial notations, Fibonacci series, Pascal triangle.*

I.Introduction

In mathematics, Pascal's triangle is a triangular array of the binomial coefficients. In much of the Western world, it is named after the French mathematician Blaise Pascal, although other mathematicians studied it centuries before him in India, Persia (Iran), China, Germany, and Italy.

Blaise Pascal was an interesting dude. He studied physics, philosophy, religion, and mathematics—with maybe just a little help from alien polynomials from a certain planet. He found a numerical pattern, called Pascal's Triangle, for quickly expanding a binomial expansions.

II.Importance of pascal triangle in binomial expansions:

Using Pascal's triangle to expand a binomial expression We will now see how useful the triangle can be when we want to expand a binomial expression. Consider the binomial expression a + b, and suppose we wish to find $(a + b)^2$.

We know that $(a + b)^2 =\binom{2}{0}a^2+\binom{2}{1}a^{2-1}b^{2-1} +\binom{2}{2}b^2$

That is,

$(a + b)^2= 1a^2+ 2ab +1b^2$

Observe the following in the final result:

1. As we move through each term from left to right, the power of a decreases from 2 down to zero.

2. The power of b increases from zero up to 2.

3. The coefficients of each term, (1, 2, 1), are the numbers which appear in the row of Pascal's triangle beginning 1,2.

4. The term 2ab arises from contributions of 1ab and 1ba, i.e. 1ab + 1ba = 2ab. This is the link with the way the 2 in Pascal's triangle is generated; i.e. by adding 1 and 1 in the previous row.

If we want to expand (a + b)3 we select the coefficients from the row of the triangle beginning 1,3: these are 1,3,3,1. We can immediately write down the expansion by remembering that for each new term we decrease the power of a, this time starting with 3, and increase the power of b. So $(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

And so on.

III.Binomial Theorem

The Binomial Theorem is a fast way of growing (or increasing out) a binomial articulation that has been raised to a few (for the most part awkwardly huge) control. For example, the articulation $(3x – 2)^{10}$ would be exceptionally agonizing to increase out by hand. Fortunately, someone made sense of a recipe for this development, and we can plug the binomial 3x – 2 and the power 10 into that equation to get that extended (duplicated out) shape.

$$\begin{aligned}(3x-2)^{10} = {}_{10}C_0\,(3x)^{10-0}(-2)^0 + {}_{10}C_1\,(3x)^{10-1}(-2)^1 + {}_{10}C_2\,(3x)^{10-2}(-2)^2 \\ + {}_{10}C_3\,(3x)^{10-3}(-2)^3 + {}_{10}C_4\,(3x)^{10-4}(-2)^4 + {}_{10}C_5\,(3x)^{10-5}(-2)^5 \\ + {}_{10}C_6\,(3x)^{10-6}(-2)^6 + {}_{10}C_7\,(3x)^{10-7}(-2)^7 + {}_{10}C_8\,(3x)^{10-8}(-2)^8 \\ + {}_{10}C_9\,(3x)^{10-9}(-2)^9 + {}_{10}C_{10}\,(3x)^{10-10}(-2)^{10}\end{aligned}$$

Note how the featured counter number checks up from zero to 10, with the variables on the closures of each term having the counter number, and the factor in the center having the counter number subtracted from 10. This example is all you truly need to think about the Binomial Theorem; this example is the means by which it works.

The formal articulation of the Binomial Theorem is as per the following:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

Or we can also expand

If n is a positive integer,

then $(a + b)^n = {}^nC0\ a^n\ b^0 + {}^nC1\ a^{n-1}\ b^1 + {}^nC2\ a^{n-2}\ b^2 + \dots + {}^nCr\ a^{n-r} b^r + \dots + {}^nCn\ a^0 b^n$

Here, nC0, nC1, nC2, … ,nCn are called binomial coefficients

And nCr = n! / r!(n – r)! for $0 \le r \le n$.

let's try using it for $n = 3$:

$$
\begin{aligned}
(a+b)^3 &= \sum_{k=0}^{3} \binom{3}{k} a^{3-k} b^k \\
&= \binom{3}{0} a^{3-0} b^0 + \binom{3}{1} a^{3-1} b^1 + \binom{3}{2} a^{3-2} b^2 + \binom{3}{3} a^{3-3} b^3 \\
&= 1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3 \cdot a^1 b^2 + 1 \cdot a^0 b^3 \\
&= a^3 + 3a^2 b + 3ab^2 + b^3
\end{aligned}
$$

IV.Properties of Binomial Theorem for Positive Integer n

(i) Total number of terms in the expansion^n of $(x + a)^n$ is $(n + 1)$.

(ii) The sum of the indices of x and a in each term is n.

(iii) The above expansion is also true when x and a are complex numbers.

(iv) The coefficient of terms equidistant from the beginning and the end are equal. These coefficients are known as the binomial coefficients and ${}^nC_r = {}^nC_{n-r}$, r = 0,1,2,…,n.

(v) General term in the expansion of $(x + a)^n$ is given by $T_{r+1} = {}^nCrx^{n-r} a^r$.

(vi) The values of the binomial coefficients steadily increase to maximum and then steadily decrease

(vii) The coefficient of x^rin the expansion of $(1+ x)^n$ is ${}^nCr$.

(viii) (a) If n is odd, then $(x + a)^n + (x – a)^n$and $(x + a)^n – (x – a)^n$ both have the same number of terms equal to (n +1) / 2.

(ix) If n is even, then $(x + a)^n + (x – a)^n$has (n +1) / 2 terms. and $(x + a)^n– (x – a)^n$ has (n / 2) terms.

(x) In the binomial expansion of $(x + a)^n$, the r th term from the end is (n – r + 2)th term

from the beginning.

(xi) If n is a positive integer, then number of terms in $(x + y + z)^n$ is (n + l)(n + 2) / 2.

(xii)Middle term in the Expansion of $(1 + x)^n$

(i) If n is even, then in the expansion of $(x + a)^n$, the middle term is (n/2 + 1)th term.

(ii) If n is odd, then in the expansion of $(x + a)^n$, the middle terms are (n + 1) / 2 th term and (n + 3) / 2 th term.

xiii) Greatest Coefficient

i)If n is even, then in $(x + a)^n$, the greatest coefficient is ${}^nCn / 2$

(ii) Ifn is odd, then in $(x + a)^n$, the greatest coefficient is ${}^nCn – 1 / 2$ or ${}^nCn + 1 / 2$ both being equal.

xiv) i) If n is a positive integer, then $(1 + x)^n$ contains (n +1) terms i.e., a finite number of terms. When n is general exponent, then the expansion of $(1 + x)^n$ contains infinitely many terms.

(ii) When n is a positive integer, the expansion of $(l + x)^n$ is valid for all values of x. If n is general exponent, the expansion of $(1+ x)^n$ is valid for the values of x satisfying the condition $|x| < 1$.

Expansion of $(1+x)^n$ for any rational index

i) $(1 + x)^{-1} = 1 – x + x^2 – x^3 + \ldots\infty$

ii) $(1 – x)^{-1} = 1 + x + x^2 + x^3 + \ldots\infty$

(iii) $(1 + x)^{-2} = 1 – 2x + 3x^2 – 4x^3 + \ldots\infty$

(iv) $(1 – x)^{-2} = 1 + 2x + 3x^2 – 4x^3 + \ldots\infty$

(v) $(1 + x)^{-3} = 1 – 3x + 6x^2 – \ldots\infty$

(vi) $(1 – x)^{-3} = 1 + 3x + 6x^2 – \ldots\infty$

(vii) $(1 + x)^n = 1 + nx$, if $x^2, x^3,\ldots$ are all very small as compared to x.

Suppose we want to find the middle term in $(3 – 2x)^5$.

Since n=5 which is odd the middle terms can be (n+1)/2 & (n+3)/2

i.e. (5+1)/2 &(5+3)/2

3rd &4th terms are the middle terms.

$T_{r+1} = {}^nCrx^{n-r} a^r$.

$T_3=\binom{5}{2}3^{5-2}(-2x)^2$

$=\frac{5*4}{2!}3^3*4x^2$

$=360x^2$

V.Binomial coefficients:

Let both *n* and *k* are natural numbers and if *k* can also have the value of 0 and $k\leq n$.The binomial coefficient is denoted with symbol$\binom{n}{k}$ and defined as:

$$\binom{n}{k} = n!/k!(n-k)!,$$

for $k\geq1$. For $k=0$ by definition, we have:

$\binom{n}{0} =1.$

When replacing *k* with 0 in the definition of the binomial coefficient we get:

$\binom{n}{0}$ $=n!/0!(n–0!)=n!/1\cdot n!=n!/n!=1.$

VI.The symmetry property

$$\binom{n}{k}=\binom{n}{n-k}, k=0,1,2,\ldots,n.$$

$$\binom{n}{n-k}=n!/(n-k)![n-(n-k)]!$$

$$=n!/(n-k)!k!=\binom{n}{k}.$$

Example: *calculate* $\binom{7}{5}$using symmetry property as above

As the above property i.e. $\binom{n}{k}=\binom{n}{n-k}$

$$\binom{7}{5}=\binom{7}{7-5}$$

$$=\binom{7}{2}=7x6/2!$$

$$=21$$

These are both ways to quickly multiply out a binomial that's being raised by an exponent. Like, say:

$(x+ a)^0 = 1$

$(x + a)^1 = x+a$

$(x+ a)^2 = x^2 + 2xa + a^2$

$(x+ a)^3 = (x + a)(x + a)^2 = (x+a)(x^2 + 2xa + a^2) = x^3 + 3x^2 a + 3xa^2 + a^3$

And so on. We could keep going all day, but eventually the exponents would get so big that it would take us all day to simplify them. We need a better way, and luckily a 17th-century French mathematician has already found one.

VII.Pascal triangle:

Pascal triangle: It is an endless, symmetrical triangle made out of numbers. The numbers that make up Pascal's triangle pursue a basic guideline: each number is the total of the two numbers above it.

Taking a gander at Pascal's triangle, you'll see that the best number of the triangle is one. The majority of the numbers in every one of the sides going down from the best are each of the ones. The numbers in the center shift, contingent on the numbers above them.

Since Pascal's triangle is boundless, there's no end. It just continues onward and going. Pascal's triangle is named for Blaise Pascal, a French Mathematician who utilized the triangle as a feature of his investigations in probability theory in the 17th century.

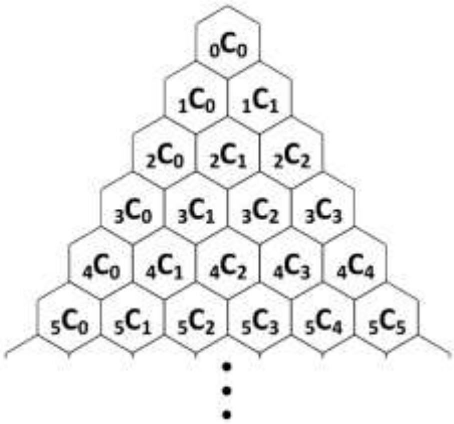
Exponent	Pascal's Triangle
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

and so on. From the way toward creating Pascal's triangle, we see any number can be produced by including the two numbers above. Scientifically, this is communicated as $nCr = {}^{n-1}Cr-1 + {}^{n-1}Cr$ this relationship has been noted by different researchers of arithmetic since forever.

VIII.Binomial coefficients ≡ Pascal triangle

Exponent	Pascal's Triangle
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

Binomialcoefficients



IX. Magic of Pascal's triangle:

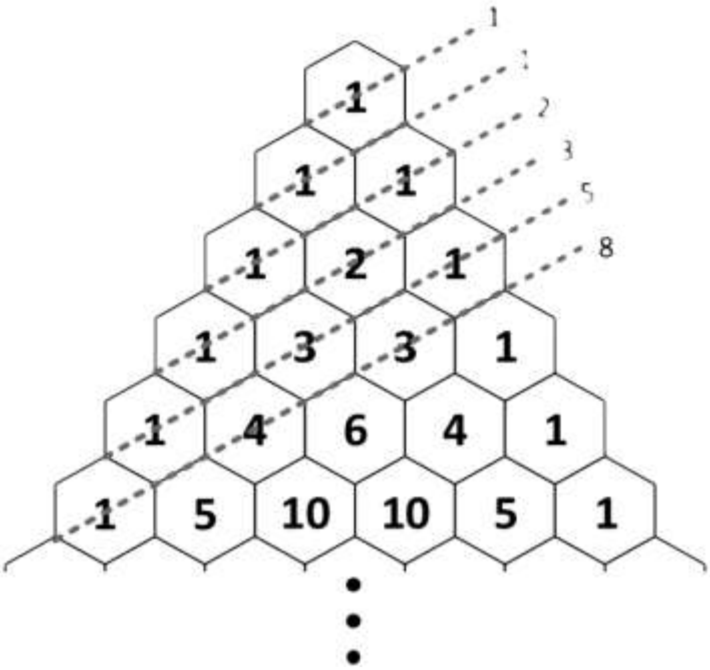
1. The numbers on each row are binomial coefficients.
2. The numbers on the second diagonal form**"COUNTING NUMBERS"**.
3. The numbers on the third diagonal are **"triangular numbers"**.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 1 5 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 26 28 8 1

4. The numbers on the fourth diagonal are“**tetrahedral numbers”**.
5. The numbers on the fifth diagonal are**pentatopenumbers**.
6. The sum of the numbers on each row are powers of 2.
The sum of the first four rows are 1, 2, 4, 8, and 16. These numbers are 2^0 , 2^1 , 2^2 , 2^3 and 2^4

7. A series of diagonals form the Fibonacci Sequence(1,1,2,3,5,8,13…….) which is discussed briefly below.

Fibonacci sequence: Pascal’s Triangle also has significant ties to number theory. The most apparent connection is to the Fibonacci sequencewhich were first noted by the medieval Italian mathematician Leonardo Pisano in his Liber abaci book.. Adding the numbers of Pascal’s triangle along a certain diagonal produces the numbers of the sequence.



Expand*: (x + 2)^4*

SOLUTION:

Using binomial theorem:

$(x + 2)^4$

$= \binom{4}{0}x^4 2^0 + \binom{4}{1}x^3 2^1 + \binom{4}{2}x^2 2^2 + \binom{4}{3}x^1 2^3 + \binom{4}{4}x^0 2^4$

$= (1)(x^4)(1) + (4)(x^3)(2) + (6)(x^2)(4) + (4)(x)(8) + (1)(1)(16)$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

To expand any binomial raised by some exponent using binomial theorem,we must have thorough knowledge on combinatorial notations ,formulas as well.

X. Binomial theorem vs Pascal triangle:

Now let us try with Pascal's Triangle:

Example:1 Suppose we want to expand $(2x + y)^3$.

We pick the coefficients in the expansion from the relevant row of Pascal's triangle: (1,3,3,1). As we move through the terms in the expansion from left to right we remember to decrease the power of 2x and increase the power of y. So,

$(2x + y)^3 = 1(2x)^3 + 3(2x)^2y + 3(2x)y^2 + 1y^3$

$= 8x^2 + 12x2y + 6xy^2 + y^3$

Either or both of the terms in the binomial expression can be negative. When raising a negative number to an even power the result is positive. When raising a negative number to an odd power the result is negative. Consider the following example.

Example:2 Expand $(3a − 2b)^5$.

We pick the coefficients in the expansion from the row of Pascal's triangle beginning 1,5;

that is 1,5,10,10,5,1. Powers of 3a decrease from 5 as we move left to right. Powers of −2b increase.

$(3a − 2b)^5 = 1(3a)^5 + 5(3a)^4(−2b) + 10(3a)^3(−2b)^2 + 10(3a)^2(−2b)^3 + 5(3a)(−2b)^4 + 1(−2b)^5$

$= 243a^5 − 810 a^4b + 1080 a^3b^2 − 720 a^2b^3 + 240ab^4 − 32b^5$

Either or both of the terms could be fractions.

Example 3: Use Pascal's Triangle to expand $(x - 2)^3$.

Solution: The coefficients of an expansion involving the power of 3 are: 1 3 3 1

Therefore, $(x – 2)^3 = 1(x)^3(-2)^0 + 3(x)^2(-2)^1 + 3(x)^1(-2)^2 + 1(x)^0(-2)^3$

$= 1x^3 + 3x^2(-2) + 3x(4) + 1(-8)$

$= x^3 - 6x^2 + 12x - 8$

Example 4: Use Pascal's Triangle to expand $(3a + b)^5$.

Solution: The coefficients of an expansion involving the power of 5 are: 1 5 10 10 5 1

Therefore,

$$(3a + b)^5 = 1(3a)^5(b)^0 + 5(3a)^4(b)^1 + 10(3a)^3(b)^2 + 10(3a)^2(b)^3 + 5(3a)^1(b)^4 + 1(3a)^0(b)^5$$

$$= 243a^5 + 5(81a^4)(b) + 10(27a^3)(b^2) + 10(9a^2)(b^3) + 5(3a)(b^4) + b^5$$

$$= 243a^5 + 405\,a^4b + 270\,a^3b^2 + 90\,a^2b^3 + 15ab^4 + b^5$$

XI.Applications of binomial theorem in real life:

Actually, there are a considerable measure of fields where the use of binomial theorem can be connected in.For example, there are a ton of regions where the utilization of binomial theorem is inescapable, even in the cutting edge world regions, for example, processing. In processing zones, binomial theorem has been exceptionally helpful such an in circulation of IP addresses. With binomial theorem , the programmed appropriation of IP addresses isn't just conceivable yet additionally the dispersion of virtual IP addresses.

Another field that utilized Binomial Theorem as the vital apparatuses is the country's financial forecast. Financial specialists utilized binomial theorem to tally probabilities that rely upon various and exceptionally appropriated factors to anticipate the manner in which the economy will carry on in the following couple of years. To have the capacity to think of sensible expectations, binomial theorem is utilized in this field.

Binomial Theorem has likewise been an extraordinary use in the engineering business in plan of framework. It permits engineers, to compute the extents of the tasks and in this manner conveying exact appraisals of the expenses as well as time required to build them. For temporary workers, it is a critical device to help guaranteeing the costing ventures is sufficiently equipped to convey benefits.

XII.USES OF PASCAL TRIANGLE:

In combinatorics, Pascal's triangle enables a direct computation of combinations from adjacent values.Basically, Pascal's triangle has astonishing associations all through numerous regions of arithmetic, including variable based math, number theory, probability, combinatorics (the science of countable designs) and fractals.Pascal s Triangle is a useful tool in finding, without tedious computations, the number of subsets of r elements that can be formed from a set with n distinct elements. Its known applications in mathematics also extend to calculus, trigonometry, plane geometry, and solid geometry**.**

XIII.CONCLUSION

In the event that we needed to extend a binomial articulation with a huge power, e.g. $(1 + x)^{29}$, utilization of Pascal's triangle would not be prescribed as a result of the need to produce countless of the triangle. An elective technique is to utilize the binomial hypothesis. The hypothesis empowers us to grow $(a + b)^n$ in expanding forces of b and diminishing forces of a. We will take a gander at growing articulations of the shape $(a+b)^2$, $(a+b)^3$, ..., $(a+b)^{32}$,..., that is the point at which the power is a positive entire number. Under specific conditions the hypothesis can be utilized when n is negative or fragmentary and this is helpful in further developed applications, however these conditions won't be considered here.

REFERENCES

[1]“Pascal triangle and binomial expansion”ShmoopuniversityInc November 11th 2008

[2]www.purplemath.com/modules/binomial.htm

[3] www.m athcentre.ac.uk/resources/uploaded/mc-ty-pascal-2009-1

[4] www.mathcentre.ac.uk 3 c

[5]www.mathcentre2009

[6]www.ncerthelp.com

[7]www.raftsac.org/ideas/resources/667/Uses for Pascal**’s** Triangle

 The Probability of Heads or Tails Pascal originally discovered the properties of this triangle by thinking of problems posed by gamblers. Investigate what happens when a coin is flipped. If you flip a coin, it will come up either heads or tails. In the following analysis the focus is on the probability of flipping heads. An equivalent could be said for flipping tails. Cover the first 1 at the top of Pascal's Triangle (row 0) with a token to represent the flip of zero coins. Next, cover the numbers on the 1st row to represent the probability of getting a head on the toss of 1 coin. There are only 2 possibilities when tossing one coin and the probability of the coin coming up heads is 1 chance out of 2, or ½. The probability of any event is expressed as a fraction between 0 and 1 inclusive. Compare the number equivalent outcomes to the numbers in the 1st row of the triangle: One Coin tossed Number of heads Number of equivalent outcomes 1st possible outcome: H 1 1 2nd possible outcome: T 0 1

Uses for Pascal's Triangle, page 2 Copyright 2011, RAFT

Suppose you toss 2 coins. Now cover the numbers in the 2nd row to represent the number equivalent outcomes when tossing 2 coins. The following are the possible outcomes:

Look at the number of heads in these possible outcomes: 1 st coin 2nd coin 1st possible outcome: H H 2nd possible outcome: H T 3rd possible outcome: T H 4th possible outcome: T T

Compare the number equivalent outcomes to the numbers in the 2nd row of the triangle: Number of heads Number of equivalent outcomes 2 1 1 2 0 1

The probability of getting 2 heads from tossing 2 coins is: 1 chance out of 4 outcomes = ¼. The probability of getting one head and one tail from tossing 2 coins is 2 chances out of 4 = ½. The probability of getting zero heads (= 2 tails) is one chance out of 4 = ¼.

Taking this further, toss 3 coins, and cover up the 3rd row of numbers. The following tree diagram represents the possible outcomes:

The probability of getting exactly 3 heads after tossing 3 coins is 1 chance out of 8, or 1/8. The probability of getting exactly 2 heads is 3 chances out of 8, or 3/8.

Uses for Pascal's Triangle, page 3 Copyright 2011, RAFT

Look at the number of heads in the possible outcomes: 1 st coin 2nd coin 3rd coin 1st possible outcome: H HH 2nd possible outcome: H H T 3rd possible outcome: H T H 4th possible outcome: H T T 5th possible outcome: T H H 6th possible outcome: T H T 7th possible outcome: T T H 8th possible outcome: T TT

Compare now the number of heads to the number of equivalent outcomes: Number of heads Number of equivalent outcomes 3 1 2 3 1 3 0 1

By now you might recognize that the number of equivalent outcomes are the same as the numbers in the third row of Pascal's Triangle!!! If we continue by tossing n coins we would end up with the number of equivalent outcomes of heads being the same as the numbers in the nth row of Pascal's Triangle! For example, if 4 coins are tossed, then the numbers of equivalent outcomes of heads are the numbers 1 4 6 4 1 which are also the numbers in the 4th row of Pascal's Triangle! Predict your odds of getting heads with coin tossing patterns in Pascal's Triangle!!!

TheFibonnacci Sequence! Fibonnacci's Sequence of numbers starts with 1, 1, and then the next number is added to the