ACCURATE DOUBLE DOMINATION IN GRAPHS

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Abstract : The concept of accurate dominating set was introduced by Kulli and Kattimani in [5]. In this paper we introduced the accurate double dominating number and we investigate some bounds for $\boldsymbol{\gamma_{2\alpha}(G)}$and its relationship with other domination parameters.

Keywords: **graph, double domination number, accurate double domination number.**
Mathematics Subject Classification:05C

I. INTRODUCTION

Domination in graphs is now well studied in graph theory and the literature on this subject has been surveyed and detailed in the two books by Haynes, Hedetniemi, and Slater [2, 3]. For a graph $G = (V, E)$, the *open neighborhood* of a vertex $v \in V$ is $N(v) = \{u \in V / uv \in E\}$ and the *closed neighborhood* is $N[v] = N(v) \cup \{v\}$. A set $S \subseteq V$ is a *dominating set* if each vertex in $V - S$ is adjacent to at least one vertex of S. Equivalently, S is a dominating set of G if for every vertex $v \in V$, $|N[v] \cap S| \geq 1$. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set. .

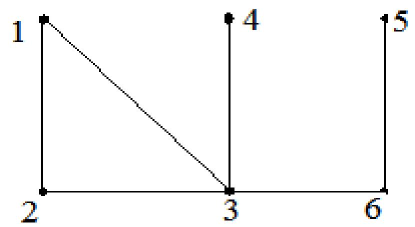
In [1] Harary and Haynes defined a generalization of domination as follows: a subset S of V is a *k-tuple dominating set* of G if for every vertex $v \in V$, $|N[v] \cap S| \geq k$, that is, v is in S and has at least $k - 1$ neighbors in S or v is in $V - S$ and has at least k neighbors in S. The *k-tuple domination number* $\gamma_{\times K}(G)$is the minimum cardinality of a k-tuple dominating set of G. Clearly, $\gamma(G) = \gamma_{\times 1}(G) = \gamma_{\times K}(G)$, while $\gamma_t(G) \leq \gamma_{\times 2}(G)$where $\gamma_t(G)$denotes the total domination number of G (see [2, 3]). For a graph to have a k-tuple dominating set, its minimum degree is at least $k - 1$. Hence for trees, $k \leq 2$. A k-tuple dominating set where $k = 2$ is called a *double dominating set* (DDS). A DDS of cardinality $\gamma_{\times 2}(G)$we call a $\gamma_{\times 2}(G)$- set.

The redundancy involved in k-tuple domination makes it useful in many applications. A dominating set D of a graph $G = (V, E)$ is an *accurate dominating set*, if $V–D$ has no dominating set of cardinality $|D|$. The *accurate domination number* $\gamma_\alpha(G)$of G is the minimum cardinality of an accurate dominating set. This concept was introduced by Kulli and Kattimani in [5]. The *upper accurate domination number* $\Gamma_\alpha(G)$of G is the maximum cardinality of an accurate dominating set.

II. ACCURATE DOUBLE DOMINATING SETS

Definition 1. A double dominating set D of a graph $G = (V, E)$ is an *accurate* double *dominating set*, if $V - D$ has no double dominating set of cardinality $|D|$. The *accurate double domination number* $\gamma_{2\alpha}(G)$of G is the minimum cardinality of an accurate double dominating set. The *upper accurate double domination number* $\Gamma_{2\alpha}(G)$of G is the maximum cardinality of an accurate double dominating set.

Example 2.



G

In figure G, The accurate double dominating sets of G are {1, 3, 4, 5}, {1,2, 4, 5, 6}.
Therefore$\gamma_{dd}(G) = \gamma_{2\alpha}(G) = 4$ and $\Gamma_{2\alpha}(G) = 5$.

III. MAIN RESULTS

Remark 1. An accurate double dominating set of a graph G may or may not be a minimal double dominating set.

Proposition 2. For any graph G,

$$\gamma_{dd}(G) \leq \gamma_{2\alpha}(G) \dots\dots\dots (1)$$

Proof: Clearly every accurate double dominating set of G is a double dominating set of G. Thus (1) holds.
The path P_3 achieves this bound.

Theorem 3. If G contains an isolated vertex., then a minimal double dominating set of G is an accurate double dominating set.

Proof: Let v be an isolated vertex of G. Then v is in every double dominating set D of G. Thus $V–D$ has no dominating set of cardinality $|D|$ and hence the desired result follows.

Corollary 4. If G contains an isolated vertex, then

$$\gamma_{dd}(G) = \gamma_{2\alpha}(G) \ldots\ldots\ldots (2)$$

We obtain the exact values of $\gamma_{2\alpha}(G)$for some standard graphs.

Proposition 5.

1. $\gamma_{2\alpha}(K_p) = \left\lfloor\frac{p}{2}\right\rfloor + 1, if\ p \geq 3.$
2. $\gamma_{2\alpha}(C_p) = \left\lfloor\frac{p}{3}\right\rfloor + 1, if\ p = 3n+1, 3n+2, n \geq 1.$
   $= \left\lfloor\frac{p}{3}\right\rfloor + 2, if\ p = 3n, n \geq 1.$
3. $\gamma_{2\alpha}(P_p) = \left\lfloor\frac{p}{3}\right\rfloor + 1, if\ p = 3n+1, 3n+2, n \geq 1.$
   $= \left\lfloor\frac{p}{3}\right\rfloor + 2, if\ p = 3n, n \geq 1.$
4. $\gamma_{2\alpha}(K_{m,n}) = m, if\ m < n$
   $= m + 1, if\ m = n.$

Theorem 6. For any graph G,$\gamma_{2\alpha}(G) \leq p - \gamma_{dd}(G) + 1$and this bound is sharp.

Proof: Let D be a minimum double dominating set of G. Then for any vertex $v \in D$, $(V - D) \cup \{v\}$ is an accurate double dominating set of G. Thus$\gamma_{2\alpha}(G) \leq |(V-D) \cup \{v\}| = p - \gamma_{dd}(G) + 1$. The cycle C_4 achieves this bound.

Theorem 7. For any graph G with $p \geq 2$ vertices, a double dominating set with $\left\lfloor\frac{p}{2}\right\rfloor + 1$ vertices is an accurate double dominating set.

Proof: Let D be a double dominating set with $\left\lfloor\frac{p}{2}\right\rfloor + 1$ vertices. Then $|V - D| < \frac{p}{2}$. Hence D is an accurate double dominating set of G.

Theorem 8. For any graph G, $\gamma_{2\alpha}(G) \leq \alpha_0(G) + 1 \ldots\ldots\ldots (1)$

Proof: Let S be a vertex cover of G. We consider the following two cases.

Case 1. Suppose $|S| < \frac{p}{2}$. Clearly S is an accurate double dominating set of G. Thus (1) holds.

Case 2. Suppose $|S| = \frac{p}{2}$. Then for any vertex $v \in V–S$, $S \cup \{v\}$ is an accurate double dominating set of G. Thus (1) holds.

Theorem 9. If T is a tree with m cutvertices and if each cutvertex except one is adjacent to at least one end vertex and the remaining cutvertex is adjacent to at least two endvertices, then$\gamma_{2\alpha}(T) = m \ldots\ldots\ldots$ (1).

Proof: Let S be the set of all cutvertices of T with $|S| = m$. Then $V–S$ is a double dominating set of all endvertices with $|V–S| > m$. Thus S is an accurate double dominating set of T. Thus (7) holds.

A dominating set D of G is a *block double dominating set* if the induced subgraph D is a block in G.

Theorem 10. If every block of a graph G has at least one non cutvertex, then any block double dominating set of G is an accurate double dominating set.

Proof: Let D be a block double dominating set of G. Let $v \in D$ be a non cutvertex of G. Since D is a block, v is not adjacent to any vertex of $V–D$. Thus D is an accurate double dominating set of G.

IV. APPENDIX I SOME OPEN PROBLEMS

In this section some of the open problems are given for further research on our work .

Prove or disprove the following :

Problem 1 : Characterize the graphs G for which $\gamma_{dd}(G) = \gamma_{2\alpha}(G)$.

Problem 2 : Characterize the graphs G for which $\gamma_{dd}(G) + \gamma_{2\alpha}(G) = p + 1$.

REFERENCES

[1] F.Harary and T.W. Haynes, Double domination in graphs ,ArsCombin. 55, 201 - 213(2000).
[2] T.W. Haynes, S.T.Hedetnimi and P.J. Slater Fundamentals of domination in Graphs. Marcel Dekker, New York, 1998.
[3] T.W. Haynes, S.T.Hedetnimi and P.J. Slater, Domination in Graphs. Advanced Topics, Marcel Dekker, New York, 1998.
[4] V.R.Kulli and M.B.Kattimani, The accurate domination number of a graph,Technical Report 2000 : 01, Gulbarga University, Gulbarga, India, 2000.
[5] V.R.Kulli, Theory of domination in graphs, Vishwa international publications, Gulbarga, India, 2010.
[6] V.R.Kulli and M.B.Kattimani, Accurate domination in graphs, Advances in Domination Theory I, Vishwa international publications, Gulbarga, India, 2012.
[7] P.Padma and P.Murugaiyan, Dr.Pauldayabaran " A note on varieties of double domination ", Asian Journal of Current Engineering and Mathematics 1 : 4 Jul - Aug (2012), 230 - 232.