"An EOQ model for deteriorating item for Special Sales Promotional Incentive Scheme with planned shortages and equivalent Holding and shortages cost."

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ABSTRACT

In this research paper, an inventory model for deteriorating items with the conclusion of the sales promotional incentive scheme with planned shortages obtainable by proportional to the order quantity when demand is certain and stock out are permitted. It is assumed that the holding cost does not change for any order quantity of the retailer. This study will help the supplier to take the decision whether to supply a regular or special stock of quantity for profit of the items without replace any item. The optimum quantity is depending on equivalent holding and shortages cost during that period. It include the numerical examples are related to the derived results. The hypothetical numerical study is also part of inventory EOQ model parameter.

KEYWORDS: EOQ model, deteriorate items, promotional incentive scheme, planned shortages, holding cost, shortages cost.

INTRODUCTION

In traditional inventory EOQ model it is indirectly assumed that when purchaser order and pay for quantity units they receive quantity only and also there is no damage or deterioration of the units while in inventory. the effect of exponential decay on order quantity was analyzed in previous research. In real life situation many times vendor announce special sales promotional scheme for a short term period.one such scheme is price discount in which vendor gives a special price reduction on the unit cost of the ordered quantity during a specified period .Sometimes vendor offer another equivalent scheme in which vendor does not give a discount on the unit cost but vendor supply some more units proportional to the order quantity free of cost. Such scheme is available for short term period and naturally customer will like to buy large amount of unit, during the short tenure of the scheme, but the purchases of too huge amount of units will significantly growth the inventory holding cost. When shortages are planned earlier then avoid the problem of deciding what optimal order size to be placed during such sales promotional scheme to get maximum profit in future.

Ghare and Schrader (1963) who developed inventory model for exponential decay in items under constant demand rate. This model expand by Covert and Philip(1973) and Philip(1974). By using Weibull distribution to derive time deterioration of an item in inventory. Dixit and Shah (2005) contributed a review article on inventory models when a temporary price discount is offered by the supplier to the seller to study the relationship between price discounts and demand policy. The all-unit extent discounts organization policy is argued by Arcelus et al. (2003), Shah et al. (2005), Bhaba and Mahmood (2006), Abad (2007), Dye et al. (2007), Shah et al. (2008), Mishra and Shah (2009). They supposed that the price discount rate is independent of the special order quantity. Though, in market, it is detected that the supplier offers a quantity discount to inspire bigger orders. For the larger order, the higher price discount rate is given by the supplier. Such as outcome, the retailer has to settle the trade-off for securing price savings in contrast to advanced total holding cost. Ouyang et al. (2009)conversed the sound of a provisional price rebate on a retailer's demand policy by assuming that the price discount rate is linked to special order quantity. They assumed that the demand is constant and deterministic.

In this EOQ model, we define through the impression of planned shortages on such variety of sales promotional incentive scheme when stock out are permitted. This study will help the vendor to take decision about stock of quantity during sales promotion tool. A mathematical model for this model is defined. Here assumed that if order for Q units during the period of sale promotional scheme the suppler supply (1+b) Q' units. The measure of effectiveness to be used will be the profit due to large amount of purchase.

Assumptions:-

1. The demand rate units per time unit is certain, known & constant.
2. Lead time is fix
3. Holding Cost (C_1) per order is per time unit constant and does not change for different order quantity.
4. Stock act are permitted and shortages or Back ordering cost (C_2) per unit known and constant.
5. Replenishment rate is infinite.
6. The ordering cost (C_3) independent and constant during the period under consideration.
7. Through the sales promotional scheme at the peak one order should be placed.
8. The unit cost (C) does not depend upon the quantity ordered or received and is constant during the period under consideration.
9. Time of a cycle with constant fraction θ ($0 \leq \theta \leq 1$) on hand inventory deteriorates per
10. Stock out are acceptable and shortages or back ordering cost per unit is known and constant
11. There is no repair of the deteriorated inventory during the period under consideration.

Unit time

Notations:

Q = Order quantity

Q' = Special order Quantity in units.

Q* = The Economic order quantity per units

C_1 = The inventory holding cost charges special of interest

Charges per unit

C_2= The inventory shortages or back-ordering cost.

C_3=The Replenishment cost per order

R =Inventory fix rate

Ic =The interest charges per rupee investment.

To= The time at which the sales promotional scheme will be Started

T_1= The time of which the sales promotional Scheme will be ended

$(1+b)Q'$ =The System received units.

D= The annual demand rate

d= uniformly demand rate

M= Maximum Inventory level for EOQ

M' = Maximum Inventory level for special order quantity

S= Maximum shortages level or maximum back-order quantity units

LT= Lead time

N= Number of Quantity received per unit

Fb= Factor for equivalent Holding cost for back ordering.

EC_1C_2 = Equivalent Holding and shortages cost when Back order are permitted under Special Sales promotional Incentive Scheme

Q-S = Shortages before quantity added.

The mathematical model:-

This EOQ model for deteriorating item with special sales promotional scheme with planned shortages

(*) **Holding Cost**:- for deteriorating items

-- During period 'To', Inventory fix rate 'R'. Q*= EOQ, Deteriorating rate θ

Order Quantity: Q and Special order Quantity: Q'

Cycle during period: D/Q

Maximum Inventory for EOQ: $M = Q^* - S = R\,T_o$

Maximum Inventory for special sale quantity: $M' = Q' - S = R\,T_o$

Average Inventory During T_o = $\dfrac{RT_0}{2} = \dfrac{Q^* - S}{2}$

Average Inventory During T_o = $\dfrac{RTo}{2} = \dfrac{Q' - S}{2}$

$M = RT_o$ & $Q = R.T$; $T = T_o + T_1$

$$\frac{M}{Q} = \frac{RT_0}{RT} = \frac{T_0}{T} \quad \text{AND} \quad \frac{M}{Q} = \frac{d.T_0}{dT} = \frac{T_0}{T}$$

$$T_o = \frac{MT}{Q} = \frac{T(Q' - S)}{Q} \quad Heres\ N = \frac{1}{T};\ NT = 1$$

$$T_o = \frac{MT}{Q} = \frac{T(Q^* - S)}{Q} \quad Heres\ N = \frac{1}{T};\ NT = 1$$

(*)Holding cost for the duration of T_o:-

$C_1(Q')$ = (Average inventory). (Inventory Holding cost per unit time T_o for special sale)

$$C_1(Q') = \left[\frac{Q'-S}{2}\right] [(C_1 + C\theta)T_0]$$

C_1 (Q^*) = (Average inventory). (Inventory Holding cost per unit time T_o for EOQ)

$$C_1(Q^*) = \left[\frac{Q^*-S}{2}\right] [(C_1 + C\theta)T_0]$$

(*) Annual Holding Cost: -(deteriorating item with special sale)

$C_1(Q')$ = (Holding cost per cycle) / (cycle per year)

$$= \left[\frac{Q'-S}{2}\right] [(C_1 + C\theta)To]\ N = \left[\frac{Q'-S}{2}\right] [C_1 + C\theta]\left[\frac{T(Q'-S)}{Q}\right] N$$

$$= \frac{(Q'-S)^2 [(C_1 + C\theta)]NT}{2Q}$$

$$= \frac{(Q'-S)^2 [C_1 + C\theta]}{2Q}$$

Annual Holding Cost: - EOQ

C_1 (Q^*) = (Holding cost per cycle) / (cycle per year)

$$= \left[\frac{Q^*-S}{2}\right] [(C_1 + C\theta)To]\ N = \left[\frac{Q^*-S}{2}\right] [C_1 + C\theta]\left[\frac{T(Q^*-S)}{Q}\right] N$$

$$= \frac{(Q^*-S)^2 [C_1 + C\theta]NT}{2Q}$$

$$= \frac{(Q^*-S)^2 [C_1 + C\theta]}{2Q}$$

(*) Shortages cost:- for deteriorating items

-- For the period of 'T1' shortages up rate 'R', Deteriorating rate θ, maximum shortages $S = R.T_1$

Average shortages during $T_1 = \frac{R.T_1}{2} = \frac{S}{2}$

$S = R.T_1$ & $Q = R.T$, $T = To + T_1$

$$\frac{S}{Q} = \frac{R.T_1}{RT} = \frac{T_1}{T} \therefore T_1 = \frac{ST}{Q}$$

C_2 (Q) = (average shortages) (shortages cost / unit for time T_1)

$$= \frac{S}{2}(C_2 + C\theta)T_1$$

(*) **Annual shortages cost**:-

$$C_2(Q) = \left(\frac{shortages\cos t}{cycle}\right)\left(\frac{cycle}{year}\right)$$

$$= \left(\frac{S}{2}\right)[(C_2 + C\theta)T_1].N$$

$$= \left(\frac{S}{2}\right)[C_2 + C\theta]\left(\frac{ST}{D}\right).N$$

$$= \frac{S^2[C_2 + C\theta]NT}{2Q}$$

$$= \frac{S^2[C_2 + C\theta]}{2Q}$$

(*) ordering cost: C_3

$$C_3(Q') = \frac{Annual\ demand}{Sp.\ order\ Quantity} = \frac{D}{Q}$$

(*) Annual order cost: $C_3(Q') = \left(\frac{Sp.\ order}{year}\right)\left(\frac{ordering\cos t}{year}\right)$

$$= \frac{D}{Q1}.C_3$$

Before the time To an order of Q* units is to be placed every $\frac{Q*}{D}$ units of time and the total cost of the system with equivalent holding and shortages cost and interest charges will be

$$K(Q^*) = C\ Q^* + \sqrt{2C_3 D\ b\ C_1\ eb}$$

$$K(Q^*) = CQ^* + \left[\frac{(Q^* - S)^2(C_1 + CIc)Q^{*2}}{2DQ}\right] + \frac{C_2 S^2}{2Q} + \frac{C_3 D}{Q}$$

The objective of units model is to determine special order size Q' to get maximum profit.

Let K (Q') denotes the total cost of the system during period To to T_1. When an order of Q' unit with deteriorating an items is placed at time T and the system receives (1+b)Q' units with planned shortages.

$$K(Q') = CQ' + (1+b)\left[\frac{(Q'-S)^2(C_1 + C\theta)Q'^2}{2DQ}\right] + \frac{CIcQ'^2}{2D} + \frac{[C_2 + C\theta]S^2}{2Q} + \frac{C_3 D}{Q}$$

$$K(Q') = CQ' + \left[\frac{(1+b)^2(Q'-S)^2(C_1 + C\theta + CIc) + C_2 S^2}{2DQ}\right]Q'^2 + \frac{C_3 D}{Q} - - - - - (1)$$

Let, K_1 (Q') denotes the total cost of the system during the period To to T_1 when no special order is placed at during the tenure of sale promotional scheme Q* units are made at every $\frac{Q*}{D}$ units of time.

$$K_1(Q^*) = C.Q^* + \left[\frac{[(Q^*-S)^2(C_1+C\theta+CIc)Q^{*2}+S^2C_2]}{2DQ}+\frac{C_3D}{Q}\right]\left[\frac{(1+b)Q'}{Q^*}\right] - - - - - (2)$$

$$K_1(Q^*) = (1+b)CQ' + (1+b)\frac{[(Q^*-S)^2\ (C_1+C\theta+CIc)+S^2C_2]Q^*Q'}{2DQ}+\frac{(1+b)C_3DQ'}{QQ^*}$$

$$= (1+b)CQ' + \frac{(1+b)Q'}{DQ}\left[\frac{[(Q^*-S)^2\ (C_1+C\theta+CIc)+S^2C_2]}{2}\right]\sqrt{\frac{2C_3DR}{C_1}}+C_3\sqrt{\frac{C_1}{2C_3DR}}$$

$$= (1+b)CQ' + \frac{(1+b)Q'}{DQ}\sqrt{2(Q^*-S)^2(C_1+C\theta+CIc+C_2S^2)C_3D}$$

The profit $P_1(Q')$ due to taking advantage of the special sale promotional scheme is

$$P_1(Q') = K_1(Q^*) - K(Q')$$

$$= (1+b)CQ' + \frac{(1+b)Q'}{DQ}\sqrt{2(Q^*-S)^2(C_1+C\theta+CIc+C_2S^2)C_3D} - CQ' - \left[\frac{(1+b)^2(Q'-S)^2C_1+C\theta+CIc+C_2S^2}{2DQ}\right]Q'^2 - \frac{C_3D}{Q}$$

$$= bCQ' + \frac{(1+b)Q'}{DQ}\sqrt{2(Q^*-S)^2(C_1+C\theta+CIc+C_2S^2)C_3D} - \left[\frac{(1+b)^2(Q'-S)^2C_1+C\theta+CIc+C_2S^2}{2DQ}\right]Q'^2 - C_3D$$

$$= bCQ' + \frac{(1+b)Q'}{DQ}\sqrt{2(Q^*-S)^2(C_1+C\theta+CIc+C_2S^2)C_3D} - \left[\frac{(1+b)^2(Q'-S)^2C_1+C\theta+CIc+C_2S^2}{2DQ}\right]Q'^2 - C_3D$$

For maximization,

$$\frac{\partial P_1(Q')}{\partial Q'} = 0 \text{ Gives}$$

$$bC + \frac{(1+b)}{DQ}\sqrt{2[(Q^*-S)^2(C_1+C\theta+CIc+C_2S^2)]C_3D} - \left[\frac{(1+b)^2[2(Q'-S)C_1+C\theta+CIc+C_2S^2]}{DQ}\right] = 0$$

$$\therefore Q^{*\prime}\ \frac{bCDQ}{(1+b)^2.[(Q'-S)^2\,C_1+C\theta+CIc+C_2S^2]} + \frac{(1+b)\ \sqrt{2[(Q^*-S)^2(C_1+C\theta+CIc+C_2S^2)]C_3D}}{(1+b)^2\,[(Q'-S)^2\,C_1+C\theta+CIc+C_2S^2]}$$

And the maximum profit will be

$$P_1(Q^{*\prime}) = \left[bC + \frac{(1+b)}{DQ}\sqrt{2[(Q^*-S)^2C_1+C\theta+CIc+C_2S^2]C_3D}\right]Q^{*\prime} -$$

$$\left[\frac{(1+b)^2.[2(Q'-S)C_1+C\theta+CIc+S^2.C_2}{2DQ}\right]Q^{*^2} - \frac{C_3D}{Q'}$$

Special cases:-

Consider the case, when an order of Q* units is placed during the short tenure of sales scheme. Then the model is

* Let – K_2 (Q*) denotes the total cost of the system during To to T_1 when an order of Q* unit is placed at time To and then at the end of $\frac{(1+b)Q*}{D}$ time unit several orders of Q* units are placed at every $\frac{Q*}{D}$ units of time

$$K_2(Q^*) = C.Q^* + \left[\frac{(1+b)^2[(Q^*-S)^2 C_1 + C\theta + CIc + C_2S^2}{2DQ}\right]Q^{*2} + \frac{C_3D}{Q}$$

$$\left[\frac{(1+b)\ (Q'-Q^*)}{2D}\right] + \left[\frac{[C.Q^*[(Q^*-S)^2 C_1 + C\theta + CIc + C_2S^2]}{2DQ}Q^{*2} + \frac{C_3D}{Q}\right]$$

$$= CQ^* + \frac{[(1+b)^2[Q^*-S)^2 C_1 + C\theta + CIc + C_2S^2]}{2DQ}Q^{*2} + \frac{C_3D}{2Q}$$

$$+(1+b)\ (CQ'-Q^*) + \frac{[(1+b)[Q^*-S)^2 C_1 + C\theta + CIc]Q^*(Q'-Q^*)]}{2DQ} + \frac{(1+b)\ C_3D(Q'-Q^*)}{QQ^*}$$

$$= (1+b)CQ' + \left[\frac{[(1+b)[Q^*-S)^2 C_1 + C\theta + CIc + C_2S^2]Q^*Q']}{2DQ}\right] + \frac{(1+b)C_3DQ'}{QQ^*} +$$

$$CQ^* + \frac{(1+b)(Q^*\_S)^2(C_1 + C\theta)Q^{*2}}{2DQ} + \frac{CIcQ^{*2}}{2D} + \frac{C_3D}{Q} - (1+b)CQ^* -$$

$$\frac{(1+b)(Q^*-S)^2(C_1 + C\theta + CIc)Q^{*2}}{2DQ} - (1+b)C_3D$$

$$= (1+b)\ CQ' + \frac{(1+b)(Q^*-S)^2(C_1 + C\theta + CIc + C_2S^2)Q^*Q'}{2DQ} +$$

$$\frac{(1+b)C_3DQ'}{QQ^*} + \frac{b(1+b)(Q^*-S)^2(C_1 + C\theta)Q^{*2}}{2DQ} -$$

$$b\left[CQ^* + \frac{CIcQ^{*2}}{2D} + \frac{C_3D}{Q}\right]$$

$$= (1+b)\ C.Q' + \frac{(1+b)Q'}{D}\sqrt{2[Q^*-S)^2 C_1 + C\theta + CIc + C_2S^2]C_3D} +$$

$$\frac{b(1+b)[Q^*-S)^2(C_1 + C\theta)Q^{*2}}{2DQ} - b\left[CQ^* + \frac{CIcQ^{*2}}{2D} + \frac{C_3D}{Q}\right]$$

$$= K_1(Q^*) - b\left[CQ^* + \frac{CIcQ^{*2}}{2D} + \frac{(Q^*-S)^2(C_1+C\theta)Q^{*2}}{2DQ} + \frac{(C_2+C\theta)S^2}{2Q} + \frac{C_3D}{Q} - \frac{(C_1+C\theta)Q^{*2}}{2D}\right]$$
$$+\frac{b(1+b)(Q^*-S)^2(C_1+C\theta)Q^{*2}}{2DQ}$$

$$K_2(Q^*) = K_1(Q^*) - b\left[K(Q^*) - \frac{(b+2)(Q^*-S)^2(C_1+C\theta)Q^{*2}}{2DQ}\right]$$

The profit due to purchasing large quantity

$P_2(Q^1) = K_2(Q^*) - K(Q^1)$

$$= K_1(Q^*) - b\left[K(Q^*) - \frac{(b+2)(Q^*-S)^2(C_1+C\theta)Q^{*2}}{2DQ}\right] - K(Q')$$

$$= P_1(Q^*) - b\left[K(Q^*) - \frac{(2+b)(Q^*-S)^2(C_1+C\theta)Q^{*2}}{2DQ}\right] ----(6)$$

$$P_2(Q') = '\left[bc + \frac{(1+b)}{DQ}\sqrt{2[(Q^*-S)^2C_1 + C\theta + CIc + C_2S^2]C_3D}\right]Q^{*'}$$
$$-\left[\frac{(1+b)^2 2(Q'-S)C_1 + C\theta + CIc + C_2S^2}{2DQ}\right]Q^{*'^2} - \frac{C_3D}{Q}$$
$$-b\left[K(Q^*) - \frac{(2+b)[(Q^*-S)^2(C_1+C\theta)]Q^{*2}}{2DQ}\right] -$$
$$\left[\frac{(1+b)^2(Q^*-S)^2C_1 + C\theta + CIc}{2DQ}\right]Q'^2 - \frac{C_3D}{Q}$$

$$P_2(Q') = \left[bC + (1+b)\sqrt{2(Q^*-S)^2(C_1 + C\theta + CIc + C_2S^2)C_3D}\right]Q' -$$
$$b\left[K(Q^*) - \frac{(b+2)(Q^*-S)^2(C_1+C\theta)Q^{*2}}{2DQ}\right] -$$
$$\left[\frac{(1+b)^2(Q'-S)^2C_1 + C\theta + CIc}{2DQ}\right]Q' - \frac{2C_3D}{Q}$$

For maximization $\dfrac{\partial P_2(Q')}{\partial Q'} = o \ Gives$

$$bc + \frac{(1+b)}{DQ}\sqrt{2[(Q^*-S)^2C_1 + C\theta + CIc + C_2S^2]C_3D} -$$

$$\left[\frac{(1+b)^2[2(Q'-S)^2C_1+C\theta+CIc+C_2S^2]}{DQ}\right]=0$$

$$\therefore\quad Q^{*\prime}=\frac{bCDQ}{(1+b)^2[2(Q'-S)^2C_1+C\theta+CIc+C_2S^2}+$$

$$\frac{(1+b)\sqrt{2\,[(Q^*-S)^2C_1+C\theta+CIc+C_2S^2]C_3D}}{(1+b)^2\;[2(Q'-S)^2C_1+C\theta+CIc+C_2S^2]}$$

And, the optimum maximum profit is

$$P_2(Q^{*\prime})\;=\;\left[bc+\frac{(1+b)}{DQ}\sqrt{2\,[(Q^*-S)^2C_1+C\theta+CIc+C_2S^2]C_3D}\right]Q^{*1}$$

$$-\,b\left[K(Q^*)-\frac{(2+b)[(Q^*-S)^2(C_1+C\theta)]Q^{*2}}{2DQ}\right]-$$

$$\left[\frac{(1+b)^2[(Q'-S)^2C_1+C\theta+CIc+C_2S^2]}{2DQ}\right]Q^{*\prime}-C_3D$$

Here, it b = 0 then, $Q^{*\prime}\;=\;\sqrt{\frac{2C_3D}{[C_1+C\theta+CIc+C_2S^2]}}$

And, P2 ($Q^{*\prime}$) = 0

Now, we define the condition for optimum. The cost of item is also include in Total cost of Q* associate for inventory.

$$K\,(Q^*)=\;C.Q^*+\left[\frac{(Q^*-S)^2C_1+C\theta+CIc}{2DQ}\right]Q^{*2}+\frac{S^2(C_2+C\theta)}{2Q}+\frac{C_3D}{Q}$$

$$\frac{\partial K(Q^*)}{\partial S}=\frac{\partial}{\partial S}\left(C.Q^*+\frac{(Q^*-S)^2C_1+C\theta+CIc}{2DQ}Q^{*2}+\frac{S^2.(C_2+C\theta)}{2Q}+\frac{C_3D}{Q}\right)$$

$$=\;0-\frac{2(Q^*-S)(C_1+C\theta+CIc)2Q^*}{2DQ}+\frac{(C_2+C\theta)(2S)}{2Q}$$

Equating it to zero

$$0=0-\frac{(Q^*-S)(C_1+C\theta+CIc)Q^*}{DQ}+\frac{(C_2+C\theta)S}{Q}$$

$$=-\frac{(Q^{*2}(C_1+C\theta+CIc)-S(C_1+C\theta+CIc)Q^*}{DQ}+\frac{(C_2+C\theta).S}{Q}$$

$0 = -\,(Q^*(C_1+C\theta+CIc)+SQ^*(C_1+C\theta+CIc)+(C_2+C\theta)S$

$Q^{*}(C_1+C\theta+CIc) = SQ^*(C_1+C\theta+CIc) + (C_2.+C\theta)S$

$= S\,Q^*.\ (C_1 + C\theta + CIc + C_2)$

$$\frac{Q^*}{S} = \frac{C_1 + C\theta + CIc + C_2 S}{C_1 + C\theta + CIc}$$

This is the optimum condition for S. Hence S is replaced by S*

Now,

$$\frac{Q^*}{S} = \frac{C_1 + C\theta + CIc + C_2 S}{C_1 + C\theta + CIc} - (1) \ \& \ \frac{S^*}{Q^1} = \frac{C_1 + C\theta + CIc}{C_1 + C\theta + CIc + C_2 S} - (2)$$

* Factor of Back – ordering:-

$$Fb = \frac{(C_2 + C\theta)S}{C_1 + C\theta + CIc + C_2 S} - (3)$$

Subtracting LHS and RHS from 1

$$1 - Fb = 1 - \frac{(C_2 + C\theta)S}{C_1 + C\theta + CIc + C_2 S} = \frac{C_1 + C\theta + CIc}{C_1 + C\theta + CIc + C_2 S} - (4)$$

Equation * (2) & (4) $1 - \text{Fb} = \dfrac{C_1 + C\theta + CIc}{C_1 + C\theta + CIc + C_2 S} = \dfrac{S^*}{Q^*}$

$\therefore\ S^* = (1 - \text{Fb})\,Q^*$ --- (5)

Now, we define the condition for optimum. The cost of item is also including in Total cost of Q' associate for inventory.

$$K(Q') = C.Q' + \left[\frac{(1+b)^2 (Q'-S)^2 C_1 + C\theta + CIc}{2DQ}\right] Q'^2 + \frac{S^2.(C_2 + C\theta)}{2Q} + \frac{C_3 D}{Q}$$

$$\frac{\partial K(Q')}{\partial S} = \frac{\partial}{\partial S}\left(C.Q' + \frac{(1+b)^2 (Q'-S)^2 C_1 + C\theta + CIc}{2DQ} Q'^2 + \frac{S^2.(C_2 + C\theta)}{2Q} + \frac{C_3 D}{Q}\right)$$

$$= 0 - \frac{2(1+b)(Q'-S)(C_1 + C\theta + CIc)2Q'}{2DQ} + \frac{(C_2 + C\theta)(2S)}{2Q}$$

Equating it to zero,

$$0 = 0 - \frac{(1+b)(Q'-S)(C_1 + C\theta + CIc)Q'}{DQ} + \frac{(C_2 + C\theta)S}{Q}$$

$$= -\frac{(bQ'^2(C_1 + C\theta + CIc) - S(C_1 + C\theta + CIc)Q'}{DQ} + \frac{(C_2 + C\theta).S}{Q}$$

$$0 = -\left(bQ'^2(C_1 + C\theta + CIc) - S(C_1 + C\theta + CIc)Q'\right) + (C_2 + C\theta)S$$

$$\left(bQ'^2(C_1+C\theta+CIc)=S(C_1+C\theta+CIc)Q'\right)+(C_2+C\theta)S$$

$$= S\,Q'\ (C_1 + C\theta + CIc + C_2)$$

$$\frac{Q'}{S}=\frac{C_1+C\theta+CIc+C_2S}{C_1+C\theta+CIc}$$

This is the optimum condition for S. Hence S is replaced by S*

Now,

$$\frac{Q'}{S}=\frac{C_1+C\theta+CIc+C_2S}{C_1+C\theta+CIc}-(1)\ \&\ \frac{S^*}{Q'}=\frac{C_1+C\theta+CIc}{C_1+C\theta+CIc+C_2S}-(2)$$

* Factor of Back – ordering:-

$$Fb=\frac{(C_2+C\theta)S}{C_1+C\theta+CIc+C_2S}-(3)$$

Subtracting LHS and RHS from 1

$$1-Fb=1-\frac{(C_2+C\theta)S}{C_1+C\theta+CIc+C_2S}=\frac{C_1+C\theta+CIc}{C_1+C\theta+CIc+C_2S}-(4)$$

Eqbs * (2) & (4) $1 - Fb = \dfrac{C_1+C\theta+CIc}{C_1+C\theta+CIc+C_2S} = \dfrac{S^*}{Q'}$

$\therefore\ S^* = (1 - Fb)\ Q'$ --- (5)

Here,

$EC_1C_2 = C_1\,(Q') + C_2\,(Q')$

$$=\frac{(Q'-S)^2(C_1+C\theta)}{2Q}+\frac{(C_2+C\theta).S^2}{2Q}$$

Put the value of S = S* = (1 – Fb)

$$=\frac{(Q'-(1-Fb)Q')^2(C_1+C\theta)}{2Q}+\frac{(C_2+C\theta)((1-Fb))Q')^2}{2Q}$$

$$=\frac{Q'^2(1-(1-Fb))^2(C_1+C\theta)}{2Q}+\frac{(C_2+C\theta)Q'^2(1-Fb)^2}{2Q}$$

$$=\frac{Q'^2}{2Q}[(1-(1-Fb))^2]C_1+C\theta+C_2\,Q'^2(1-Fb)^2$$

$$=\frac{Q'^2}{2Q}[Fb^2\,C_1+C\theta+C_2(1-Fb)^2]$$

Putting Fb & (1-Fb) in terms of C_1 & C_2 from (3) & (4)

$$= \frac{Q'^2}{2Q}\left[\left(\frac{(C_2 + C\theta)S}{C_1 + C\theta + CIc + C_2 S}\right)^2 C_1 + C_2\left(\frac{C_1 + C\theta + CIc}{C_1 + C\theta + CIc + C_2 S}\right)^2\right]$$

$$= \frac{Q'^2}{2Q}\left[\frac{C_1 C_2 S^2 + (C_1 + C\theta + CIc)^2 C_2}{(C_1 + C\theta + CIc + C_2)^2}\right]$$

$$= \frac{Q'^2}{2Q}\left[\frac{(C_1 + C\theta + CIc)C_2 S\ (C_1 + C\theta + CIc)C_2 S}{(C_1 + C\theta + CIc + C_2 S)^2}\right]$$

$$= \frac{Q'^2}{2Q}\left[\frac{(C_1 + C\theta + CIc)C_2 S}{C_1 + C\theta + CIc + C_2 S}\right]$$

$$= \frac{Q'^2}{2Q}\ C_1 + C\theta + CIc\left[\frac{(C_2 + C\theta)S}{C_1 + C\theta + CIc + C_2 S}\right] - (6)$$

$$= \frac{Q'^2}{2Q}\ C_1 + C\theta + CIc.Fb \quad - \quad (7)$$

Observe RHS of equation (7). It is annual holding and shortages cost for EOQ model under consideration. Difference is just factor of back ordering or shortages. For EOQ model annual holding cost:

$$Fb = 1\ \ then\ \ \frac{Q'^2}{2Q}\ C_1 + C\theta + CIc$$

EOQ model do not have shortages cost. So, we define by analogy equation equivalent holding cost for EOQ.EC_1C_2 for EOQ model is the product of holding cost with interest charges and factor of back ordering.

C_1 eb notes equivalent holding cost for EOQ model.

$$EC_1C_2 = \frac{Q'^2}{2Q}\ C_{1eb} + C\theta + CIc$$

Total annual inventory for EOQ with equivalent holding and shortages cost

$$K(Q) = \frac{Q'^{\,2}}{2Q}\ (C_{1eb} + C\theta + CIc) + \frac{C_3 D}{Q}$$

In order to minimize the annual total cost, the partial derivative of total cost, K (Q) with respect to Q

$$\frac{\partial K(Q)}{\partial Q} = \frac{\partial}{\partial Q}\left[\frac{Q'^2}{2}(C_{1eb} + C\theta + CIc) + \frac{C_3 D}{Q}\right]$$

$$= \frac{Q'^2}{2}(C_{1eb} + C\theta + CIc) + (-1)\frac{C_3 D}{Q}$$

Equating it to zero

$$\frac{Q'^2}{2}(C_{1eb}+C\theta+CIc)+(-1)\frac{C_3D}{Q}=0$$

$$\therefore Q^2=\frac{2C_3D}{Q'^2(C_{1eb}+C\theta+CIc)}$$

Hence, the quantity (Q) found is the minimum cost point optimum quantity denoting it as Q*

For EOQ model with EC_1C_2

$$\therefore Q^*=\sqrt{\frac{2C_3D}{Q'^2(C_{1eb}+C\theta+CIc)}}$$

Before the time To an order of Q* units is to be placed every $\frac{Q^*}{D}$ units of time and the total cost of the system with equivalent holding and shortages cost and interest charges will be

$$K(Q)=CQ^*+\frac{Q'^2(C_{1eb}+C\theta+CIc)Q^{*2}}{2DQ}+\frac{C_3D}{Q}$$

The objective of unit model is to determine special order size Q' to get maximum profit.

Let K (Q') denotes the total cost of the system during period To to T_1. When an order of Q' units is placed at time T and the system receives (1+b)Q' units with planned shortages.

$$K(Q')=CQ'+(1+b)^2\left[\frac{Q'^2(C_{1eb}+C\theta+CIc)Q'^2}{2DQ}\right]+\frac{C_3D}{Q'}$$

Let, K_3 (Q') denotes the total cost of the system during the period To to T_1 when no special order is placed at during the tenure of sale promotional scheme Q* units are made at every $\frac{Q^*}{D}$ units of time.

$$K_3(Q^*)=\left[CQ^*+\frac{Q'^2(C_{1eb}+C\theta+CIc)Q^{*2}}{2DQ}+\frac{C_3D}{Q}\right]\left[\frac{(1+b)Q'}{Q^*}\right]$$

$$=(1+b)CQ'+\frac{(1+b)Q'^2(C_{1eb}+C\theta+CIc)Q^*Q'}{2DQ}+\frac{(1+b)C_3DQ'}{QQ^*}$$

$$=(1+b)CQ'+\frac{(1+b)Q'}{DQ}\left[\frac{Q'^2(C_{1eb}+C\theta+CIc)Q^*}{2}+\frac{C_3D}{Q^*}\right]$$

$$=(1+b)CQ'+\frac{(1+b)Q'}{DQ}\sqrt{2Q'^2((C_{1eb}+C\theta+CIc)Q^*)C_3D}$$

$$P_3(Q')=K_3(Q^*)-K(Q')$$

$$=(1+b)CQ'+\frac{(1+b)Q'}{DQ}\sqrt{2Q'^2(C_{1eb}+C\theta+CIc)C_3D}-CQ'-(1+b)^2\left[\frac{Q'^2(C_{1eb}+C\theta+CIc)Q'^2}{2DQ'}\right]-\frac{C_3D}{Q'}$$

$$= bCQ' + \frac{(1+b)Q'}{DQ}\sqrt{2Q'^2(C_{1eb} + C\theta + CIc)C_3 D} - (1+b)^2\left[\frac{Q'^2(C_{1eb} + C\theta + CIc)Q'}{2D}\right] - \frac{C_3 D}{Q'}$$

For maximization $\frac{\partial P_3(Q')}{\partial Q'} = 0$ Gives

$$bC + \frac{(1+b)}{DQ}\sqrt{4Q'(C_{1eb} + C\theta + CIc)C_3 D} - \left[\frac{(1+b)^2 4Q'(C_{1eb} + C\theta + CIc)}{2DQ'}\right] = 0$$

$$bC + \frac{(1+b)}{DQ}\sqrt{4Q'(C_{1eb} + C\theta + CIc)C_3 D} - \left[\frac{(1+b)^2(C_{1eb} + C\theta + CIc)}{D}\right]$$

$$\therefore Q^{*\prime} = \frac{bCDQ}{(1+b)^2 C_{1eb} + C\theta + CIc} + \frac{(1+b)\sqrt{4Q'(C_1 + C\theta + CIc)C_3 D}}{(1+b)^2 C_{1eb} + C\theta + CIc} = 0$$

And maximum profit will be

$$P_3(Q^{*\prime}) = \left[bC\frac{(1+b)}{DQ}\sqrt{4Q'(C_{1eb} + C\theta + CIc)C_3 D}\right]Q^{*\prime} - \left[\frac{(1+b)^2 C_1 + C\theta + CIc}{D}\right]Q^{*\prime} - \frac{C_3 D}{Q'}$$

$$P_3(Q^{*\prime}) = \left[bC + \frac{(1+b)}{DQ}\sqrt{4Q'(C_{1eb} + C\theta + CIc)C_3 D}\right]Q^{*\prime} - \left[\frac{(1+b)^2 C_1 + C\theta + CIc}{D}\right]Q^{*\prime} - \frac{C_3 D}{Q'}$$