AVDTC of splitting graph formed from the cartesian product of cycle and path graphs

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Abstract

In this article, we determined the AVD chromatic number of the splitting graph which is formed from the cartesian product of path and cycle graphs. Also, the relationship between the chromatic number and AVD- chromatic number are investigate and illustrated with some examples.

Keywords: Path graph, cycle graph, cartesian product of simple graph, splitting graph, proper colouring, chromatic number, AVD-total colouring, AVD-total chromatic number.

1.Introduction

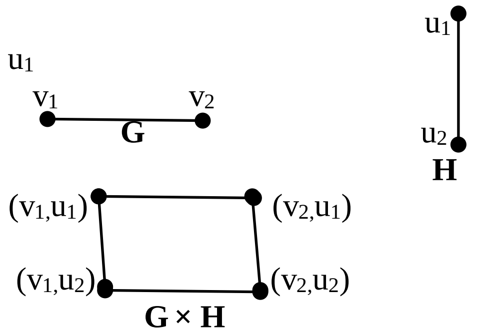
In this paper, we have taken the graph to be undirected, finite and simple graph. A colouring of vertices of G is a mapping f : V(G) $\rightarrow$ {1,2,3…..k} for every vertex v, the integer $f(v)$ is called the colour of v. If no two adjacent vertices have the same colours then f is called proper colouring. A proper total colouring of a graph G is a mapping f from V(G) $\cup$E(G) to {1,2,3...,k} such that,

a) For all u,v$\in$V(G) if uv$\in$E(G) then f(u)$\neq$f(v)
b) For all $e_1, e_2 \in$ E(G), $e_1 \neq e_2$ if e_1, e_2 have a common end vertex then $f(e_1) \neq f(e_2)$
c) For all u$\in$V(G), e$\in$E(G) if u is the end vertex, then f(u)$\neq$f(e)
d) It is called a avd total colouring if $\phi[u]=\phi[v]$ where $\phi[u]=\{f(e) \mid e$ is incident to v$\}\cup$ {f(v)}.

The avd total chromatic number of G denoted by $\chi_{at}(G)$, is the minimum number of colours needed is an avd total colouring of G. Therefore, $\chi_{at}(G)=\{k \mid G$ is avd total k-colourable}. A cut vertex is a vertex the removal of which disconnect the remaining graph. M. pilsniak and M. Wozniak first introduced that a proper total colouring of ϕ is a proper total colourings distinguishing adjacent vertices by sums if for a vertex $v \in V(\phi)$, the total sum of colours of the edges incident to v and the colour of v, denoted by f(v), are distinct for adjacent vertices. Here we constructed the splitting graph $S^{'}(G)$ formed from the cartesian product of cycle and path graphs and we obtained the bounds for $S^{'}(G)$ using AVD-total colouring for various non-negative values of m and n. We begin with some basic definitions and notations.

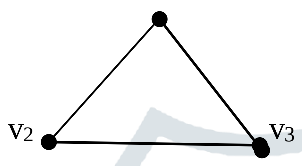
Definition 1.1. The cartesian product of simple graphs G and H is denoted by G × H whose vertex set is V(G)×V(H) and $v=(v_1,v_2)$ and $u=(u_1,u_2)$ are adjacent if $v_1=u_1$ and v_2 is adjacent to u_2 in H or v_1 is adjacent to u_1 in G and $v_2=u_2$ in H.

Example 1.2

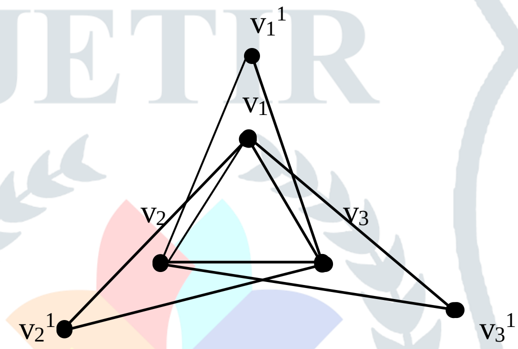


Definition 1.3. For a graph G, the splitting graph $S^{'}(G)$ of a graph G is obtained by adding a new vertex $v^{'}$ corresponding to each vertex v of G such that $N(v) = N(v^{'})$

Example 1.4. a) **G**



b) $S^{'}(G)$



Definition 1.5. Let G be a graph and let V(G) be the set of all vertices of G and let {1,2,3…k} denotes the set of all colours which are assigned to each vertex of G. A proper vertex colouring of a graph G is a mapping C : V(G)$\rightarrow$ {1,2,3…..k}such that C(u) $\neq$ C(v) for all arbitrary adjacent vertices u,v $\in$ V(G).

Definition 1.6. If G has a proper vertex colouring then the chromatic number of G is the minimum number of colours needed to colour G. The chromatic number of G is denoted by $\chi(G)$

Definition 1.7. Let G be a simple graph and ϕ is total colouring of G. ϕ is an AVD-total colouring if for all u,v $\in$V(G), uv adjacent we have C(u) $\neq$ C(v). Here C(u) : set of colours that occur in a vertex u.

2.AVD-total chromatic number of $S^{'}(C_m \times C_n)$

Theorem 2.1

Let C_m and C_n be two cycle graphs of order m and n respectively. Let G=$C_m \times C_n$ be the cartesian product of two cycle graphs and let $S^{'}(G)$ be the splitting graph then the AVD-chromatic number of $S^{'}(G)$is given by $\chi_a[S^{'}(G)] = \Delta + 2 \forall m, n = 3$.

Proof: Let C_m and C_n be two cycle graphs of order m and n respectively. Let G=$C_m \times C_n$ be the cartesian product of two cycle graphs and let $S^{'}(G)$ be the splitting graph. Let W={w_1,w_2,w_3,…..w_m} be the vertex set of C_m and U={u_1,u_2,u_3.....u_n} be the vertex set in C_n. The graph G has mn vertices and 2mn edges. By definition of splitting graph, $S^{'}(G)$ has 2mn vertices and it has 6mn edges. This Theorem can be proved using two cases.

Case1: when m=n=even, i.e., when $m = n = 2l + 2 \forall l = 1,2,3...$ Then,

$$V(G) = \begin{Bmatrix} (w_1, u_1) & (w_1, u_2) & . & . & . & . & (w_1, u_{2l+2}) \\ (w_2, u_1) & (w_2, u_2) & . & . & . & . & (w_1, u_{2l+2}) \\ . & . & & & & & . \\ . & . & & & & & . \\ . & . & & & & & . \\ . & . & & & & & . \\ (w_{2l+2}, u_1) & (w_{2l+2}, u_2) & . & . & . & . & (w_{2l+2}, u_{2l+2}) \end{Bmatrix}$$

be the vertex set of order $4l^2 + 8l + 4$ and $E(G) = \{e_1, e_2, ...e_{8l^2+16l+8}\}$ (where $e_i = v_iv_{i+1} : 1 \le i \le n-1$) be the edge set of order $8l^2 + 16l + 8$. By the definition of splitting graph, adding the new vertices $\{v_i' : 1 \le i \le n-1\}$ corresponding to the vertices $\{v_i : 1 \le i \le n\}$ of G, which are added to obtain $S'(G)$. In $S'(G)$, the vertex set is given by V($S'(G)$)= $\{v_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\}$ with $8l^2 + 16l + 8$ vertices and the edge set E($S'(G)$)={ $v_iv_{i+1} : 1 \le i \le n-1\} \cup \{ v_iv_{i+1}' : 1 \le i \le n-1\} \cup \{ v_i' v_{i+1} : 1 \le i \le n\}$ with $24l^2 + 48l + 24$ edges. In, particular, when m=4,n=4 , the graph G has 16 vertices and 32 edges and the graph $S'(G)$ has 32 vertices and 96 edges. Here V($S'(G)$)= $\{v_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\}$ and E($S'(G)$)={ $v_iv_{i+1} : 1 \le i \le n-1\} \cup \{ v_iv_{i+1}' : 1 \le i \le n-1\} \cup \{ v_i' v_{i+1} : 1 \le i \le n\}$. Now we assign the AVD total colour the graph $S'(G)$ as follows: we construct total colouring with distinguishable vertices,(i.e.,) ϕ is the total colouring of G, if ϕ is an AVD total colouring if $\forall$ uv $\in$ V(G) uv adjacent. We have C(u) $\neq$C(v). Here C(u): Set of colours that occur in a vertex u. The colour sets corresponding to each vertices are as follows :

C(v_1)=C(v_7)={2,3,5,6,7,8,9,10},C(v_2)=C(v_4)=C(v_5)=C(v_{10})=C(v_{13})=C(v_{15})={1,2,3,4,5,7,8,9, 10},C(v_3)=C(v_{12})={1,2,3,5,6,7,8,9,10},C(v_6)=C(v_9)=C(v_{11})={1,3,4,5,6,7,8,9,10},C(v_8)= C(v_{14})={1,2,3,4,6,7,8,9,10}C(v_{16})={1,2,4,5,6,7,8,9,10}C(v_1')=C(v_2')=C(v_3')=C(v_4')=C(v_5') C(v_6')=C(v_7')=C(v_8')=C(v_9')=C(v_{10}')=C(v_{11}')=C(v_{12}')=C(v_{13}')=C(v_{14}')=C(v_{15}')=(v_{16}')={6,7,8,9, 10}. The adjacent vertices have distinct colour sets. which satisfies the condition of AVD-total colouring. Hence the minimum number of colour needed to AVD-total colour the graph S'(G) is $\Delta + 2$.(i.e.,) $\chi_a[S'(G)] = \Delta + 2$, when $m = n = 2l + 2 \forall l = 1,2,3...$

case:2 when m and n are odd and distinct. (i.e.,) if $m = 2l + 1, n = 2l + 3 \ \forall \ l = 1,2....$

$$V(G) = \begin{Bmatrix} (w_1, u_1) & (w_1, u_2) & . & . & . & (w_1, u_{2l+3}) \\ (w_2, u_1) & (w_2, u_2) & . & . & . & (w_2, u_{2l+3}) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ (w_{2l+1}, u_1) & (w_{2l+1}, u_2) & . & . & . & (w_{2l+1}, u_{2l+3}) \end{Bmatrix}$$

be the vertex set of order $4l^2 + 8l + 3$ and $E(G) = \{e_1, e_2, ...e_{8l^2+16l+8}\}$ be the edge set of order $8l^2 + 16l + 6$. By definition of splitting graph, V($S'(G)$)= $\{v_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\}$ be the vertex set of order $8l^2 + 16l + 6$ and E($S'(G)$)={ $v_iv_{i+1} : 1 \le i \le n-1\} \cup \{ v_iv_{i+1}' : 1 \le i \le n-1\} \cup \{ v_i' v_{i+1} : 1 \le i \le n\}$ be the edge set of order $24l^2 + 48l + 18$. Suppose m=3, n=5 we have the graph G has 15 vertices and 30 edges and $S'(G)$ has 30 vertices and 90 edges. Here V[$S'(G)$]=30, E[$S'(G)$]=90, we assign the colours as follows:

C(v_1)=C(v_4)=C(v_8)={1,2,3,4,6,7,8,9,10},C(v_2)=C(v_6)={1,2,3,4,5,7,8,9,10},C(v_3)=C(v_{11})=C(v_{14})={1,2,4,5,6, 7,8,9,10},C(v_5)=C(v_{12})={2,3,5,6,7,8,9,10},C(v_7)=C(v_{10})=C(v_{13})={1,3,4,5,6,7,8,9,10},C(v_9)=C(v_{15})={1,2,3,5 ,6,7,8,9,10}C(v_1')=C(v_2')=C(v_3')=C(v_4')=C(v_5')C(v_6')=C(v_7')=C(v_8')=C(v_9')=C(v_{10}')=C(v_{11}')=C(v_{12}')= C(v_{13}')=C(v_{14}')=C(v_{15}')=(v_{16}')={6,7,8,9,10}. The adjacent vertices have distinct colour sets. which satisfies the total colouring with distinguishable vertices. We need $\Delta + 2$ colour for proper AVD total colouring. Therefore $\chi_a[S'(G)] = \Delta + 2$, proceeding in this manner, for m and n are odd and distinct we have the AVD-chromatic number of $S^{'}(G)$ is $\Delta+2$. (i.e.,) $\chi_a[S^{'}(G)]=\Delta+2$, for $m=2l+1$, $n=2l+3$ $\forall$ $l=1,2....$ from case 1 and 2 we have $\chi_a[S^{'}(G)]=\Delta+2$, m,n≥3.

Example 2.2. when m=3 and n=3. The cartesian product of G=$C_3 \times C_3$ is given in figure 1.

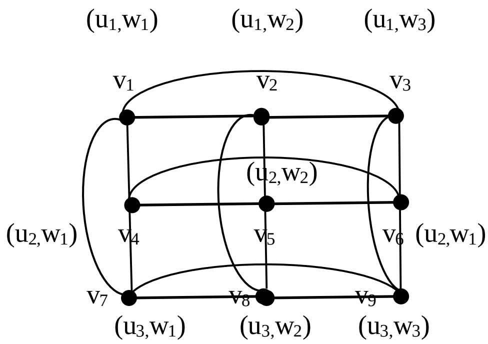


Figure 1. G=$C_3 \times C_3$

The Splitting graph of G=$C_3 \times C_3$ is denoted by $S^{'}(G)$ is shown in figure

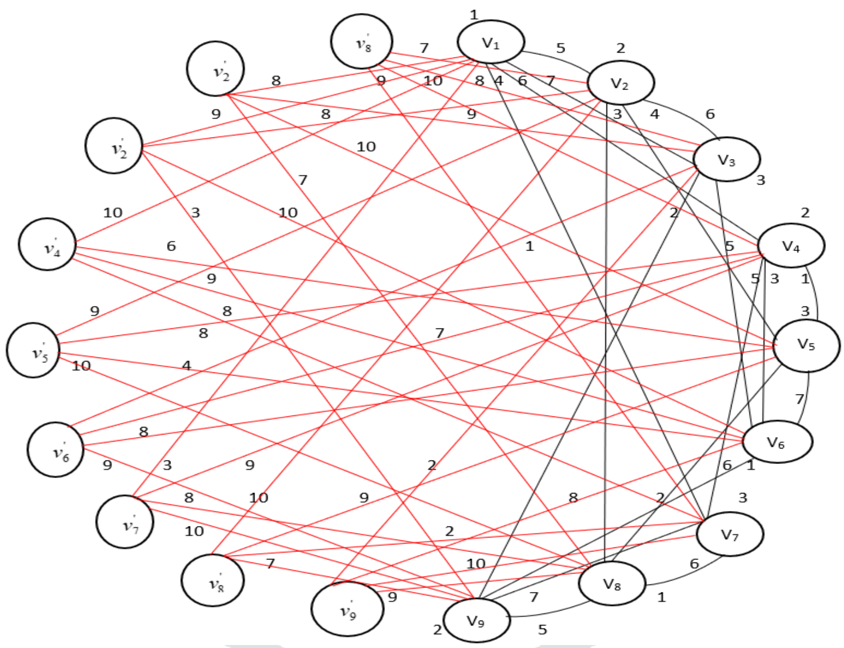


Figure 2. $S^{'}(G)$

From figure 1 and 2 the graph G has 9 vertices and 18 edges and the graph $S^{'}(G)$ has 18 vertices and 54 edges. By proper AVD-total colouring we have, C(v_1)={1,3,4,5,6,7,8,9,10} C(v_2)={2,3,4,5,6,7,8,9,10} C(v_3)={1,2,3,4,5,6,7,8,9} C(v_4)={1,2,3,5,6,7,8,9,10} C(v_5)={1,2,3,4,6,7,8,9,10} C(v_6)={1,3,4,5,6,7,8,9,10} C(v_7)={2,3,4,5,6,7,8,9,10} C(v_8)={1,2,3,5,6,7,8,9,10} C(v_9)={1,2,3,4,5,6,7,9,10} C($v_1^{'}$)=C($v_2^{'}$)= C($v_3^{'}$)=C($v_4^{'}$)=C($v_5^{'}$) C($v_6^{'}$)=C($v_7^{'}$)=C($v_8^{'}$)={3,6,7,8}

3. AVD-total chromatic number of $S^{'}(P_m \times P_n)$

Theorem: 3.1.

Let P_m and P_n be two path graphs of order m and n respectively. Let G= $P_m \times P_n$ be the cartesian product of two path graphs and let $S^{'}(G)$ be the splitting graph then the AVD-chromatic number of $S^{'}(G)$ is given by $\chi_a[S^{'}(G)] = \Delta + 2 \forall m, n = 3$.

Proof:

Let P_m and P_n be the path graphs of order m and n respectively. Let G= $P_m \times P_n$ be the cartesian product of two graphs. Let $S^{'}(G)$ be the splitting graph, the vertex set of P_m be W={$w_1, w_2, \dots, w_m$} and the vertex set of P_n be U={$u_1, u_2, \dots, u_n$}. The graph G has mn vertices and 2mn-(m+n) edges and the splitting graph $S^{'}(G)$ has 2mn vertices and 4mn edges. This theorem can be proved in the following cases

Case(i): when m=n are equal and odd i.e., $m = n = 2l + 1 \ \forall \ l = 1, 2 \dots.$

$$V(G) = \begin{Bmatrix} (w_1, u_1) & (w_1, u_2) & . & . & . & (w_1, u_{2l+1}) \\ (w_2, u_1) & (w_2, u_2) & . & . & . & (w_2, u_{2l+1}) \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ (w_{2l+1}, u_1) & (w_{2l+1}, u_2) & . & . & . & (w_{2l+1}, u_{2l+1}) \end{Bmatrix}$$

be the vertex set and $E(G) = \{e_1, e_2, \dots \dots e_{8l^2+16l+8}\}$ be the edge set in G. By the definition of splitting graph, V($S^{'}(G)$)= {v_i:1≤i≤n}∪{$v_i^{'}$:1≤i≤n} with $8l^2 + 8l - 4$ vertices and the edge set is E($S^{'}(G)$)={ $v_i v_{i+1}$:1≤i≤n-1}∪{ $v_i v_{i+1}^{'}$:1≤i≤n-1}∪{ $v_i^{'} v_{i+1}$:1≤i≤n} with $16l^2 + 16l - 8$ edges. In particular, for m,n=3 the graph G has 9 vertices and 12 edges and the splitting graph $S^{'}(G)$ has 18 vertices and 36 edges. Here V($S^{'}(G)$)={($v_1, v_2, \dots \dots v_n$) ∪ ($v_1^{'}, v_2^{'} \dots . v_n^{'}$)} E($S^{'}(G)$)={($e_1, e_2, \dots e_n$)∪($e_1^{'}, e_2^{'} \dots . e_n^{'}$)} where $e_i = v_i v_{i+1}$ and $e_i^{'} = (v_i^{'} v_{i+1})$∪($v_i v_{i+1}^{'}$). Now we assign the AVD-total colour the graph $S^{'}(G)$ as follows: (i) we construct total colouring with distinguishable vertices. The colour sets corresponding to each vertices are given below :

C(v_1)={1,3,4,8,10},C(v_2)={2,3,4,5,7,8,9}C(v_3)={3,4,2,9,8}C(v_4)={2,4,1,5,8,7,9}C(v_5)={3,1,4,5,6,7,8,9,10} C(v_6)={1,4,5,2,7,10,9}C(v_7)={1,3,5,7,9}C(v_8)={2,3,4,6,7,8,10}, C(v_9)={3,4,5,8,10}C($v_1^{'}$)=C($v_3^{'}$)={1,8,7}C($v_2^{'}$)={1,7,8,9}=C($v_4^{'}$)={1,7,8,10}C($v_5^{'}$)={1,7,8,9, 10}C($v_6^{'}$)=C($v_8^{'}$){1,8,9,10} C($v_7^{'}$)={17,9} C($v_9^{'}$)={1,9,10}. The adjacent vertices have distinct colour sets. Thus the proper AVD-total colouring, Therefore $\chi_a[S^{'}(G)] = \Delta + 2 \forall m, n = 3$ and l=1 proceeding in this way for m and n are equal and odd. We have the AVD-chromatic number of $S^{'}(G)$ is $\chi_a[S^{'}(G)] = \Delta + 2$ when $m = n = 2l + 1 \ \forall \ l = 1, 2 \dots.$

Example 3.2 when n=3,m=3 The cartesian product of G=$P_3 \times P_3$ is given in figure 3.

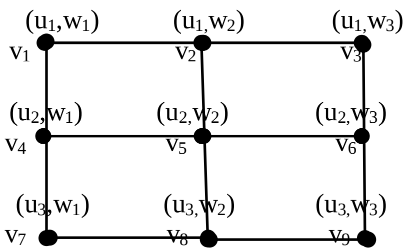
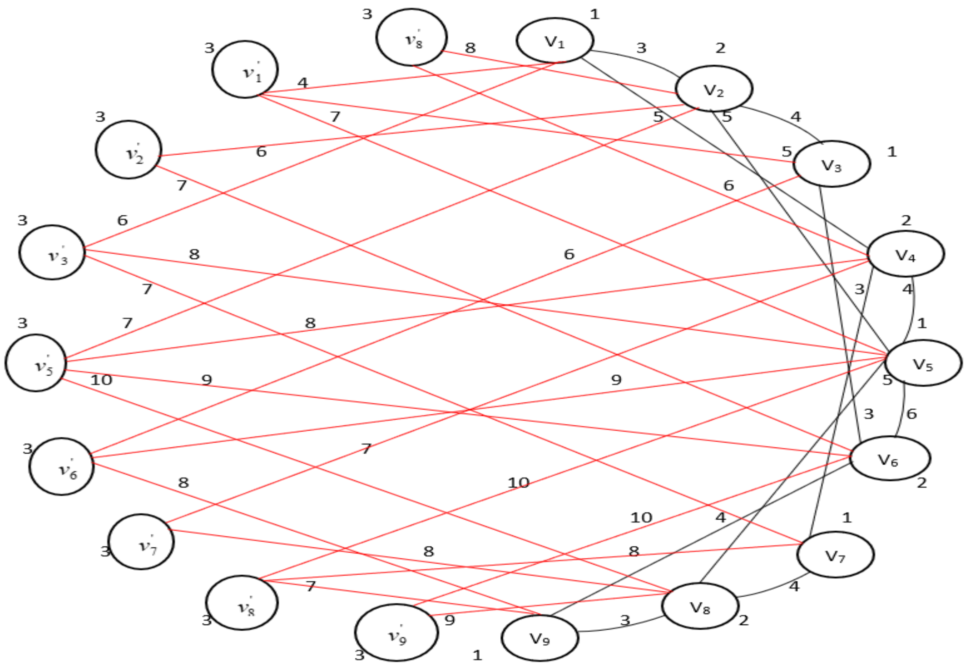


Figure 3. G=$P_3 \times P_3$

Figure 4. The splitting graph of G=$P_3 \times P_3$ is denoted by $S^{'}(G)$ is shown in figure 4.



From figure 1 and 2 the graph G has 9 vertices and 18 edges and the graph $S^{'}(G)$ has 18 vertices and 36 edges. By proper AVD-total colouring we have, C(v_1)={1,3,4,5,6} C(v_2)={2,3,4,5,6,7,8} C(v_3)={1,3,4,5,6,} C(v_4)={2,3,4,5,6,7} C(v_5)={1,3,4,5,6,7,8,9,10} C(v_6)={2,3,4,6,7,9,10} C(v_7)={1,3,4,7,8} C(v_8)={2,3,4,5,8,9,10} C(v_9)={1,3,4,7,8} C($v_1^{'}$)= {3,6,8}C($v_2^{'}$)={3,4,5,7}C($v_3^{'}$)={3,6,7} C($v_4^{'}$)={3,6,7,8}C($v_5^{'}$)={3,7,8,9,10}C($v_6^{'}$)={3,6,8,9}C($v_7^{'}$)={3,7,8}C($v_8^{'}$)={3,7,8,10} C($v_9^{'}$)={3,9,10}

Theorem 3.3.

Let P_m and P_n be two path graphs of order m and n respectively. Let G= $P_m \times P_n$ be the cartesian product of two path graphs and let $S^{'}(G)$ be the splitting graph then the AVD-chromatic number of $S^{'}(G)$ is given by $\chi_a[S^{'}(G)] = \Delta + 2 \forall m, n = 2$.

Proof:

Let P_m and P_n be the two path graphs of order m and n respectively. Let G= $P_m \times P_n$ be the cartesian product of two path graphs and let $S^{'}(G)$ be the splitting graph. Let W={$w_1, w_2, \ldots, w_m$} be the vertex set of P_m and U={$u_1, u_2, \ldots, u_n$} be the vertex set of P_n. The graph G has mn vertices. We prove this theorem by the following cases.

Case1: when m is even and n is odd, i.e., if $m = 2l, n = 2l+1 \; \forall \; l = 1,2....$ $V(G) = \{(w_i, u_j) / \; 1 \le i \le 2l \; ; \; 1 \le j \le 2l+1 \; ; \; \forall \; l = 1,2.... \}$ be the vertex set and the edge set of G is $E(G) = \{e_1, e_2, ...e_{4l^2+4l-1}\}$ By the definition of splitting graph adding the new vertices $\{v_i^{'} : 1 \le i \le n\}$ corresponding to the vertices $\{v_i : 1 \le i \le n\}$ of G. which are added to obtain $S^{'}(G)$. In particular, when m=2, n=5 the graph G has 10 vertices and 13 edges and the graph $S^{'}(G)$ has 20 vertices and 39 edges. Here V($S^{'}(G)$)={($v_1, v_2, \ldots v_n$) ∪ ($v_1^{'}, v_2^{'} \ldots v_n^{'}$)} E($S^{'}(G)$)={($e_1, e_2, \ldots e_n$)∪($e_1^{'}, e_2^{'} \ldots e_n^{'}$)} where $e_i = v_i v_{i+1}$ and $e_i^{'} = (v_i^{'} v_{i+1})$∪($v_i v_{i+1}^{'}$). Now we assign the AVD-total colour the graph $S^{'}(G)$ as follows: (i) we construct total colouring with distinguishable vertices. The colour sets corresponding to each vertices are given below :

C(v_1)={1,3,4,6,7},C(v_2)={2,3,4,6,7},C(v_3)=C(v_6)=C(v_7)={2,3,4,5,6,7,8}C(v_4)=C(v_5)=C(v_6)=C(v_8)={1,3,4,5, 6,7,8},C(v_9)={1,3,5,6,7},C(v_{10})={2,3,5,7,8},C($v_1^{'}$)={3,8,7}C($v_2^{'}$)=C($v_9^{'}$)=C($v_{10}^{'}$)={3,6,7}C($v_3^{'}$)= C($v_4^{'}$)=C($v_5^{'}$)C($v_6^{'}$)=C($v_7^{'}$)=C($v_8^{'}$)={3,6,7,8}.

Case 2: when m and n are even and distinct, i.e., if $m=2l$, $n=2l+2\ \forall\ l=1,2....$ $V(G)=\ \{(w_i,u_j)/\ \ 1\le i\le 2l\ \ ;\ \ 1\le j\le 2l+2\ \ ;\ \ \forall\ \ l=1,2....\ \}$ be the vertex set and the edge set of G is $E(G)=\{e_{1,}e_{2,}......e_{4l^2+6l}\}$. By the definition of splitting graph, For m=4, n=2, we have the graph G has 8 vertices and 10 edges and the splitting graph $S^{'}(G)$ has 16 vertices and 30 edges. Here V($S^{'}(G)$)= {v_i:1≤i≤n} ∪{$v_i^{'}$:1≤i≤n} and E($S^{'}(G)$)={ v_iv_{i+1} :1≤i≤n-1}∪{ $v_iv_{i+1}^{'}$:1≤i≤n-1}∪{ $v_i^{'}v_{i+1}$:1≤i≤n}. By proper AVD-total colouring we have, First we construct total colouring with distinguishable vertices. The colour sets corresponding to each vertices are given below C(v_1)={1,3,4,6,7}, C(v_2)={2,3,4,6,7} C(v_3)=C(v_6)={2,3,4, 5,6,7,8}C(v_4)=C(v_5)={1,3,4,5,6,7,8}C(v_7)={2,3,4,7,8},C(v_8)={1,3,4,6,7},C($v_1^{'}$)=C($v_2^{'}$)= C($v_7^{'}$)={3,6,7}, C($v_3^{'}$)=C($v_4^{'}$)=C($v_5^{'}$) = C($v_6^{'}$)={3,6,7,8},C($v_8^{'}$)={3,7,8}. From case 1 and case 2 we have the adjacent vertices have distinct colour sets. Which satisfies the condition of AVD-total colouring and the minimum number of colour needed to AVD-total colour the graph $S^{'}(G)$ is $\Delta+2$. Therefore proceeding in this way for all m and n are even and distinct, the AVD-total chromatic number of $S^{'}(G)$ is $\Delta+2$. Hence $\chi_a[S^{'}(G)]=\Delta+2\forall m,n=2$.

Example 4.2 when m=4, n=2. The cartesian product of G= $P_4\times P_2$ is given in figure 5.

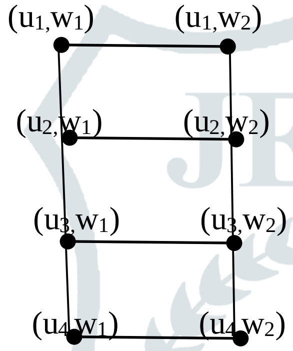


Figure 5. G= $P_4\times P_2$ From figure 5, the graph G has 8 vertices and 10 edges. The splitting graph of figure 5 is shown in figure

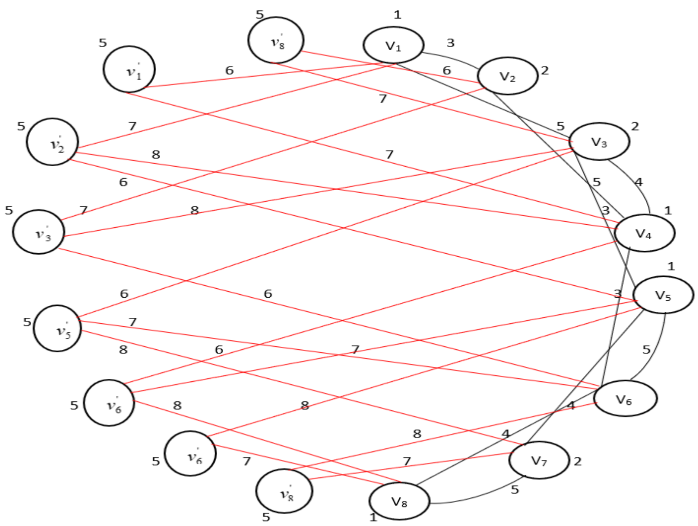


Figure 6. $S^{'}(G)$

From figure 1 and 2 the graph G has 8 vertices and 10 edges and the graph $S^{'}(G)$ has 16 vertices and 30 edges. By proper AVD-total colouring we have, C(v_1)={1,3,5,6,7} C(v_2)={2,3,5,6,7} C(v_3)={2,3,4,5,6,7,8} C(v_4)={1,4,3,5,6,7,8}C(v_5)={1,3,4,5,6,7,8} C(v_6)={2,3,4,5,6,7,8} C(v_7)={2,4,5,7,8} C(v_8)={1,3,5,7,8} C($v_1^{'}$)=C($v_2^{'}$)={5,6,7}C($v_3^{'}$)=C($v_4^{'}$)=C($v_5^{'}$) C($v_6^{'}$)={5,6,7,8}C($v_7^{'}$)=C($v_8^{'}$)={5,7,8}

4. AVD-total chromatic number of $S^{'}(P_m\times C_n)$

In this section we obtain the splitting graph of G formed from the cartesian product of path and cycle graphs and we determined the bounds for the splitting graphs using the concept of AVD-total colouring.

Theorem 4.1:

Let P_m and C_n be two graphs of vertices m and n respectively and let G= $P_m \times P_n$ be the cartesian product of two graphs and let $S^{'}(G)$ be the splitting graph then the AVD-chromatic number of $S^{'}(G)$ is given by $\chi_a[S^{'}(G)] = \Delta + 2 \forall m = 2$ and n>2.

Proof:

Let P_m and C_n be two graphs of vertices m and n respectively. Let G= $P_m \times P_n$ be the cartesian product of two graphs and let $S^{'}(G)$ be the splitting graph. Let W={$w_1, w_2, \ldots. w_m$} be the vertex set in P_m and U={$u_1, u_2, \ldots.. u_n$} be the vertex set in C_n.. The graph G has mn vertices and 2mn-n edges. The splitting graph $S^{'}(G)$ has 2mn vertices and 4mn+n edges. We shall prove this theorem in different cases.

Case 1: when m and n are even and distinct. i.e., $m = 2l, n = 2l + 1 \forall\ l = 1,2....$ $V(G) = \{(w_i, u_j) /\ 1 \le i \le 2l\ ;\ 1 \le j \le 2l + 2\ ;\ \forall\ l = 1,2....\}$ be the vertex set and the edge set of G is $E(G) = \{e_1, e_2, ...e_{8l^2+6l-2}\}$. By the definition of splitting graph, V($S^{'}(G)$)= {v_i:1≤i≤n}∪{$v_i^{'}$:1≤i≤n} and E($S^{'}(G)$)={ v_iv_{i+1} :1≤i≤n-1}∪{ $v_iv_{i+1}^{'}$:1≤i≤n-1}∪{ $v_i^{'}v_{i+1}$:1≤i≤n}. In particular, when m=2, n=4 and when l=1; we have G has 8 vertices and 12 edges and $S^{'}(G)$ has 16 vertices and 36 edges. Now we assign AVD-total colour the graph $S^{'}(G)$ as follows: First we construct total colouring with distinguishable vertices. The colour sets corresponding to each vertices are given below: C(v_1)=C(v_3)=C(v_6)=C(v_8)={1,3,4,5,6,7,8} C(v_2)=C(v_4)=C(v_5)=C(v_7)={2,3,4,5,6,7,8};C($v_1^{'}$)=C($v_2^{'}$)=C($v_3^{'}$)=C($v_4^{'}$)=C($v_5^{'}$)C($v_6^{'}$)= C($v_7^{'}$)=C($v_8^{'}$)={3,6,7,8}. Hence $\chi_a[S^{'}(G)] = \Delta + 2 \forall m = 2$, n=4 and $l = 1$. Proceeding in this manner, For n and m are even and distinct. we have the AVD-chromatic number of $S^{'}(G)$ is $\Delta + 2$. i.e., $\chi_a[S^{'}(G)] = \Delta + 2$ when $m = 2l, n = 2l + 1 \forall\ l = 1,2....$

Case 2: when m is even and n is odd, i.e., $m = 2l, n = 2l + 1 \forall\ l = 1,2....$ $V(G) = \{(w_i, u_j) /\ 1 \le i \le 2l\ ;\ 1 \le j \le 2l + 2\ ;\ \forall\ l = 1,2....\}$ be the vertex set and the edge set of G is $E(G) = \{e_1, e_2, ...e_{8l^2+2l-1}\}$. By the definition of splitting graph, V($S^{'}(G)$)= {v_i:1≤i≤n}∪{$v_i^{'}$:1≤i≤n} and E($S^{'}(G)$)={ v_iv_{i+1} :1≤i≤n-1}∪{ $v_iv_{i+1}^{'}$:1≤i≤n-1}∪{ $v_i^{'}v_{i+1}$:1≤i≤n}. In particular, suppose m=2, n=5 and $l = 1$; The graph G has 10 vertices and 15 edges and the graph $S^{'}(G)$ has 20 vertices and 45 edges. Now we assign AVD-total colour the graph $S^{'}(G)$ as follows. First we construct total colouring with distinguishable vertices. The colour sets corresponding to each vertices are given below : C(v_1)=C(v_3)=C(v_7)= C(v_9)={1,3,4,5,6, 7,8};C(v_2)={2,3,4,5,6,7,8};C(v_4)={1,2,3,4,6,7,8};C(v_5)=C(v_8)={2,3,4,5,6,7,8};C(v_6)={1,2,3,56,7,8}C(v_{10})= {1,2,3,4,5,6,8};C($v_1^{'}$)=C($v_2^{'}$)=C($v_4^{'}$)=C($v_7^{'}$)=C($v_8^{'}$)={3,6,7,8};C($v_3^{'}$)=C($v_9^{'}$)= {3,1,7,8}; C($v_5^{'}$)={3,2,7,8}; C($v_6^{'}$)={3,4,7,8}; C($v_{10}^{'}$)={3,2,5,8}. Hence the minimum number of colours needed to AVD-total colour this graph is $\Delta + 2$. Therefore, $\chi_a[S^{'}(G)] = \Delta + 2$ for all m=2, n>2. Proceeding in this way for m is even and n is odd we have the AVD-chromatic number of $S^{'}(G)$ is $\Delta + 2$ ie., $\chi_a[S^{'}(G)] = \Delta + 2$ when $m = 2l, n = 2l + 1$ $\forall\ l = 1,2....$

Example 4.2 when m=2 and n=4. The cartesian product of G= $P_2 \times C_4$ is shown in figure 7

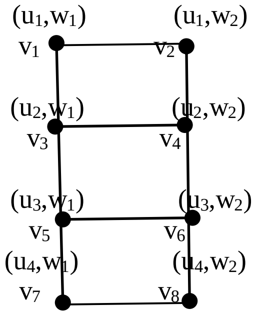


Figure 7. G= $P_2 \times C_4$ The splitting graph of figure 7 is shown in figure 8.

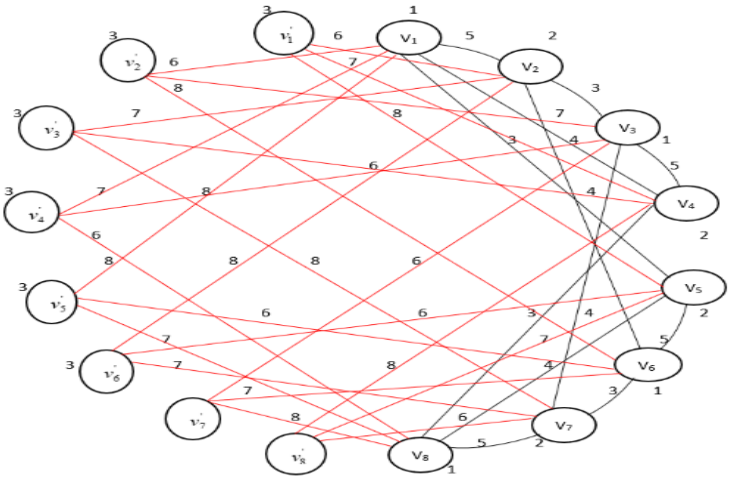


Figure 8. $S^{'}(G)$

From figure 1 and 2 the graph G has 9 vertices and 18 edges and the graph $S^{'}(G)$ has 18 vertices and 54 edges. By proper AVD-total colouring we have, $C(v_1)=\{1,3,4,5,6,7,8\}$ $C(v_2)=\{2,3,4,5,6,7,8\}$ $C(v_3)=\{1,3,4,5,6,7,8\}$ $C(v_4)=\{2,3,4,5,6,7,8\}$ $C(v_5)=\{2,3,4,5,6,7,8\}$ $C(v_6)=\{1,3,4,5,6,7,8\}$ $C(v_7)=\{2,3,4,5,6,7,8\}$ $C(v_8)=\{1,3,4,5,6,7,8\}$ $C(v_1^{'})=C(v_2^{'})=C(v_3^{'})= C(v_4^{'})=C(v_5^{'})$ $C(v_6^{'})=C(v_7^{'})=$ $C(v_8^{'})=\{3,6,7,8\}$

Conclusion :

In this paper, we have established the chromatic and AVD-chromatic number of splitting graphs formed from the cartesian product of path and cycle. We discussed the relationship between the AVD-chromatic number and chromatic number with different parameters. Subsequently, this work can be additionally extended to simple graphs formed from various graph products.

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