NEW CLASSES OF TOPOLOGICAL MAPPINGS

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Abstract : In this paper we introduce and study a new class of function called αDδg continuous and αDδg irresolute and obtain a decomposition of continuity, α − semi continuity and αδgˆ continuity in topological spaces.

I. INTRODUCTION

Levine [15], Noiri [22], Balachandran et al [4], Dontchev.J and Ganster [5] in- troduced generalised closed sets,δ-continuity, generalised continuous function and δ- generalised continuous (beiefly δg-contineous) & δ-generalised irresolute functions respectively. Devi et al [4] and Veerakumar [26] introduced semi-generalised conti- nuity and ˆg-continuity in topological spaces. Ganster and Reilly [9] introduced and studied the notion of LC-continuous functions. Dontchev [7] presented a new notion of continuous function called contra-continuity. This notion is a stronger form of LC-continuity. Dontchev and Noiri [6] introduced a weaker form of contra-continuity called contra-semi-continuity. The purpose of this present chapter is to define a new class of generalised continuous functions called αDδg-continuous functions and in- vestigate their relationships to other generalised continuous functions. We further introduce and study a new class of functions namely αDδg-irresolute. Also we define a new class of generalized continuous functions called contra-αDδg-continuous func- tions and investigate their relationships to other functions. We further introduce and study two new spaces called αDδg-Hausdorff spaces and αDδg-normal spaces and obtain some new results.

1.1. Preliminaries. Throughout this thesis (X,τ) (or simply X) represent topolog- ical spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 1.1. A subset A of a space (X,τ) is called a
(i) semi-open set [14] if A ⊆ cl(int(A)).
(ii) pre-open set [17] if A ⊆ int(cl(A)).
(iii) α-open set [21] if A ⊆ int(cl(int(A))).
The complement of a semi-open (resp. pre-open, α-open) set is called semi-closed (resp. pre-closed,α-closed).

Definition 1.2. The δ-interior [27] of a subset A of X is the union of all regular open set of X contained in A and is denoted by intδ(A). The subset A is called δ- open [27] if A =intδ(A), i.e. a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set A⊆ (X,τ) is called δ-closed [27] if A=clδ(A), where clδ(A)={ x∈X: int(cl(U))∩A6= φ,U∈ τ and x∈ U}.

Definition 1.3. A subset A of (X,τ) is called
(i) generalized closed (briefly g-closed) [15] if cl(A)⊆U whenever A ⊆U and U is open in (X,τ).
(ii) generalized alpha-semi-closed (briefly αgs-closed) [2] if scl(A)⊆ U whenever A ⊆ U and U is αopen in (X,τ).
(iii) α- generalized closed (briefly αg-closed) [16] if αcl(A)⊆ U whenever A ⊆ U and U is open in (X,τ).
(iv) αδ-generalized closed (briefly αδg-closed) [5] if clδ(A) ⊆ U whenever A ⊆ U and U is α open in (X,τ).
(v) ˆg-closed [26] if cl(A)⊆U whenever A ⊆U and U is semi-open in (X, τ). The complement of gˆ-closed set is called gˆ-open.
(vi) α-δ-gˆ-closed (briefly αδgˆ-closed) [11] if αclδ(A) ⊆U whenever A ⊆U and U is gˆ- open in (X,τ).
The complement of a g-closed (resp. gs-closed, αg-closed, δg-closed, gˆ-closed and δgˆ-closed) set is called g-open (resp. gs-open,αg-open, δg-open, gˆ-open and δgˆ-open).

Definition 1.4. [8] A subset A of a space (X, τ) is called a αB-set if A = G ∩ F where G is α open and F is t- set in X.

Definition 1.5. A space (X,τ) is called (i) T1/2-space [15] if every g-closed set in it is closed. (ii) T3/4-space [5] if every δg-closed set in it is δ-closed. (iii) Tˆ 3/4-space [11] if every δgˆ-closed set in it is δ- closed.

Definition 1.6. A function f : (X, τ) → (Y, σ) is called
(i) α semi-continuous [14] if f^{-1} (V) is α semi-closed in (X, τ) for every closed set V of (Y, σ).
(ii) g-continuous [4] if f^{-1} (V) is g-closed in (X, τ) for every closed set V of (Y, σ).

(iii) αgs-continuous [3] if f^{-1} (V) is αgs-closed in (X, τ) for every closed set V of (Y, σ).
(iv) αg-continuous [4] if f^{-1} (V) is αg-closed in (X, τ) for every closed set V of (Y, σ).
(v) Super continuous [22] if f^{-1} (V) is δ-open in (X, τ) for every open set V of (Y, σ).
(vi) ˆg-continuous [26] if f^{-1} (V) is gˆ-closed in (X, τ) for every gˆ-closed set V of (Y, σ).
(vii) δ-continuous [22] if f^{-1} (V) is δ-open in (X, τ) for every δ-open set V of (Y, σ).
(viii) δ-closed [22] if f(V) is δ-closed in (Y, σ) for every δ-closed set V of (X, τ).
(ix) αδg-continuous [5] if f^{-1} (V) is αδg-closed in (X, τ) for every closed set V of (Y, σ). (x) αδgˆ- continuous [11] if f^{-1} (V) is αδgˆ-closed in (X, τ) for every closed set V of (Y, σ).

Definition 1.7. A function f : (X, τ) → (Y, σ) is called
(i) contra-continuous [7] if f−1 (V) is closed in (X, τ) for every open set V in (Y, σ).
(ii) contra-αgs-continuous [6] if f−1 (V) is αgs-closed in (X, τ) for every open set V of (Y, σ).
(iii) contra-αδgˆ-continuous [13] if f−1 (V) is αδgˆ-closed in (X, τ) for every open set V in (Y, σ).

Definition 1.8. A map f : (X, τ) → (Y, σ) is called
(i) generalized closed (briefly g-closed) (resp. g-open) [18] if the image of every closed (resp. open) set in (X, τ) is g-closed (resp. g-open) in (Y, σ).
(ii) α generalized semi-closed (briefly αgs-closed) (resp. αgs-open) [?] if the image of every closed (resp. open) set in (X, τ) is αgs-closed (resp. αgs-open) in (Y, σ).
(iii) ˆg-open [26] if f(V) is gˆ-open in (Y, σ) for every open set V of (X, τ).
(iv) δ-closed [22] if f(V) is δ-closed in (Y, σ) for every δ-closed set V of (X, τ).
(v) αδgˆ-closed [11] if the image of every closed set in (X, τ) is αδgˆ-closed in (Y, σ).

Definition 1.9. A Topological Space is said to be Ultra normal [25] if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

Proposition 1.10. Every αDδg-closed set is αδg-closed (resp.g-closed, αg-closed, αgs-closed).

2. αDδg-continuous

Definition 2.1. A function f : (X, τ) → (Y, σ) is called αDδg-continuous if f−1(V)is αDδg-closed in (X, τ) for every closed set V of (Y, σ).

Example 2.2. Let X = {a, b, c} and Y= {p, q, r} with the topologies τ = {φ, {a},{b}, {a, b}, X} and σ = {φ, {q}, {p, q}, Y}. Define a function f : (X, τ) → (Y, σ)by f(a) = p, f(b) = q and f(c) = r. Clearly f is αDδg-continuous.

Definition 2.3. A function f : (X, τ) → (Y, σ) is called αDδg-irresolute if f−1(V) is αDδg-closed in (X, τ) for every αDδg-closed set V of (Y, σ).

Example 2.4. Let X = {a, b, c}, Y= {p, q, r} with the topologies τ={φ, {a}, X}and σ = {φ, {q}, {q, r}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = p, f(b) = r andf(c) = q. Clearly f is αDδg-irresolute.

Proposition 2.5. If f : (X, τ) → (Y, σ) is αDδg-continuous then f is g-continuous,αg-continuous, αgs-continuous and αδg-continuous maps.

Proof It is true that every αDδg-closed set is g-closed, αg-closed, αgs-closed and αδg-closed.

Remark 2.6. The converse of the above proposition is not true as seen from the following examples.
Example 2.7. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ={φ, {c},{b,c}, X} and σ ={ Φ, {q, r}, Y}. Define the map f : (X, τ) → (Y, σ) by f(a) = q,f(b) = p and f(c) = r. Clearly f is not αDδg-continuous because {p} is closed in(Y, σ) but f−1({p}) = {b} is not αDδg-closed in (X, τ). However f is g-continuous.

Example 2.8. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {c},X} and, σ = { Φ, {p}, {p, q}, {p, r}, Y}. Let f : (X, τ) → (Y, σ) be a function definedby f(a) = p, f(b) = q and f(a) = r. Then f is αg-continuous and sg-continuous. But f is not αDδg-continuous. Since for the closed set {q} of (Y, σ), f−1({q}) = {b} is not αDδg-closed in (X, τ).

Example 2.9. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ,{c}, {b, c}, X} and σ = { Φ, {p}, {q}, {p, q}, Y}. Define f : (X, τ) → (Y, σ) byf(a) = r, f(b) = r and f(c) = p. Then f is not αDδg-continuous because {r} isclosed in (Y, σ) but f−1({r}) = {a, b} is not αDδg-closed in (X, τ). However f is δg-continuous functions.

Theorem 2.10. Every super continuous is αDδg-continuous.
Proof It is true that every δ-closed set is αDδg-closed.

Remark 2.11. The converse of Theorem 2.10 need not be true as shown in the following example.

Example 2.12. Let X = {a, b, c, d} and Y = {p, q, r} with the topologies τ = {φ,{d}, {a, c}, {a, c, d}, X} and σ = { Φ, {p, q}, Y}. Define f : (X, τ) → (Y, σ) be afunction defined by f(a) = p, f(b) = r and f(c) = q. Then f is αDδg-continuous. But f is not super continuous because {r} is closed in (Y, σ) but f−1({r}) = {b} is not δ-closed in (X, τ).

Remark 2.13. The following examples show that αDδg-continuity is independent of continuity, α − semi-continuous and αδgˆ-continuity.

Example 2.14. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {a},{b, c}, X} and σ = { Φ, {p, q}, Y}. Let f : (X, τ) → (Y, σ) be a function defined by f(a) = q, f(b) = r and f(c) = p. Then f is neither continuous nor α-semi-continuous also not αδgˆ-continuous but αDδg-continuous.

Example 2.15. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {c},{b, c} X} and σ = { Φ, {q}, Y}. Define a function f : (X, τ) → (Y, σ) by f(a) = p,f(b) = r and f(c) = q. Then f is continuous, α-semi-continuous and αδgˆ-continuous but not αDδg-continuous.

3. Properties and Characterizations

Theorem 3.1. A function f : (X, τ) → (Y, σ) is αDδg-continuous iff f−1(U) is αDδg-open in (X, τ) for every open set U in (Y, σ).

Proof Let f : (X, τ) → (Y, σ) be an αDδg-continuous function and U be an open set in (Y, σ). Then f−1(U^c) is αDδg-closed set in (X, τ). But f−1(U^c) = [f−1(U)]cand hence f−1(U) is αDδg-open in (X, τ). Conversely f−1(U) is αDδg-open in (X, τ)for every open set U in (Y, σ). Then Ucis closed set in (Y, σ) and $[f−1(U)]^c$ is αDδg-closed in (X, τ). But $[f−1(U)]^c$ = f−1(U^c), so f−1(U^c) is αDδg-closed set in (X, τ).Hence f is αDδg-continuous.

Theorem 3.2. Let f : (X, τ) → (Y, σ) be a αDδg-irresolute and g : (Y, σ) → (Z, η) a αDδg-irresolute. Then their composition is g ◦f : (X, τ) → (Z, η) is αDδg-irresolute.

Proof Let F be αDδg-closed set in (Z, η). Then g−1(F) is αDδg-closed in (Y, σ).Since f is αDδg-irresolute, (g ◦ f)−1(F) = f−1(g−1(F)) is αDδg-closed set of (X, τ)and so g ◦ f is αDδg-irresolute function.

Remark 3.3. The composition of two αDδg-continuous functions need not be αDδg-continuous as the following examples shows.

Example 3.4. Let X = {a, b, c} = Y = Z with the topologies τ ={φ, {b}, {c}, {a,b}, {b, c}, X} and σ = {φ, {b}, {a, c}, Y} and η = {φ, {a}, {b}, {a, b}, Z}. Letf : (X, τ) → (Y, σ) be the identity function. and define a function g : (Y, σ) → (Z, η)by g(a) = a,g(b) = c and g(c) = b . Clearly f and g are αDδg-continuous. But g ◦f : (X, τ) → (Z, η) is not an αDδg-continuous function because (g ◦f)−1({b, c}) =f−1(g−1({b, c})) = f−1({b, c}) = {c} is not an αDδg-closed in (X, τ) where {b, c} is a closed set of (Z, η).

Theorem 3.5. Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be two functions.Then
(i) g ◦ f : (X, τ) → (Z, η) is αDδg-continuous, if g is continuous and f is αDδg continuous.
(ii) g ◦ f : (X, τ) → (Z, η) is αDδg-continuous, if g is αDδg-continuous and f is α Dδg-irresolute.

Proof (i) Let F be closed set in (Z, η). Since g is continuous, g−1(F) is closedin (Y, σ). But f is αDδg-continuous, (g ◦ f)−1(F) = f−1(g−1(F)) is αDδg-closed set of (X, τ) and hence g ◦ f is αDδg-continuous function. (ii) Let G be any closed set in (Z, η). Then g−1(G) is αDδg-closed in (Y, σ). Since f is αDδg-irresolute,(g ◦ f)−1(G) = f−1(g−1(G)) is αDδg-closed set of (X, τ) and so g ◦ f is αDδg-continuous functions.

Theorem 3.6. Let f : (X, τ) → (Y, σ) be continuous and δ -closed map. Then for every αDδg-closed subset A of (X, τ), f(A) is αDδg-closed in (Y, σ).

Proof Let A be αDδg-closed in (X, τ). Let f(A) ⊆ O where O is open in (Y, σ).Since A⊆ f−1(O) is open in (X, τ), f−1(O) is αB-set in (X, τ). Since A is αDδg-closed and since f−1(O) is αB-set in (X, τ), then clδ(A) ⊆ f−1(O). Thus f(clδ(A)) ⊆ O.Hence clδ(f(A)) ⊆ clδ(f(clδ(A)) = f(clδ(A)) ⊆ O, since f is δ-closed. Hence f(A) is αDδg-closed in (Y, σ).

Remark 3.7. αDδg-continuity and αDδg-irresoluteness are independent notions as seen in the following examples.

Example 3.8. Let X = {a, b, c}, Y = {a, b, c} with the topologies τ = {φ, {c}, {a,b}, {b, c}, X} and σ = { Φ, {b}, {a, c}, Y}. Define a function f : (X, τ) → (Y, σ) by f(a) = a, f(b) = c and f(c) = b. Then f is αDδg-continuous but it is not αDδg-irresolute function because f−1({c}) = {b} is not αDδg-closed in (X, τ), where {c} is αDδg-closed in (Y, σ).

Example 3.9. Let X = {a, b, c}, Y = {p, q, r} with the topologies τ = {φ, {a}, {b},{a, b}, {a, c}, X} and σ = { Φ, {r}, {q, r}, Y}. Define a function f : (X, τ) → (Y, σ)by f(a) = p, f(b) = q and f(c) = r. Then f is αDδg-irresolute but it is not αDδg-continuity function because f−1({p}) = {a} is not αDδg-closed in (X, τ), where {p}is closed in (Y, σ).

Proposition 3.10. The product of two αDδg-open sets of two spaces is αDδg-openset in the product space.

Proof Let A and B be two αDδg-open sets of two spaces (X, τ) and (Y, σ) respectively and V = A×B ⊆ X ×Y. Let F⊆ V be a complement of αB-set in X ×Y, then there exists two complement of αB-sets F1 ⊆ A and F2 ⊆ B. So, F_1 ⊆ intδ(A) andF_2 ⊆ intδ(B). Hence $F_1 \times F_2$ ⊆ A×B and $F_1 \times F_2$ ⊆ intδ(A)×intδ(B) = intδ(A×B).Therefore A × B is αDδg-open subset of the space X × Y.

Theorem 3.11. Let f_i: $X_i \rightarrow Y_i$ be αDδg-continuous functions for each i ∈ {1, 2}and let f : $X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined by f((x_1, x_2)) = (f(x_1), f(x_2)). Then f is αDδg-continuous.

Proof Let V1 and V2 be two open sets in Y1 and Y2 respectively. Since f_i: $X_i \rightarrow Y_i$ are αDδg-continuous functions, for each i∈ {1,2}, $f_1^{-1}(V_1)$ and $f_2^{-1}(V_2)$ are αDδg-open sets in X1 and X2 respectively. By the Proposition 4.5, $f_1^{-1}(V_1) \times f_2^{-1}(V_2)$ is αDδg-open set in X1 × X2. Therefore f is αDδg-continuous.

Theorem 3.12. Let f : (X, τ) → (Y, σ) be a function. Then the following statements are equivalent.
(i) f is αDδg-irresolute.
(ii) For x ∈ (X, τ) and any αDδg-closed set V of (Y, σ) containing f(x) there exists an αDδg-closed set U such that x ∈ U and f(U) ⊂ V.
(iii) Inverse image of every αDδg-open set of (Y, σ) is αDδg-open in (X, τ).

Proof (i) ⇒ (ii). Let V be an αDδg-closed set of (Y, σ) and f(x) ∈ V . Since f is αDδg-irresolute, f−1(V) is αDδg-closed in (X, τ) and x ∈ f−1(V). Put U = f−1(V).Then x ∈ U and f(U) ⊆ V . (ii) ⇒ (i). Let V be an αDδg-closed set of (Y, σ) andx ∈ f−1(V). Then f(x) ∈ V. Therefore by (ii) there exists an αDδg-closed set Uxsuch that x ∈ U_x and f(U_x) ⊆ V. Hence x ∈ U_x⊆ f−1(V). This implies that f−1(V) is a union of αDδg-closed sets of (X, τ), f−1(V) is αDδg-closed set. This shows thatf is αDδg-irresolute. (i) ⇔ (iii). It is true that f−1(Y − V) = X−f−1(V).

4. Properties and Characterizations

We introduce the following definitions.

Definition 4.1. A function f : (X, τ) → (Y, σ) is called contra-αDδg-continuous if $f^{-1}(V)$ is αDδg-closed in (X, τ) for every open set V in (Y, σ).

Example 4.2. Let X = {a, b, c}, Y = {p, q, r} with the topologies τ = {φ, {a}, {b},{a, b}, {a, c}, X} and σ = { Φ, {q}, {p, r}, Y}. Define a function f : (X, τ) → (Y, σ) by f(a) = p, f(b) = q and f(c) = r. Clearly f is contra-αDδg-continuous function.

Definition 4.3. A map f : (X, τ) → (Y, σ) is said to be contra-αDδg-irresolute iff−1(V) is αDδg-closed in (X, τ) for every αDδg-open set V in (Y, σ).

Example 4.4. Let X = {a, b, c}, Y={p, q, r} with the topologies τ = {φ, {a},{b}, {a, b}, X} and σ = { Φ, {r}, {p, r}, Y}. Define a function f : (X, τ) → (Y, σ)by f(a) = r, f(b) = q and f(c) = p. The family of all αDδg-open sets of (X, τ) is denoted by αDδgO(X). The set αDδg − O(X, x) = {V ∈ αDδg − O(X)/x ∈ V } for x ∈ X.

Proposition 4.5. The product of two αDδg-open sets of two spaces is αDδg-openset in the product space.

Proof Let A and B be two αDδg-open sets of two spaces (X, τ) and (Y, σ) respectively and V = A×B ⊆ X ×Y. Let F⊆ V be a complement of αD-set in X ×Y, then there exists two complement of αD-sets F_1 ⊆ A and F_2 ⊆ B. So, F_1 ⊆ intδ(A) and F_2 ⊆

intδ(B). Hence $F_1 \times F_2$ ⊆ A×B and $F_1 \times F_2$ ⊆ intδ(A)×intδ(B) = intδ(A×B).Therefore A × B is αDδg-open subset of the space X × Y.

Theorem 4.6. Let f : (X, τ) → (Y, σ) be a map. Then the followings are equivalent.
(i) f is contra-αDδg-continuous.
(ii) The inverse image of each closed set in (Y, σ) is αDδg-open in (X, τ).
(iii) For each x ∈X and each F ∈ C(Y, f(x)), there exists U ∈ αDδgO(X, x) suchthat f(U) ⊆ F.

Proof (i) ⇒ (ii), (ii) ⇒ (i) and (ii) ⇒ (iii) are obvious. (iii) ⇒ (ii) Let F be any closed set of (Y, σ) and x ∈ f^{-1} (F). Then f(x) ∈ F and there exists U_x ∈ αDδgO(X, x) such that f(U_x) ⊆ F. Hence we obtain f−1(F) = U{U_x/x ∈f−1(F)} ∈ αDδgO(X). Thus the inverse of each closed set in (Y, σ) is αDδg-open in(X, τ).

Remark 4.7. The concepts of αDδg-continuity and contra-αDδg-continuity are independent as shown in the following examples.

Example 4.8. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {a},{b}, {a, b}, X} and σ = { Φ, {r}, {p, r}, Y}. Define the map f : (X, τ) → (Y, σ) by f(a) = r, f(b) = p and f(c) = q. Clearly f is αDδg-continuous function, but f is not contra-αDδg-continuous because f^{-1}({r}) = {a} is not αDδg-closed in (X, τ) where{r} is open in (Y, σ).

Example 4.9. Let X = {a, b, c} and Y={p, q, r} with the topologies τ = {φ, {a},X} and σ = { Φ, {p, q}, Y}. Define a function f : (Y, σ) → (X, τ) by f(p) = b,f(q) = c, f(r) = a. Then f is contra-αDδg-continuous function, but f is not αDδg-continuous because

f^{-1}({b c}) = {p, q} is not αDδg-closed in (X, τ) where {r} is closed in (Y, σ).A map f : (X, τ) → (Y, σ) is said to be contra-αDδg-irresolute iff^{-1}(V) is αDδg-closed in (X, τ) for every αDδg-open set V in (Y, σ).

Example 4.4. Let X = {a, b, c}, Y={p, q, r} with the topologies τ = {φ, {a},{b}, {a, b}, X} and σ = { ϕ, {r}, {p, r}, Y}. Define a function f : (X, τ) → (Y, σ)by f(a) = r, f(b) = q and f(c) = p. The family of all αDδg-open sets of (X, τ) isdenoted by αDδgO(X). The set αDδg − O(X, x) = {V ∈ αDδg − O(X)/x ∈ V } forx ∈ X.

Proposition 4.5. The product of two αDδg-open sets of two spaces is αDδg-openset in the product space.

Proof Let A and B be two αDδg-open sets of two spaces (X, τ) and (Y, σ) respectively and V = A×B ⊆ X ×Y. Let F⊆ V be a complement of αD-set in X ×Y, then there exists two complement of αD-sets F_1 ⊆ A and F_2 ⊆ B. So, F1 ⊆ intδ(A) andF_2 ⊆ intδ(B). Hence F_1×F_2 ⊆ A×B and F_1×F_2 ⊆ intδ(A)×intδ(B) = intδ(A×B).Therefore A × B is αDδg-open subset of the space X × Y.

Theorem 4.6. Let f : (X, τ) → (Y, σ) be a map. Then the followings are equivalent.
(i) f is contra-αDδg-continuous.
(ii) The inverse image of each closed set in (Y, σ) is αDδg-open in (X, τ).
(iv)For each x ∈X and each F ∈ C(Y, f(x)), there exists U ∈ αDδgO(X, x) such that f(U) ⊆ F.
(v)

Proof (i) ⇒ (ii), (ii) ⇒ (i) and (ii) ⇒ (iii) are obvious. (iii) ⇒ (ii) LetF be any closed set of (Y, σ) and x ∈ f^{-1}(F). Then f(x) ∈ F and there exists U_x ∈ αDδgO(X, x) such that f(U_x) ⊆ F. Hence we obtain f^{-1}(F) = U{Ux/x ∈f^{-1}(F)} ∈ αDδgO(X). Thus the inverse of each closed set in (Y, σ) is αDδg-open in(X, τ).

Remark 4.7. The concepts of αDδg-continuity and contra-αDδg-continuity are independent as shown in the following examples.

Example 4.8. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {a},{b}, {a, b}, X} and σ = { ϕ, {r}, {p, r}, Y}. Define the map f : (X, τ) → (Y, σ) byf(a) = r, f(b) = p and f(c) = q. Clearly f is αDδg-continuous function, but f is not contra-αDδg-continuous because f^{-1}({r}) = {a} is not αDδg-closed in (X, τ) where{r} is open in (Y, σ).

Example 4.9. Let X = {a, b, c} and Y={p, q, r} with the topologies τ = {φ, {a},X} and σ = { ϕ, {p, q}, Y}. Define a function f : (Y, σ) → (X, τ) by f(p) = b,f(q) = c, f(r) = a. Then f is contra-αDδg-continuous function, but f is not αDδg-continuous because f^{-1}({b c}) = {p, q} is not αDδg-closed in (X, τ) where {r} is closed in (Y, σ).

Remark 4.10. A function f : (X, τ) → (Y, σ) is αDδg-continuous if for each x ∈ X and each open set V of Y containing f(x), there exists U ∈ αDδgO(X, x) such that f(U) ⊂ V .

Theorem 4.11. If a function f : (X, τ) → (Y, σ) is contra-αDδg-continuous and (Y, σ) is regular then f is αDδg-continuous.
Proof Let x be an arbitrary point of (X, τ) and V be any open set of (Y, σ)containing f(x). Since (Y, σ) is regular, there exists an open set W of (Y, σ) containingf(x) such that cl(W) ⊆ V . Since f is contra-αDδg-continuous, by Theorem 4.6 there exists U ∈ αDδgO(X, x) such that f(U) ⊆ cl(W). Then f(U) ⊆ cl(W) ⊆ V . Hence by Remark 4.10 f is αDδg-continuous.

Remark 4.12. The concepts of contra-continuous and contra-αDδg-continuous are independent as shown in the following examples.

Example 4.13. X = {a, b, c} and Y={p, q, r} with the topologies τ = {φ, {b}, {a,c}, X} and σ = { ϕ, {p}, {r}, {p, r}, {q, r}, Y}.Define a function f : (X, τ) → (Y, σ)by f(a) = p, f(b) = r, f(c) = q. Then clearly f is contra-αDδg-continuous but f is not contra-continuous because f^{-1}({q, r})={b,c} is not closed in (X, τ) where {q, r} is open in (Y, σ).

Example 4.14. X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {b},{a, b}, {b, c}, X} and σ = { ϕ, {p}, {p, r}, Y}. Define a map f : (X, τ) → (Y, σ)by f(a) = r, f(b) = q, f(c) = p. Then clearly f is contra-continuous. But f is not contra-αDδg-continuous because f^{-1}({p, r}) = {a, b} is not αDδg-closed in (X, τ) where {a, b} is open in (Y, σ).

Remark 4.15. The concept of contra-αDδg-continuous and contra-δgˆ-continuous are independent of each other as shown in the following examples.

Example 4.16. X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {a},{b}, {a, b}, {a, c}, X} and σ = { ϕ, {r}, {q, r} ,Y}. Define a map f : (X, τ) → (Y, σ)by f(a) = p, f(b) = r, f(c) = q. Then clearly f is contra-δgˆ-continuous but f isnot contra-αDδg-continuous because f^{-1}({q, r}) = {b, c} is not αDδg-closed in (X, τ) where {q, r} is open in (Y, σ).

Example 4.17. X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {c}, {a,b}, X} and σ ={ ϕ, {q}, {r}, {p, q}, {q, r}, Y}. Define a map f : (X, τ) → (Y, σ) byf(a) = r, f(b) = p, f(c) = q. Then clearly f is contra-αDδg-continuous but f is not contra-αδgˆ-continuous because f^{-1}({r}) = {a} is not αδgˆ-closed in (X, τ) where {r} is open in (Y, σ).

Remark 4.18. The composition of two contra-αDδg-continuous functions need not be contra-αDδg-continuous as the following example shows.

Example 4.19. Let X ={a, b, c} = Y = Z; τ = {φ, {a}, {b}, {a, b}, {a, c} ,X}, σ ={ Φ, {b}, {a, c}, Y} and η = { Φ, {a}, {c}, {a, b}, {a, c}, Z}. Let f : (X, τ) → (Y, σ)and g : (Y, σ) → (Z, η) be two identity functions. Then both f and g are contra-α Dδg-continuous but g ∘ f : (X, τ) → (Z, η) is not contra-αDδg-continuous because$(g \circ f)^{-1}$({a, b})= {a,b} is not αDδg-closed in (X, τ) where {a, b} is open in (Z, η).

Theorem 4.20. If f : (X, τ) → (Y, σ) is contra-α D δ g-continuous function and g : (Y, σ) → (Z, η) is a continuous function. Then g ∘ f : (X, τ) → (Z, η) is contra-α Dδg-continuous.

Proof Let V be open set in (Z, η). Since g is continuous, g−1(V) is open in (Y, σ).Since f is contra-αDδg-continuous, $f^{-1}(g^{-1}(V))$ is αDδg-closed in (X, τ). That is$(g \circ f)^{-1}(V)$ is αDδg-closed in (X, τ). Hence (g ∘ f) is contra-αDδg-continuous.

Theorem 4.21. If f : (X, τ) → (Y, σ) is α D δ g-irresolute and g : (Y, σ) → (Z, η) is contra-αDδg-continuous function. Then g ∘ f : (X, τ) → (Z, η) is contra-αDδg-continuous.

Proof Let V be open in (Z, η). Since g is contra-αDδg-continuous, $g^{-1}(V)$ is α Dδg-closed in (Y, σ). Since f is αDδg-irresolute, $f^{-1}(g^{-1}(V))$ is αDδg-closed in (X, τ). That is $(g \circ f)^{-1}(V)$ is αDδg-closed in (X, τ). Hence (g ∘ f) is contra-αDδg-continuous.

Theorem 4.22. Let f : (X, τ) → (Y, σ) be a map then the followings are equivalent.
(i) f is contra-αDδg-irresolute.
(ii) For x ∈ X and any α D δ g-open set V of (Y, σ) containing f(x) there exists an αDδg-closed set U such that x ∈ U and f(U)⊂ V .
(iii) Inverse image of every αDδg-closed set in (Y, σ) is αDδg-open in (X, τ).

Proof (i) ⇒ (ii). Let V be a αDδg-open set in (Y, σ) and f(x) ∈ V. Since f is contra-αDδg-irresolute, $f^{-1}(V)$ is α D δ g-closed in (X, τ) and x ∈ $f^{-1}(V)$. Put U = $f^{-1}(V)$. Then x ∈ U and f(U) ⊂ V . (ii) ⇒ (i). Let V be a αDδg-open set in(Y, σ) and x ∈ $f^{-1}(V)$. Then f(x) ∈ V . Hence by (ii), there exists an α D δ g-closed set U_x such that x ∈ U_x and $f(U_x)$ ⊂ V . Thus x ∈ U_x ⊂ $f^{-1}(V)$. This implies that $f^{-1}(V)$ is a union of αDδg-closed sets of (X, τ). $f^{-1}(V)$ is αDδg-closed set of (X, τ). This shows that f is contra-αDδg-irresolute. (i) ⇔ (iii). Let V be a αDδg-closed in (Y, σ). Then Y − V is αDδg-open in (Y, σ). Since f is contra-α Dδg-irresolute, $f^{-1}(Y - V)$ is αDδg-closed in (X, τ). Also $f^{-1}(Y - V) = X - f^{-1}(V)$.Therefore $X - f^{-1}(V)$ is αDδg-closed in (X, τ). Hence $f^{-1}(V)$ is αDδg-open in (X, τ).

Remark 4.23. Contra-αDδg-irresolute map and contra-αDδg-continuous are independent notions as shown below.
Example 4.24. Let X = {a, b, c}, Y = {p, q, r} with the topologies τ = {φ, {a}, {b},{a, b}, {a, c}, X} and σ = { Φ, {p, q}, Y}. Define the function f : (X, τ) → (Y, σ)by f(a) = p, f(b) = r, f(c) = q. Then clearly f is contra-αDδg-continuous but itis not contra-αDδg-irresolute because f^{-1}({p}) = {a} is not αDδg-closed in (X, τ)where {p} is αDδg-open in (Y, σ).

Example 4.25. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ, {a},{a, b}, X} and σ = { Φ, {r}, {p, r}, {q, r}, Y}. Define a map f : (X, τ) → (Y, σ)by f(a) = r, f(b) = p,f(c) = q. Clearly f is contra-αDδg-irresolute but not contra-α Dδg-continuous because f^{-1}({p, r}) = {a, b} is not αDδg-closed in (X, τ) where{p, r} is open in (Y, σ).

Remark 4.26. αDδg-irresoluteness and contra-αDδg-irresoluteness are independentnotions as shown in the following examples.

Example 4.27. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ,{b}, {a, c}, X} and σ = { Φ, {p}, {q}, {p, q}, Y}. Define a map f : (X, τ) → (Y, σ)by f(a) = q, f(b) = r and f(c) = p. Then clearly f is contra-αDδg-irresolute but not α Dδg irresolute because f^{-1}({r}) = {b} is not αDδg-closed in (X, τ) where {r} is α Dδg-closed in (Y, σ).

Example 4.28. Let X = {a, b, c} and Y = {p, q, r} with the topologies τ = {φ,{a}, X} and σ = { Φ, {q}, {q, r}, Y}. Define a mapf : (X, τ) → (Y, σ) by f(a) = p,f(b) = r and f(c) = q. Then clearly f is αDδg-irresolute but not contra-αDδg-irresolute because f^{-1}({r}) = {c} is not αDδg-closed in (X, τ) where {r} is αDδg-open in (Y, σ).

Theorem 4.29. Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be any two maps such that g ∘ f : (X, τ) → (Z, η).
(i) If f is contra-αDδg-irresolute and g is αDδg-continuous then g ∘ f is contra-α Dδg-continuous.
(ii) If f is αDδg-irresolute and g is contra-αDδg-irresolute then g ∘ f is contra- α Dδg-irresolute.

Proof (i). Let V be an open set in (Z, η). Since g is αDδg-continuous, $g^{-1}(V)$ is α Dδg-open in (Y, σ). Since f is contra-αDδg irresolute, $f^{-1}(g^{-1}(V))$ is αDδg-closedin (X, τ). That is $(g \circ f)^{-1}(V)$ is αDδg-closed in (X, τ). Hence g ∘ f is contra-αDδg-continuous. (ii). Let U be an αDδg-open in (Z, η). Since g is contra-αDδg-irresolute,$g^{-1}(U)$ is αDδg-closed in (Y, σ). Since f is αDδg-irresolute, $f^{-1}(g^{-1}(U))$ is αDδg-closed in (X, τ). This implies that $(g \circ f)^{-1}(U)$ is αDδg-closed in (X, τ). Hence g ∘ f is contra-αDδg-irresolute.

5. Applications

Theorem 5.1. Let f : (X, τ) → (Y, σ) be αDδg-irresolute. Then f is δ-continuous if (X, τ) is αDTδg-space.
Proof Let V be a δ-closed subset of (Y, σ). Every δ-closed is αDδg-closed and hence V is αDδg-closed in (Y, σ). Since f is αDδg-irresolute, $f^{-1}(V)$ is αDδg-closedin (X, σ). Since X is αDTδg-space, $f^{-1}(V)$ is δ-closed in (X, τ). Thus f is δ-continuous.

Theorem 5.2. Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be two functions. Let(Y, σ) be αDTδg-space. Then g ◦ f is αDδg-continuous if g is αDδg-continuous and fis αDδg-continuous.

Proof Let G be any closed set in (Z, η). Then $g^{-1}(G)$ is αDδg-closed in (Y, σ).Since (Y, σ) is αDTδg-space, $g^{-1}(G)$ is closed in (Y, σ). Since f is αDδg-continuous,$(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is αDδg-closed in (X, τ). Hence g ◦ f is αDδg-continuous function.

Theorem 5.3. Let f : (X, τ) → (Y, σ) be onto, αDδg-irresolute and δ-closed. If(X, τ) is a αDTδg-space, then (Y, σ) is also a αDTδg-space.

Proof Let B be a αDδg-closed subset of (Y, σ). Since f is αDδg-irresolute, then$f^{-1}(B)$ is αDδg-closed set in (X, τ). Since (X, τ) is αDTδg-space, $f^{-1}(B)$ is δ-closed in(X, τ). Also since f is surjective, B is δ-closed in (Y, σ). Hence (Y, σ) is αDTδg-space.

Theorem 5.4. If f : (X, τ) → (Y, σ) is bijection, open and αDδg-continuous, then f is αDδg-irresolute.

Proof Let V be αDδg-closed in (Y, σ) and let $f^{-1}(V) \subseteq U$ where U is open in(X, τ). Since f is open, f(U) is open in (Y, σ). Every open set is αB-set and hencef(U) is αB-set. Clearly V ⊆ f(U). Then clδ(V) ⊆ f(U) and hence $f^{-1}(cl\delta(V)) \subseteq U$. Since f is αDδg-continuous and since clδ(V) is a closed subset of (Y, σ), then$cl\delta(f^{-1}(V)) \subseteq cl\delta(f^{-1}(cl\delta(V)) = f^{-1}(cl\delta(V)) \subseteq U$. U is open and hence αB-set in (X, τ). Thus we have $cl\delta(f^{-1}(V)) \subseteq U$ whenever $f^{-1}(V) \subseteq U$ and U is D-set set in(X, τ). This shows that $f^{-1}(V)$ is αDδg-closed in (X, τ). Hence f is αDδg-irresolute.

Theorem 5.5. If (Y, σ) is αDTδg-space. Then every contra-αDδg-continuous map is contra-αDδg-irresolute.

Proof Let f : (X, τ) → (Y, σ) be a contra-αDδg-continuous map. Let V be a αDδg-open in (Y, σ). Since (Y, σ) is αDTδg-space, V is open in (Y, σ). By hypothesis,$f^{-1}(V)$ is αDδg-closed in (X, τ). Hence f is contra-αDδg-irresolute.

Theorem 5.6. Let f : (X, τ) → (Y, σ) be surjective αDδg-irresolute and weakly-αDδg-closed function where (X, τ) is αDTδg-space and g : (Y, σ) → (Z, η) be func-tion. Then g ◦ f : (X, τ) → (Z, η) is contra-αDδg-continuous iff g is contra-αDδg-continuous.

Proof Let V be a open in (Z, η) and g be contra-αDδg-continuous function. Then $g^{-1}(V)$ is αDδg-closed in (Y, σ). Since f is αDδg-irresolute, $f^{-1}(g^{-1}(V))$ is αDδg-closed in (X, τ). That is $(g \circ f)^{-1}(V)$ is αDδg-closed in (X, τ). Hence (g ◦ f) is contra-αDδg-continuous. Conversely, assume that g ◦ f : (X, τ) → (Z, η) is contra-αDδg-continuous function. Let U be a open set in (Z, η). Then $(g \circ f)^{-1}(U)$ is αDδg-closed in (X, τ). That is $f^{-1}(g^{-1}(U))$ is αDδg-closed in (X, τ). Since (X, τ) is DTδg-space, $f^{-1}(g^{-1}(U))$ is δ-closed in (X, τ). Also since f is weakly-αDδg-closed,$f(f^{-1}(g^{-1}(U)))$ is αDδg-closed in (Y, σ). Since f is surjective, $g^{-1}(U)$ is αDδg-closed in (Y, σ). Hence g is contra-αDδg-continuous.

Theorem 5.7. Let f : (X, τ) → (Y, σ) be function and g : X → X × Y be the graph function, given by g(x) = (x, f(x)) for every x ∈ X. Then f is contra-αDδg-continuous iff g is contra-αDδg–continuous.

Proof Necessity: Let x ∈ X and let V be a closed subset of X × Y such that $x \in g^{-1}(V)$. That is g(x) = (x, f(x)) ∈ V. Then V∩ ({x} × Y) is closed in {x} × Ycontaining g(x). Also {x} × Y is homeomorphic to Y, hence {y ∈ Y/(x, y) ∈ V} isa closed subset of Y. Since f is contra-αDδg-continuous, $\cup\{f^{-1}(y)/(x, y) \in V\}$ is an αDδg-open subset of X. Further $x \in \cup\{f^{-1}(y)/(x, y) \in V\} \subseteq g^{-1}(V)$. Hence $g^{-1}(V)$ is αDδg-open. Thus g is contra-αDδg-continuous.
Sufficiency: Let U be a closed subset of Y. Then X×U is a closed subset of X × Y. Since g is contra-αDδg-continuous, $g^{-1}(X \times U)$ is αDδg-open subset of X. Also$g^{-1}(X \times U) = f{-}1(U)$. Hence f is contra-αDδg-continuous.

We introduce the following definition.
Definition 5.8. A topological space (X, τ) is said to be αDδg-Hausdorff space if for each pair of distinct points x and y in X there exists U ∈ αDδgO(X, x) and V ∈ αDδgO(X, y) such that U ∩ V = ϕ.

Example 5.9. Let X = {a, b, c}, τ = {φ, {a}, {b}, {a, b}, {b, c}, X}. Let a and b be two distinct points of X, there exists an αDδg-open neighbourhood of a and b respectively such that {a} ∩ {b, c} = ϕ. Hence (X, τ) is αDδg-Hausdroff space.

Theorem 5.10. If X is a topological space and for each pair of distinct points x_1 and x_2 in X, there exists a function f of X into Uryshon topological space Y such that $f(x_1)$ 6= $f(x_2)$ and f is contra-αDδg-continuous at x_1 and x_2, then X is αDδg-Hausdroff space.

Proof Let x_1 and x_2 be any distinct points in X. Then by hypothesis, there isa Uryshon space Y and a function f : X → Y such that $f(x_1)$ 6= $f(x_2)$ and f is contra-αDδg-continuous at x_1 and x_2. Let $y_i = f(x_i)$ for i = 1, 2 then y_1 6= y_2. Since Y is Uryshon, there exists an open sets Uy_1 and Uy_2 containing y_1 and y_2 respectively in Y such that $cl(Uy_1) \cap cl(Uy_2) = \varphi$. Since f is contra-αDδg-continuous at x_1 and x_2, there exists an αDδg-open sets Vx_1 and Vx_2 containing x_1 and x_2 respectively in X such that $f(Vx_i) \subseteq cl(Uy_i)$ for i = 1, 2. Therefore we get $Vx_1 \cap Vx_2 = \Phi$. Hence X is αDδg-Hausdroff.

Corollary 5.11. If f is contra-αDδg-continuous injective of a topological space X into a Uryshon space Y, then X is αDδg-Hausdroff.

Proof Let x1 and x2 be distinct points in X. Then by hypothesis, f is a contra-αDδg-continuous function of X into a Uryshon space Y such that f(x1) 6= f(x2) because f is injective. Hence by Theorem 5.10 X is αDδg-Hausdroff.

Theorem 5.12. Let f1 : X1 → Y and f2 : X2 → Y be two contra-αDδg-continuous functions. If Y is a Uryshon space then $\{(x_1, x_2)/f1(x_1) = f2(x_2)\}$ is αDδg-closed in the product space $X_1 \times X_2$.

Proof Let A denote the set {(x1, x2)/f1(x1) = f2(x2)}. We have to prove that A is αDδg-closed in the product space $X_1 \times X_2$, we show that $(X_1 \times X_2) - A$ is αDδg-open. Let (x1, x2) ∈/ A. Thenf1(x1) 6= f2(x2). Since Y is Uryshon space, there exists an open sets V1 and V2 containing f1(x1) and f2(x2) respectively such that $cl(V_1) \cap cl(V_2) = \Phi$. Since f1 and f2 are contra-αDδg-continuous, $f_1^{-1}1(cl(V_1))$ and $f_2^{-1}(cl(V_2))$ are αDδg-open sets containing x1 in X1 and x2 in X2 respectively.$f_1^{-1}(cl(V_1))\times f_2^{-1}(cl(V_2))$ is αDδg-open in X1×X2. Further,$(x_1, x_2) \in f_1^{-1}(cl(V_1))\times f_2^{-1}(cl(V_2)) \subset (X_1\times X_2)-A$. This implies that (X1×X2)−A is αDδg-open in (X1 × X2). Hence A is αDδg-closed in $X_1 \times X_2$.

Definition 5.13. A topological space (X, τ) is said to be αDδg-normal if each pair of nonempty disjoint closed sets in (X, τ) can be separated by disjoint αDδg-open sets in (X, τ).

Example 5.14. Let X = {a, b, c} and τ = {φ, {a}, {b}, {a, b}, {a, c}, X}. Then{b} and {c} are nonempty disjoint closed sets in (X, τ). Then there exists two αDδg-open sets {b} and {a, c} such that {b} ⊆{b},{c}⊆ {a, c} and {b} ∩ {a, c} = φ. Thus (X, τ) is a αDδg-normal space.

Theorem 5.15. If f : X → Y is a contra-αDδg-continuous, closed, injective and Y is Ultra normal, then X is a αDδg-normal.

Proof Let U and V be disjoint closed subsets of X. Since f is closed and injective,f(A) and f(B) are disjoint closed subsets of Y. Since Y is Ultra-normal, there exists disjoint closed sets A and B such that $f(U) \subset A$ and $f(V) \subset B$. Hence $U \subseteq f^{-1}(A)$ and $V \subseteq f^{-1}(B)$. Since f is contra-αDδg-continuous and injective, $f^{-1}(A)$and $f^{-1}(B)$ are disjoint αDδg-open sets in X. Hence X is αDδg-normal.

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