TANGENT SIMILARITY MEASURE OF INTUITIONISTIC FUZZY SETS BASED ON HAUSDORFF DISTANCE

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ABSTRACT. In this paper, we propose a Tangent Similarity measure of Intuitionistic fuzzy sets based on Hausdorff distance. Similarity is an important tool in decision making in Intuitionistic Fuzzy sets. To demonstrate the effciency of the proposed tangent similarity measure, it is applied in medical diagnosis.

Key words

Intuitionistic Fuzzy Sets(IFSs), Similarity Measure, Hausdorff Distance, Tangent Similarity Measure.

1. Introduction

The concept of Intuitionistic Fuzzy Sets, introduced by K.T.Attanasov(1986), is a generalization for L. Zadeh's fuzzy sets. It has many interesting and useful applications in various areas. The main advantage of IFS is its capability of handling uncertainty that may exist due to information imprecision. ie, it assigns to each element a membership degree, non-membership degree and a hesitation degree. The theory of IFS has applied in a variety of areas such as logic programming, decision making problems and in medical diagnosis etc.

In many instances measuring the similarity between IFSs is an important tool in decision making. Similarity measure gives the degree of similarity between two objects. There exists several similarity measures for Intuitionistic Fuzzy Sets. Hung and Yang [5] used Hausdorff distance for constructing similarity measures. In this paper we propose a tangent similarity measure for IFS based on Hausdorff distance, which is more simple than the tangent similarity measure proposed by Pramanik and Mondal [2]. To show the efficiency of the proposed tangent similarity measure, it is applied in medical diagnosis. Rest of the paper is arranged as follows:

In section 2, we present the preliminaries. In section 3, we propose Tangent Similarity Measure based on Hausdorff distance and its properties. Section 4 is the illustration of proposed formula in the field of Medical Diagnosis. Section 5 is the conclusion.

2. Preliminaries

Definition 2.1 [1] Let X be a given set. An Intuitionistic Fuzzy Set A in X is given by A= $\left((x, \mu(x), \upsilon(x)) / x \in X\right)$ where $\mu_A, \upsilon_A : X \to [0,1]$, and $0 \le \mu_A(x) + \upsilon_A(x) = 1$. $\mu_A(x)$ is the degree of membership of the element x in A and $\upsilon_A(x)$ is the degree of non-membership of x in A. For each $x \in X, \pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ is called the degree of hesitation.

Definition 2.2 [5] A Similarity Measure for IFS(X) is a real function S so that $S : IFS(X) \times IFS(X) \rightarrow [0,1]$ and satisfies the following properties, where IFS(X) denotes the class of all IFSs of X.

(P_1) $0 \leq S(A,B) \leq 1, \forall A,B \in IFS(X)$

(P_2) $S(A,B) = S(B,A), \forall A,B \in IFS(X)$

(P_3) $S(A,B) = 1 \Leftrightarrow A = B$

(P_4) If $A \subseteq B \subseteq C$ then $S(A,C) \leq S(A,B)$ and $S(A,C) \leq S(B,C)$ for all A, B and $C \in$ IFS(X).

Definition 2.3 [3] The Hausdorff Distance Measure between two IFSs A,B is given by

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} Max(|\mu_A(x_i) - \mu_B(x_i)|, |\upsilon_A(x_i) - \upsilon_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)|)$$

Definition 2.4 [2] The Intuitionistic Fuzzy Tangent Similarity Measure between two IFSs P, Q is given by

$$(T_{IFS})(P,Q) = \frac{1}{n}\sum_{i=1}^{n}[1 - \tan(\frac{\pi(|\mu_P(x_i) - \mu_Q(x_i)| + |\upsilon_P(x_i) - \upsilon_Q(x_i)| + |\pi_P(x_i) - \pi_Q(x_i)|)}{12})]$$

Properties 2. 1

The Intuitionistic Fuzzy Tangent Similarity Measure between two IFSs P, Q satisfies the following four properties

(1) $0 \leq T_{IFS}(P,Q) \leq 1, \forall P,Q$

(2) $T_{IFS}(P,Q) = 1 \Leftrightarrow P = Q$

(3) $T_{IFS}(P,Q) = T_{IFS}(P,Q), \forall P,Q$

(4) If $P \subseteq Q \subseteq R$ then $T_{IFS}(P,R) \leq T_{IFS}(P,Q)$ and $T_{IFS}(P,R) \leq T_{IFS}(Q,R)$ for all P,Q and $R \in IFS(X)$

3. Tangent Similarity Measure based on Hausdorff Distance

Definition 3.1 Let A= $(\mu_A(x_i), \upsilon_A(x_i), \pi_A(x_i))$ and $B = (\mu_B(x_i), \upsilon_B(x_i), \pi_B(x_i))$ be two Intuitionistic Fuzzy Sets. Then the tangent similarity measure based on Hausdorff Distance between A and B, denoted by $T_H(A,B)$ is

$$T_H(A,B) = 1 - \frac{1}{n}\tan(\frac{\pi}{12}\sum_{i=1}^{n}\lambda_i)$$

Where $\lambda_i = Max(|\mu_A(x_i) - \mu_B(x_i)|, |\upsilon_A(x_i) - \upsilon_B(x_i)|, |\pi_A(x_i) - \pi_B(x_i)|)$ for every i.

Properties 3.1

The defined Tangent Similarity measure $T_H(A,B)$ between A and B satisfies the following properties.

(T_1) $0 \leq T_H(A,B) \leq 1$

(T_2) $T_H(A,B) = 1 \Leftrightarrow A = B$

(T_3) $T_H(A,B) = T_H(A,B)$

(T_4) If $A \subseteq B \subseteq C$ then $T_H(A,C) \leq T_H(A,B)$ and $T_H(A,C) \leq T_H(B,C)$ for all A,B and $C \in IFS(X)$

Proof

(**T1**) Since

$$0 \le |\mu_A(x_i) - \mu_B(x_i)| \le 1$$
$$0 \le |\upsilon_A(x_i) - \upsilon_B(x_i)| \le 1$$
$$0 \le |\pi_A(x_i) - \pi_B(x_i)| \le 1 \text{ , for every } i \text{ ,}$$

so we get $0 \le \lambda_i \le 1$ for every i.

Therefore, $0 \le 1 - \frac{1}{n}\tan(\frac{\pi}{12}\sum_{i=1}^{n}\lambda_i) \le 1$

i.e. $0 \le T_H(A,B) \le 1$

i.e. Since the membership, nonmembership and hesitancy functions of the Intuitionistic Fuzzy Sets are in [0,1], each $\lambda_i \in [0,1]$ and the value of the tangent function is also within [0,1]. So the proposed Tangent Similarity Measure is also within [0,1]. Hence $0 \le T_H(A,B) \le 1$

(**T2**) Let A=B.

This implies $\mu_A(x_i) = \mu_B(x_i)$, $\upsilon_A(x_i) = \upsilon_B(x_i)$ and $\pi_A(x_i) = \pi_B(x_i)$ for every x_i.

Therefore $\lambda_i = 0$ for every i.

Thus $T_H(A,B) = 1$

Conversely, Let $T_H(A,B) = 1$

$$\Rightarrow \frac{1}{n}\tan(\frac{\pi}{12}\sum_{i=1}^{n}\lambda_i) = 0$$

$$\Rightarrow \tan(\frac{\pi}{12}\sum_{i=1}^{n}\lambda_i) = 0$$

$$\Rightarrow \sum_{i=1}^{n}\lambda_i = 0$$

$$\Rightarrow \lambda_i = 0, \forall i$$

$\Rightarrow \mu_A(x_i) = \mu_B(x_i)$, $\upsilon_A(x_i) = \upsilon_B(x_i)$ and $\pi_A(x_i) = \pi_B(x_i)$ for every x_i.

$\Rightarrow$ A= B.

(**T3**) Clearly, $T_H(A,B) = T_H(B,A)$

(**T4**) Let A $\subseteq$ B $\subseteq$ C.

Then $\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$

$\upsilon_A(x_i) \ge \upsilon_B(x_i) \ge \upsilon_C(x_i)$

$\pi_A(x_i) \ge \pi_B(x_i) \ge \pi_C(x_i)$, for every x_i.

Now, $\mu_B(x_i) \le \mu_C(x_i) \Rightarrow |\mu_A(x_i) - \mu_B(x_i)| \le |\mu_A(x_i) - \mu_C(x_i)|$

$\mu_A(x_i) \le \mu_B(x_i) \Rightarrow |\mu_B(x_i) - \mu_C(x_i)| \le |\mu_A(x_i) - \mu_C(x_i)|$

$\upsilon_B(x_i) \ge \upsilon_C(x_i) \Rightarrow |\upsilon_A(x_i) - \upsilon_B(x_i)| \le |\upsilon_A(x_i) - \upsilon_C(x_i)|$

$\upsilon_A(x_i) \ge \upsilon_B(x_i) \Rightarrow |\upsilon_B(x_i) - \upsilon_C(x_i)| \le |\upsilon_A(x_i) - \upsilon_C(x_i)|$

Similarity, $|\pi_A(x_i) - \pi_B(x_i)| \le |\pi_A(x_i) - \pi_C(x_i)|$

$|\pi_B(x_i) - \pi_C(x_i)| \le |\pi_A(x_i) - \pi_C(x_i)|$

Since tangent function is an increasing function in $\left[0, \frac{\pi}{4}\right]$, $T_H(A,C) \le T_H(A,B)$ and $T_H(A,C) \le T_H(B,C)$.

4. Case Study

The data given by Mondal and Pramanik in [2] is adopted for the illustration of the proposed Tangent Similarity Measure.

Let P= {P_1, P_2, P_3, P_4} be a set of patients, D= {Viral Fever, Malaria, Typhoid, Stomach Problem, Chest Problem} be the set of diseases and S= {Temperature, Head Ache, Stomach Pain, Cough, Chest Pain} be the set of symptoms. The relation between patients and symptoms are presented in Table 1. The relation between symptoms and diseases is presented in Table 2. The tangent similarity measure between the sets A and B using definition 3.1 is given in Table 3.

TABLE 1. The relation between patient and symptoms (A)

Relation1	Temperature	Head Ache	Stomach Pain	Cough	Chest Pain
P_1	(0.8, 0.1, 0.1)	(0.6, 0.1, 0.3)	(0.2, 0.8, 0.0)	(0.6, 0.1, 0.3)	(0.1, 0.6, 0.3)
P_2	(0.0, 0.8, 0.2)	(0.4, 0.4, 0.2)	(0.6, 0.1, 0.3)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
P_3	(0.8, 0.1, 0.1)	(0.8, 0.1, 0.1)	(0.0, 0.6, 0.4)	(0.2, 0.7, 0.1)	(0.0, 0.5, 0.5)
P_4	(0.6, 0.1, 0.3)	(0.5, 0.4, 0.1)	(0:3, 0.4, 0.3)	(0.7, 0.2, 0.1)	(0.3, 0.4, 0.3)

TABLE 2. The relation among Symptoms and Diseases (B)

Relation2	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4, 0.0, 0.6)	(0.7, 0.0, 0.3)	(0.3, 0.3, 0.4)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
Head Ache	(0.3, 0.5, 0.2)	(0.2, 0.6, 0.2)	(0.6, 0.1, 0.3)	(0.2, 0.4, 0.4)	(0.0, 0.8, 0.2)
Stomach Pain	(0.1, 0.7, 0.2)	(0.0, 0.9, 0.1)	(0.2, 0.7, 0.1)	(0.8, 0.0, 0.2)	(0.2, 0.8, 0.0)
Cough	(0.4, 0.3, 0.3)	(0.7, 0.0, 0.3)	(0.2, 0.6, 0.2)	(0.2, 0.7, 0.1)	(0.2, 0.8, 0:0)
Chest Pain	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.7 0.1)	(0.8, 0.1, 0.1)

TABLE 3. The tangent similarity measure between the sets A and B using definition 3.1

Relation2	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
P_1	0.9232	**0.9350**	0.9232	0.8292	0.8199
P_2	0.8845	0.8465	0.9109	**0.9629**	0.8774
P_3	0.8914	0.8701	**0.9109**	0.8465	0.8199
P_4	**0.9232**	0.9171	0.8914	0.8701	0.8292

The highest similarity measure yields the proper medical diagnosis. Therefore patient P_1 suffers from Malaria, P_2 suffers from Stomach Problem, P_3 suffers from Typhoid and P_4 suffers from Viral fever.

5. Conclusion

In this paper, we have developed a Tangent Similarity Measure for IFSs, which is more simple than [2]. The efficiency of the proposed similarity measure is established by applying it in Medical Diagnosis. The result which is evaluated using the new tangent similarity measure coincides with the earlier approach.

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