An Introduction of Lagrangian and Eulerian Approach

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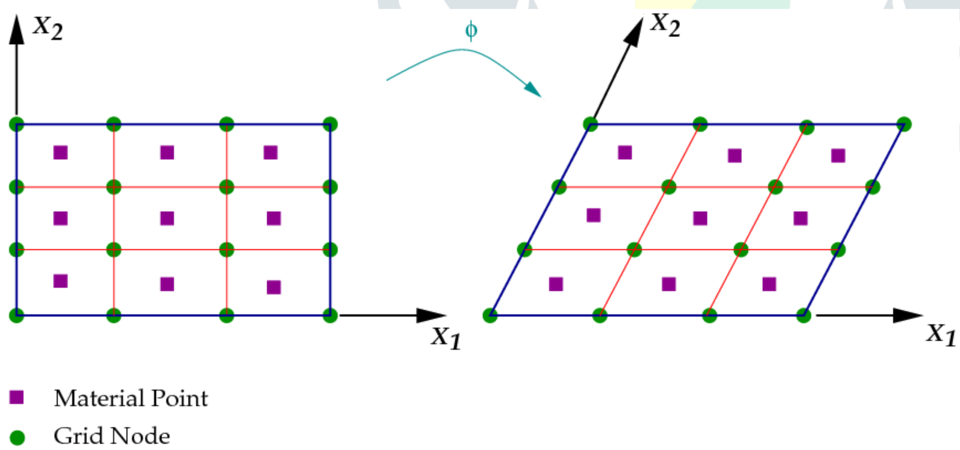
Abstract

To solve flow related problem we have either lagrangian and Eulerian approach. To define a flow, we use either the Lagrangian description or the Eulerian description.

In Lagrangian description the picture of fluid flow where each fluid particle carries its own properties such as density, momentum etc. As the particle advance its properties may change in time. The procedure of describing the entire flow by recording the detailed histories of each fluid particle is the langrangian description. The particle properties density, velocity, pressure etc. can be mathematically represented as follow: ρ(t), v(t), Ƥ(t).

In Eulerian description rather than following each fluid particle we can record the evolution of the flow properties at every point in the space as time varies. This is the Eulerian description this means that the flow properties at a specified location depend at a specified location & on time. For example The density, velocity, pressure…... can be mathematically represented by v(x,t), Ƥ(x,t), ρ(x,t),



Eulerian description is harder to understand (how do we apply conservation law?) For this reason, in fluid mechanics we use mainly the Eulerian description. Whereas the lagrangian description is simple to understand: conservation of mass and Newton's law apply directly to each fluid particle. However, it is computationally expensive to keep track of the trajectories of all the fluid particle is a flow and therefore the lagrangian description is used only in some numerical simulations.

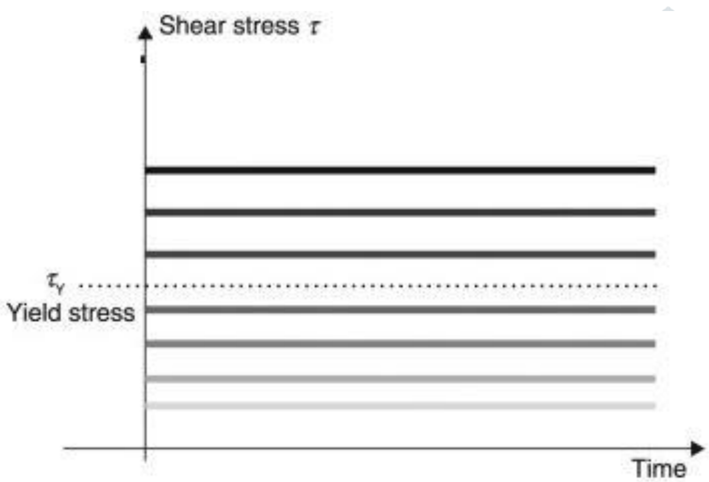
Keywords

Eulerian, Lagrangian, Conservation of mass, Newton's Law, Fluid,

Introduction

A steady flow is strictly Eulerian concept: first we assume that a steady flow is observed from a fixed position. This is like watching from a river bank i.e. $\frac{\delta}{\delta t} = 0$

Be careful not to confuse this with $\frac{D}{Dt}$ which is more like following a twin in water. Note that $\frac{D}{Dt} = 0$ does not means steady. Since the flow could speed up at some points and slow down at others.



An Incompressible flow is strictly Lagrangian concept: Assume a flow where the density of every fluid particle is constant in time. Be careful not to confuse this with $\frac{\delta \rho}{\delta t} = 0$ Which means that the density at a particular point is the flow is constant and would allow particle to change density as they flow from point to point.

In Lagrangian derivative we know how to measure the time derivative of a physical quantity associated with the fluid, for example that of the temperature T(x,t) at a fixed point in a space (the Eulerian derivative), it is $\frac{\delta T}{\delta t}$. This quantity will describe the change of temperature at a fixed location, for example air temperature in Jaipur. However, this quantity will not be a measure of how a mass of air heats up or become colder. The reason is that air is swept away prevailing flow. In other words to describe the change of temperature of a piece of air, we need to consider the rate of change of T(x,t), following a fluid particle with trajectories. This is called the Lagrangian derivative and is denoted as $\frac{D}{Dt}$.

So how do we mathematically express 'differentiation following the motion' . In order to follow particle in a continuum a special type of differentiation is required. Arbitrary small variation of T(x,y,z,t) a function of position and time.

The fluid velocity is the rate of change of position of the fluid element, following that element along the variation of a property T following an element of fluid is thus derived by $\delta x = u\,\delta t$, $\delta y = v\,\delta t$, $\delta z = w\,\delta t$, where u,v,w are the speed in x,y,z direction respectively.

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