VISCO-ELASTIC FLOW THROUGH POROUS MEDIUM

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ABSTRACT

The effect of unsteady, two dimensional free convective flow during the motion of a visco-elastic fluid through a highly porous medium has been investigated analytically. The porous medium is bounded by a vertical plane surface of constant temperature, which absorbs the fluid with constant velocity. The free stream velocity of fluid vibrates at a mean constant value. The analytical expressions for the velocity of the fluid have been presented. Also the expression for skin frictions has been given. The effects of rotation and permeability parameters and the visco-elastic parameter on the axial and transverse components of velocity have been discussed in detail with the help of graphs.

I. **INTRODUCTION**:

Today, different branches of engineering and meteorology have been highly enriched with analysis of flow of fluids in rotation. Turkyilmazoglu[1] has recently discussed free convection flow about a heated vertically stretching permeable surface placed in a porous medium under the influence of a temperature depended internal heat generation or absorption. Earlier, Rott and Lewellen [2] have given an extensive survey of rotating flows and their various applications. Hari Priya and associates[3] studied the effect of radiation on MHD free convection flow in a vertical channel filled with porous medium. Cheng and Minkowycz [4] found out solutions for the free convective flow in a porous medium adjacent to a vertical plate with wall temperature being a power function of distance from the leading edge. A theoretical analysis of a two-dimensional free convective flow of a viscous incompressible fluid through porous medium bounded by a porous and isothermal plate, has been presented by Raptis et.al.[5]. They [5] have also studied the steady convective flow and the mass transfer through a very porous medium bounded by an infinite vertical plate. Raptis and his associates [6,7] have extended their studies to unsteady free convective flow through a porous medium under different aspects. Baghel and his associates [9] have studied the effects of unsteady two-dimensional free convective flow of a viscous incompressible fluid through a rotating porous medium.

In this paper the effects of unsteady two-dimensional free convective flow of a visco-elastic fluid through a rotating porous medium has been taken for discussion. Here the model characterized by Walters liquid (Model B/) has been taken. It is assumed that the visco-elastic fluid occupies major portion of the porous medium so that viscous effects may be prominent. The porous medium is bounded by a vertical surface which is at a constant temperature. This surface absorbs fluid with a constant velocity. The free stream velocity of the fluid vibrates at a mean constant value about z-axis in unison with the free vertical plate.

Let us consider the constitutive equation

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e^{ik}$$

where σ^{ik} is the stress tensor, p is an isotropic pressure, g_{ik} is the metric tensor of a fixed coordinate system x^i , v^i being the velocity vector. e^{ik} is deformation rate, defined by

$$2e_{ik} = v_{i,k} + v_{k,i}$$

Here, η_0 is the limiting viscosity at small rate of shear which is given by integration of relaxation spectrum $N(\tau)$ as introduced by Walter.

$$\eta_0 = \int_0^\infty N(\tau)d\tau \quad \text{and} \quad k_0 = \int_0^\infty \tau N(\tau)d\tau$$

This idealized model is a valid approximation with very short memories.

II. MATHEMATICAL FORMULATION:

Let us consider a a coordinate system with the x-axis taken along a vertical infinite plane surface with a direction opposite to the direction of gravity and the y-axis is taken normal to the surface. We will take the unsteady two-dimensional flow of a visco-elastic fluid through highly porous medium bounded by the plane surface. The surface absorbs the fluid with a constant velocity, and that the velocity of the fluid far away vibrates at a mean value with a direction parallel to x-axis. All the fluid properties are assumed to be constant, excepting that the influence of the density variation with temperature is considered only in the body force term.

When the velocities and temperature are the functions of y and time t, the equations of continuity, momentum and energy for free convection flow, in this case, are

$$v_y = 0 \tag{1}$$

$$u_t + vu_y - 2\Omega' v = -\frac{1}{\rho} p_x - g + \nu_0 u_{yy} - K_0{}'\left(u_{tyy} + vu_{yyy}\right) - \frac{\nu_0}{k'} u \tag{2}$$

$$v_t + v'v_y - 2\Omega' u = \nu_0 v_{yy} - 2K_0'\left(v_{tyy} + vv_{yyy}\right) - \frac{\nu_0}{k'} v \quad \ldots(3)$$

$$T_t + vT_y = \frac{\kappa}{\rho C_p} T_{yy} \quad \ldots(4)$$

Where $\nu_0 = \dfrac{\eta_0}{\rho}$, $K_0' = \dfrac{k_0}{\rho}$

The boundary conditions are

$$u = 0, v = -V = \text{constant}, \; T = T_w' \text{ at } y = 0$$

$$u = U' \to U\left(1 + \varepsilon e^{iwt'}\right), T \to T_\infty' \quad \text{at } y \to \infty \quad \ldots(5)$$

Where u and v are components of velocity along x and y directions respectively; ρ, p and g are density, pressure and acceleration due to gravity; respectively. k' is the permeability of the porous medium. T, T_w' and T_∞' are the temperatures of the fluid, the surface far and the fluid at away from the surface. κ is the thermal conductivity of the fluid; C_p ,the specific heat of the fluid at constant pressure; U, the constant velocity; w', the frequency of vibration of the fluid; Ω', the rotation parameter of the fluid; $\varepsilon(\langle 1)$, the constant quantity and K_0' the visco-elastic parameter.

The equation (2) for free stream is reduced to

$$U'_t = -\frac{1}{\rho} p_x - \frac{g}{\rho}\rho_\infty - \frac{\nu_0}{k'} U' \quad \ldots(6)$$

Eliminating p_x from (2) and (6), we have

$$u_t + vu_y - 2\Omega' v = \frac{dU'}{dt} + \frac{g}{\rho}(\rho_\infty - \rho) + v_0' u_{yy} + \frac{v_0}{k'}(U' - u) - K_0' \left(u_{tyy} + vu_{yyy}\right) \quad \ldots(7)$$

Combining equations (7) and (3), using the complex function $f' = u' + iv'$, we get

$$f_t + vf_y + 2\Omega' if = \frac{dU'}{dt'} + g\beta(T - T_\infty') + v_0 f_{yy} - K_0'\left(f_{tyy} + vf_{yyy}\right) + \frac{v_0}{k'}(U' - f) \quad \ldots(8)$$

where we have used the constitutive relation

$$\rho_\infty - \rho = \beta\rho(T - T_\infty') \quad \ldots(9)$$

where β is the volumetric coefficient of thermal expansion and ρ_∞, the density of the fluid far away from the surface.

Since the surface absorbs the fluid with a constant velocity, the continuity equation (1) gives $v = -v_0 =$ constant. Using this equation (8), we get

$$f_t - v_0 f_y + 2\Omega' if = \frac{dU'}{dt'} + g\beta(T - T_\infty') + v_0 f_{yy} + \frac{v_0}{k'}(U - f) - K_0' \left(f_{tyy} + vf_{yyy}\right) \quad \ldots(10)$$

Using non-dimensional quantities, the equations (4) and (10) may be reduced to

$$P(T_t - T_y) = T_{yy} \quad \ldots(11)$$

$$f_t - f_y + i\Omega f = \frac{dU^*}{dt} + G_r T + f_{yy} + \frac{1}{k}(U^* - f) - K_0\left(f_{tyy} - f_{yyy}\right) \quad \ldots(12)$$

where $P = \dfrac{P v_0 C_p}{\kappa}$, Prandtf number

$Gr = \dfrac{v_0 g\beta\left(T_w' - T_\infty'\right)}{Uv_0^2}$, Grashof number

$k = \dfrac{v_0^2}{v_0^2} k'$, Permeability parameter

$K_0 = K_0' \dfrac{v_0^2}{v_0^2}$, visco-elastic parameter.

The corresponding boundary conditions reduce to:

$$f = 0, T = 1 \text{ at } y = 0$$
$$f \to 1 + \varepsilon e^{iwt}, T \to 0 \text{ at } y \to \infty \quad \ldots(13)$$

Where we have taken into account from equation (5) that $U' = U\left(1 + \varepsilon e^{iw't'}\right)$.

III. SOLUTION OF EQUATIONS:

To solve the equations (11) and (12) subject to boundary conditions (13) we assume that

$$f(y,t) = f_0(y) + \varepsilon e^{iwt} f_1(y) + \ldots$$
$$T(y,t) = T_0(y) + \varepsilon e^{iwt} T_1(y) + \ldots \quad \ldots(14)$$

Substituting the expressions (14) in equations (11) and (12) and equating the coefficients of various powers of ε we get the following system of different equations.

$$K_0 f_0''' + f_0'' + f_0' - \left(\frac{1}{k} + i\Omega\right) f_0 = -\frac{1}{k} - GrT_0 \quad \ldots(15)$$

$$K_0 f_1''' + (1 - iwK_0) f_1'' + f_1' - \left[\frac{1}{k} + i(w + \Omega)\right] f_1 = -\left(\frac{1}{k} + iw\right) - GrT_1 \quad \ldots(16)$$

$$T_0'' + PT_0' = 0 \quad \ldots(17)$$

$$T_1'' + PT_1' - iwPT_1 = 0 \quad \ldots(18)$$

The corresponding boundary conditions are

$$f_0 = 0,\ f_1 = 0, T_0 = 0, T_1 = 0 \ \text{at} \ y = 0$$

$$f_0 \to 1,\ f_1 \to 1, T_0 \to 0 \ \text{and} \ T_1 \to 0 \ \text{at} \ y \to \infty \quad \ldots(19)$$

In order to solve equations (15) and (16), if we consider very small values of non-Newtonian-parameter K_0, then substituting,

$$f_i(y) = f_{i0}(y) + K_0 f_{i1}(y) + 0(K_0^{\,2}) \quad \ldots(20)$$

into equations (15) and (16) for $i = 0$ and 1 respectively and the corresponding boundary conditions upto the 1st order of K_0 and equating the coefficients of powers of K_0, we obtain the following set of differential equations and the boundary conditions as –

$$f_{00}'' + f_{00}' - \left(\frac{1}{k} + i\Omega\right) f_{00} = -\frac{1}{k} - GrT_0 \quad \ldots(21)$$

$$f_{00}''' + f_{01}'' + f_{01}' - \left(\frac{1}{k} + i\Omega\right) f_{01} = 0 \quad \ldots(22)$$

$$f_{10}'' + f_{10}' - \left[\frac{1}{k} + i(w + \Omega)\right] f_w = -\left(\frac{1}{k} + iw\right) - GrT_1 \quad \ldots(23)$$

$$f_{10}''' + f_{11}'' - iwf_{10}'' + f_{11}' - \left[\frac{1}{k} + i(w + \Omega)\right] f_{11} = 0 \quad \ldots(24)$$

and

$$f_{00} = 0,\ f_{01} = 0,\ f_{10} = 0,\ f_{11} = 0 \quad \text{at} \quad y = 0$$

$$\ldots(25)$$

$$f_{00} \to 1,\ f_{01} \to 0,\ f_{10} \to 1,\ f_{11} \to 0 \quad \text{at} \quad y \to \infty$$

Solutions of (17) and (18) consistent with the boundary conditions (19) are

$$T_0 = e^{-Py} \quad \ldots(27)$$

$$T_1 = 0 \quad \ldots(28)$$

Using these, and the boundary conditions (25) we get the solutions of (21) to (24) as

$$f_{00} = \left[\frac{Gr}{(P + R_1)(P + R_2)} - \frac{1}{1 + i\Omega k}\right] e^{R_{1y}} + \frac{1}{1 + i\Omega k} - \frac{Gre^{-P_y}}{(P + R_1)(P + R_2)} \quad \ldots(29)$$

$$f_{01} = \frac{P^3 Gr}{(P+R_1)^2 (P+R_2)^2}\left(e^{-P_y} - e^{R_{1y}}\right) + \left[\frac{Gr}{(P+R_1)(P+R_2)} - \frac{1}{1+i\Omega k}\right]\frac{R_1^{\ 3}}{(2R_1+1)} y e^{R_{1y}} \quad \ldots(30)$$

$$f_{10} = (1-i\Omega)\left(1-e^{R_{3y}}\right) \quad \ldots(31)$$

$$f_{11} = \frac{(1-i\Omega)(R_3 - iw)R_3^{\ 2}}{2R_3+1} y e^{R_{3y}} \quad \ldots(32)$$

where $R_1 = \dfrac{-1-\sqrt{\left(1+\frac{4}{k}\right)+i4\Omega}}{2}$

$$R_2 = \frac{-1+\sqrt{\left(1+\frac{4}{k}\right)+i4\Omega}}{2}$$

$$R_3 = \frac{-1-\sqrt{\left(1+\frac{4}{k}\right)+4i(w+\Omega)}}{2} \quad \ldots(33)$$

Thus we have got

$$f = f_0 + \varepsilon e^{iwt} f_1 = f_{00} + K_0 f_{01} + \varepsilon e^{iwt}(f_{10} + K_0 f_{11}) = \left(f_{00} + \varepsilon e^{iwt} f_{10}\right) + K_0\left(f_{01} + \varepsilon e^{iwt} f_{11}\right) \quad \ldots(34)$$

where f_{00}, f_{01}, f_{10}, f_k are given by equations (29) , (30) , (31) and (32).

and $T = T_0 + \varepsilon e^{iwt} T_1 = e^{-P_y}$ …(35)

Substituting

$$f_0 = u_1 + iv_1 \quad \text{and} \quad f_1(y) = u_2 + iv_2 \quad \ldots(36)$$

and using the equation (14), we have

$$u = u_1 + \varepsilon(u_2 \cos\omega t - v_2 \sin\omega t) \quad \ldots(37)$$

$$v = v_1 + \varepsilon(v_2 \cos\omega t - u_2 \sin\omega t) \quad \ldots(38)$$

If τ_x and τ_y are the axial and transverse components of the skin-friction in the non-dimensional form, we can obtain them by the following formula

$$\tau_x + i\tau_y = -\left.\frac{\partial f}{\partial y}\right|_{y=0}$$

IV. DISCUSSION:

The main purpose of this discussion is to observe the non-Newtonian effects. The corresponding results for the Newtonian fluid can be deduced from the above results by setting $K_0 = 0$. The primary and secondary velocity profiles for various values of Gvashof number (Gr), permeability parameter (k)and rotation parameter (Ω) have been analysed in figures 1 to 8. For convenience we have considered $P = \cdot 7, wt = \frac{\pi}{2}, w = 5, \varepsilon = 0\cdot 2$. It is observed from the figures that the primary velocity component decreases while the secondary velocity component increases with an increase of permeability parameter, Grashof number and the rotation parameter in both Newtonian and non-Newtonian cases. It is noted from the figures

1 to 4 that the velocity component u decreases with the increasing values of the non-Newtonian parameter K_0 as compared to their values for Newtonian fluid. From figures 5 to 8 it is concluded that the velocity component v increases with the increasing values of K_0.

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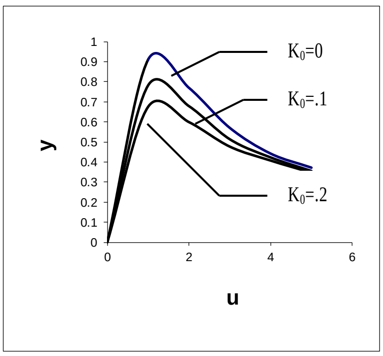

Fig. 1 (k=3,Gr=3, Ω =1)

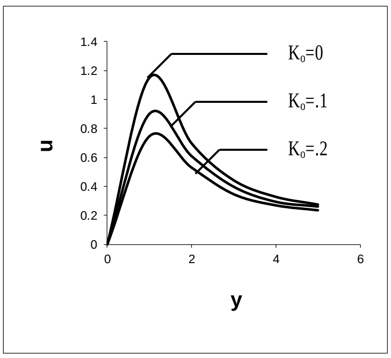

Fig. 2 (k=5,Gr=3, Ω =1)

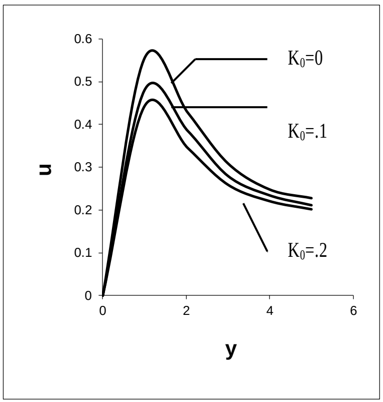

Fig. 3 (k=3,Gr=3, Ω =2)

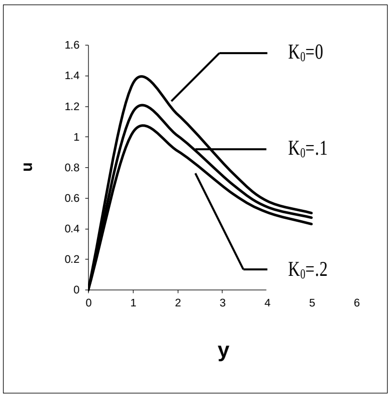

Fig. 4 (k=3,Gr=5, Ω =1)

I. Primary velocity profiles at P=0.7, ε =0.2 and w =5

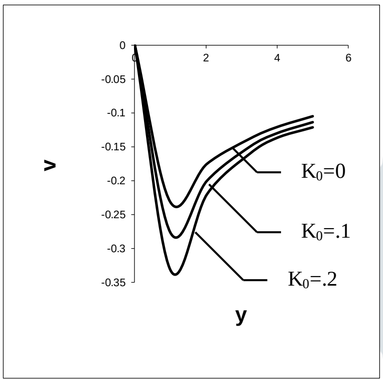

Fig. 5 (k=3,Gr=3, Ω =1)

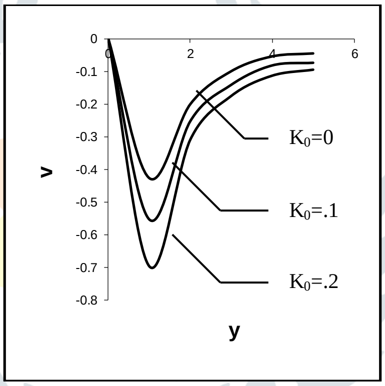

Fig. 6 (k=5,Gr=3, Ω =1)

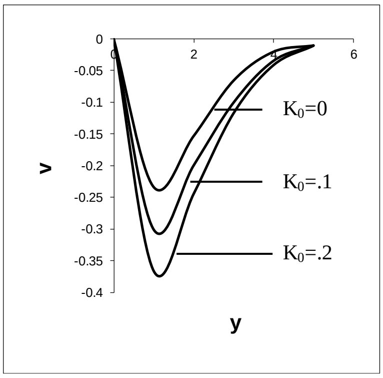

Fig. 7 (k=3,Gr=3, Ω =2)

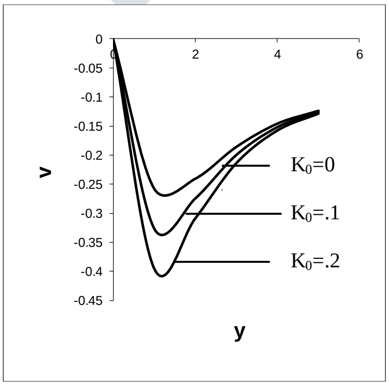

Fig. 8 (k=3,Gr=5, Ω =1)

II. Secondary velocity profiles at P=0.7, ε =0.2 and w =5