Convection in a Nanofluid Layer through Porous Medium

[1] Manu Devi, [2]Sandeep Kumar Tiwari
[1] Research Scholar, [2] Associate Professor, Department of Mathematics
[1]Department of Mathematics, Motherhood University, Roorkee, INDIA

Abstract :

The problem of triple diffusive convection in a horizontal nanofluid layer in a porous medium heated and salted from below studied analytically. Following the linear stability theory and normal made technique, the effects of the solute Rayleigh numbers, concentration Rayleigh number, Lewis number, modified Darcy number, porosity and modified diffusivity ratio on the stability of the system is investigated analytically. Thus, concentration Rayleigh number, modified Darcy number and porosity destabilize the stationary convection while the solute Rayleigh number stabilizes the stationary convection. The critical Rayleigh number for the onset of instability is determined numerically, and results are depicted graphically. The sufficient conditions for the non-existence of over-stability are also derived.

***IndexTerms* – Triple-diffusive convection; Brownian motion; oscillatory convection; porous medium; Nanofluid**

I. INTRODUCTION

The study of nanofluid has become of increasing importance in the last few years. Some fluids, like water, engine oil and ethylene glycol mixture, have low thermal conductivity, so they have limited heat transfer capability. The flow of nanofluid is of great importance due to their applications in electronics, automotive, high flux solar collectors, and nuclear applications. Choi [1] coined the term "nanofluid" to refer to a fluid containing a dispersion of nanoparticles. The term nanofluids are colloidal suspensions of nanoparticles with typical dimensions of about 1-100 nm dispersed in a non-conducting carrier liquid like water, kerosene, ester and hydrocarbons etc. Nanofluids are synthesized in the laboratory; they do not occur naturally. Characteristic features of nanofluid are the formation of a very stable colloidal system with minimal settling and anomalous enhancement of the thermal conductivity compare to the base fluid [2-10]. Buongiorno [11] focused on heat transfer enhancement of nanofluid in convective situations and concluded that in the absence of destructive effects, only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. Buongiorno [11] gave the conservation equation of a nonhomogeneous equilibrium model of nanofluid for mass, momentum and heat transport. Based on the transport equation, many researchers have studied the stability of the onset of convection in a nanofluid. Tzou [12] has studied the thermal instability of nanofluids in natural convection. The problem of onset of convection in a horizontal nanofluid layer of finite depth is studied by Nield and Kuznetsov [13]. They obtained that oscillatory instability is possible when nanoparticles concentrate near the bottom of the layer. The onset of double-diffusive convection in a nanofluid layer is studied by Nield and Kuznetsov [14]. On using one-term Galerkin approximation, they obtained that the stability boundaries for both non-oscillatory and oscillatory cases.

The growing importance of the use of non-Newtonian fluids in several applied fields oil recovery, food processing, the spread of contaminants in the environment and various processes in the chemical and materials industry has led various researchers to attempt diverse flow problems related to several non-Newtonian fluids. The problem on the thermal convection in a viscoelastic fluid was studied by many authors [15-18]. It is commonly believed that oscillatory convection is not possible in viscoelastic fluids under realistic experimental conditions [19]. Those experiments triggered new interest in convection by applying binary aspects to viscoelastic fluids. Rayleigh-Benard convection in binary viscoelastic fluids was studied by some researchers [20-22]. Laroze, Martinez-Mardones, Bragardc and Peirez-Garcia [23] have studied the problem of realistic rotating convection in a DNA suspension. The problem of thermal instability of rotating nanofluid layer is studied by Yadav, Agrawal and Bhargava [24]. The problem linear stability analysis of in a viscoelastic nanofluid layer is studied by Sheu [25]. He found that Oscillatory convection is possible for both bottoms heavy and top-heavy nanoparticle distributions. Chand and Rana [26] have studied the problem on the onset of thermal convection in rotating nanofluid layer saturating Darcey-Brinkman porous medium and found that Lewis number and modified diffusivity ratio stabilize the stationary convection while concentration Rayleigh number destabilize the stationary convection. Gupta et al. [27] studied the effect of magnetic field on the onset of convection in a nanofluid layer and found that Lewis number increases the stability in nanofluid layer system and decreases it for oscillatory motions.

The double-diffusive convection is convection in which the fluid contains two components with different molecular diffusivities. However, there are many situations where more than two components are involved like the solidification of molten alloys, geothermally heated lakes, magmas and their laboratory models and seawater. The triple-diffusive convection in nanofluid has been studied by Rionero [28] and Khan et al. [29].

In recent years, the theory of nanofluids became more attractive due to the inexpensive price and easy production. Also, the presence of more than one chemical dissolved in fluid mixtures is very often requested for describing natural phenomena like contaminant transport, underground water flow, acid rain effects, worming of the stratosphere. These situations can also occur in a nanofluid mixture if we dissolve different salts in a nanofluid. Therefore, the triple-diffusive convection in a nanofluid layer has been studied in the present paper, taking the effects of Brownian diffusion and thermophoresis in the account.To the best of our knowledge, the effect of triple-diffusive convection in stability of the nanofluid layer has not been investigated yet. Employing the linear stability analysis, the analytical Rayleigh number has been obtained using normal mode technique. We have also discussed the stationary, oscillatory convection and case of overstability.

II. Assumptions

The following assumptions have been taken for the mathematical treatment of the considered problem.

(i) The nanofluid is taken as incompressible and Newtonian, and the flow is considered to be laminar.

(ii) There is no chemical reactions take place during the thermal convection.

(iii) The density of the nanofluid is considered to be constant except in the term for the external force in momentum equation while other thermophysical properties of nanofluid (viscosity, specific heat, thermal conductivity, solute diffusivities etc.)are assumed to be constant (Boussinesq Hypothesis).

(iv) The nanoparticles and fluid phase are in a thermal equilibrium state.

(v) Nano particles are considered to be spherical.

(vi) In comparison with the other modes of heat transfer, the Radiative heat transfer between the sides of the wall is extremely small; therefore, the effect has been neglected.

(vii) Nanoparticles are being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these on the porous matrix.

(viii) The reference temperature is assumed to be high as compared to fluid temperature.

III. Problem Formulation

We consider an infinite horizontal layer of fluid confined between two horizontal planes situated at $z^*=0$ and $z^*=d$ acted upon by gravity field $\mathbf{g}(0,0,-g)$. We use the asterisks to denote the variables in dimensional form. In the present problem, z -axis-aligned vertically upward. The boundary walls are assumed to be perfectly thermally conducting and nanoparticles concentrated. At the bottom surface, the temperature (T^*) is, and solute concentrations $(C^{(1)*}, C^{(2)*})$ are taken as $T_0^*+\Delta T^*$, $C_0^{(1)*}+\Delta C^{(1)*}$ and $C_0^{(2)*}+\Delta C^{(2)*}$ while at the top surface, these values are assumed to be T_0^*, $C_0^{(1)*}$ and $C_0^{(2)*}$, respectively. In the present analysis, we use the Oberbeck-Boussinesq approximation, and a mixture of nanofluid and concentrations (salts) is homogeneous and is in local thermal equilibrium. The temperature change is assumed to be small as compared to T_0^*, and similarly, the changes in concentrations are assumed to be small in comparison with $C_0^{(1)*}$ and $C_0^{(2)*}$, respectively.

The governing equation for the nanofluid layer (Buongiorno [11], Tzou [12])can be written as

$$\nabla.\mathbf{q}^*=0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left(\frac{\partial\mathbf{q}^*}{\partial t^*}+\frac{1}{\varepsilon}(\mathbf{q}^*.\nabla^*)\mathbf{q}^*\right)=-\nabla^*p^*+\tilde{\mu}\nabla^{*2}\mathbf{q}^*-\frac{\mu}{K}\nabla^{*2}\mathbf{q}^*+\left[\phi^*\rho_p+(1-\phi^*)\rho_0\right.$$
$$\left.\left[1-\beta_T(T^*-T_0^*)+\beta_{C_1}(C^{(1)*}-C_0^{(1)*})+\beta_{C_2}(C^{(2)*}-C_0^{(2)*})\right]\right]\mathbf{g}, \tag{2}$$

$$(\rho c)_m\frac{\partial T^*}{\partial t^*}+(\rho c)_f(\mathbf{q}^*.\nabla^*)T^*=\kappa\nabla^{*2}T^*+\varepsilon(\rho c)_p\left[D_B\nabla^*\phi^*.\nabla^*T^*+\left(D_T/T_0^*\right)\nabla^*T^*.\nabla^*T^*\right], \tag{3}$$

$$\frac{\partial C^{(1)*}}{\partial t^*}+\frac{1}{\varepsilon}(\mathbf{q}^*.\nabla^*)C^{(1)*}=D_{S_1}\nabla^{*2}C^{(1)*}, \tag{4}$$

$$\frac{\partial C^{(2)*}}{\partial t^*}+\frac{1}{\varepsilon}(\mathbf{q}^*.\nabla^*)C^{(2)*}=D_{S_2}\nabla^{*2}C^{(2)*}, \tag{5}$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}(\mathbf{q}^*.\nabla^*)\phi^* = D_B \nabla^{*2}\phi^* + \left(D_T / T_0^*\right)\nabla^{*2} T^* . \tag{6}$$

Here, flow velocity $\mathbf{q}^* = (u^*, v^*, w^*)$ and is assumed that solute concentrations and volume fraction of the nanoparticles are constant on both the boundaries. Therefore, the boundary conditions can be written as

$$\text{At } z^* = 0 \begin{cases} w^* = 0, \quad \dfrac{\partial w^*}{\partial z^*} + \lambda_1 d \dfrac{\partial^2 w^*}{\partial z^{*2}} = 0, \quad T^* = T_0^* + \Delta T^* \\ C^{(1)*} = C_0^{(1)*} + \Delta C^{(1)*}, \quad C^{(2)*} = C_0^{(2)*} + \Delta C^{(2)*}, \quad \phi^* = \phi_0^* \end{cases} \tag{7}$$

and

$$\text{At } z^* = d \begin{cases} w^* = 0, \quad \dfrac{\partial w^*}{\partial z^*} - \lambda_2 d \dfrac{\partial^2 w^*}{\partial z^{*2}} = 0, \quad T^* = T_0^* \\ C^{(1)*} = C_0^{(1)*}, \quad C^{(2)*} = C_0^{(2)*}, \quad \phi^* = \phi_1^* \end{cases} , \tag{8}$$

where λ_1 and λ_2 are the parameters which take the value zero for the case of a rigid boundary and infinity for the case of a free boundary.

We accept that in some context, the choice of boundary conditions imposed on ϕ is somewhat arbitrary. It could be claimed that on the boundaries, zero particle flux is more realistic physically, but then one is faced with the problem that it appears that no steady-state solution for the basic conduction equations is then possible (we have tried to find one and met a contradiction) so that in order to find the analytical solution for considered problem it is necessary to restrict the basic profile for ϕ and at that stage our choice of boundary conditions is seen to be quite realistic.

To introduce non-dimensional variables, we define $(x, y, z) = (x^*, y^*, z^*)/d$, $(u, v, w) = (u^*, v^*, w^*)d/\alpha_f$, $p = p^* K/\mu\alpha_f$, $T = \left(T^* - T_0^*\right)/\Delta T^*$, $\phi = \left(\phi^* - \phi_0^*\right)/\left(\phi_1^* - \phi_0^*\right)$, $t = t^*\alpha_f / \sigma d^2$, $C^{(1)} = \left(C^{(1)*} - C_0^{(1)*}\right)/\Delta C^{(1)*}$, $C^{(2)} = \left(C^{(2)*} - C_0^{(2)*}\right)/\Delta C^{(2)*}$, where $\alpha_f = \kappa/(\rho c)_f$ and $\sigma = (\rho c)_m/(\rho c)_f$.

The non-dimensional form of the equations (1) - (6) can be written as

$$\nabla.\mathbf{q} = 0 , \tag{9}$$

$$\frac{Da}{\varepsilon\sigma \Pr}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \tilde{D}a\nabla^2\mathbf{q} - \mathbf{q} - Rm\hat{e}_z + RaT\hat{e}_z - \frac{Rs_1}{Le_1}C^{(1)}\hat{e}_z - \frac{Rs_2}{Le_2}C^{(2)}\hat{e}_z - Rn\phi\hat{e}_z , \tag{10}$$

$$\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)T = \nabla^2 T + \frac{N_B}{Ln}\nabla\phi.\nabla T + \frac{N_A N_B}{Ln}\nabla T.\nabla T , \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial C^{(1)}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)C^{(1)} = \frac{1}{Le_1}\nabla^2 C^{(1)} , \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial C^{(2)}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)C^{(2)} = \frac{1}{Le_2}\nabla^2 C^{(2)} , \tag{13}$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\phi = \frac{1}{Ln}\nabla^2\phi + \frac{N_A}{Ln}\nabla^2 T . \tag{14}$$

The dimensionless boundary conditions are

$$\text{At } z = 0 \left\{ w = 0, \quad \frac{\partial w}{\partial z} + \lambda_1 d \frac{\partial^2 w}{\partial z^2} = 0, \quad T = 1, C^{(1)} = 1, \quad C^{(2)} = 1, \phi = 0 \right. \tag{15}$$

and

$$\text{At } z=1\left\{ w=0,\quad \frac{\partial w}{\partial z}-\lambda_2 d\frac{\partial^2 w}{\partial z^2}=0,\quad T=0,\ C^{(1)}=0,\quad C^{(2)}=0,\quad \phi=1, \right. \tag{16}$$

Here, dimensionless parameters as follows;

$\Pr=\frac{\mu}{\rho_0\alpha_f}$; the Prandtl number, $Da=K/d^2$; the Darcy number, $\tilde{D}a=\tilde{\mu}K/\mu d^2$; the Brinkman-Darcy number,

$Ln=\frac{\alpha_f}{D_B}$;the nanofluid Lewis number, $Le_1=\frac{\alpha_f}{D_{S_1}}$; the familiar Lewis number, $Le_2=\frac{\alpha_f}{D_{S_2}}$; the analogous Lewis number,

$Ra=\frac{\rho_0 g\beta_T K d\Delta T^*}{\mu\alpha_f}$;the familiar thermal Rayleigh number, $Rs_1=\frac{\rho g\beta_{C_1} K d\Delta C^{(1)*}}{\mu D_{S_1}}$;the familiar solute Rayleigh number,

$Rs_2=\frac{\rho g\beta_{C_2} K d\Delta C^{(2)*}}{\mu D_{S_2}}$; the analogous solute Rayleigh number. The parameters $Rm=\frac{\left[\rho_p\phi_1^*+\rho\left(1-\phi_1^*\right)\right]gKd}{\mu\alpha_f}$ and

$Rn=\frac{\left[\left(\rho_p-\rho\right)\left(\phi_1^*-\phi_0^*\right)\right]gKd}{\mu\alpha_f}$ may be regarded as a basic density Rayleigh number and a nanoparticle concentration

Rayleigh number, respectively. The parameter $N_A=\frac{D_T\Delta T^*}{D_B T_0^*\left(\phi_1^*-\phi_0^*\right)}$ is a modified diffusivity ratio and

$N_B=\frac{\varepsilon(\rho c)_p\left(\phi_1^*-\phi_0^*\right)}{(\rho c)_f}$ is a modified particle-density increment.

As we assumed the case of small thermal and solutal gradients in a dilute suspension of nanoparticles, the linearization of equation (11) will be done by the neglect of terms, one proportional to the product of ϕ and T another proportional to the product of ϕ, $C^{(1)}$ and ϕ, $C^{(2)}$. For the linearization, we use the concept of Oberbeck-Boussinesq approximation.

IV. Basic State

We assume that thebasic state of nanofluid layer is timeindependent and is described by

$$\mathbf{q}=(0,\ 0,\ 0),\ p=p_b(z),\ T=T_b(z),\ C^{(1)}=C_b^{(1)}(z),\ C^{(2)}=C_b^{(2)}(z),\ \phi=\phi_b(z), \tag{17}$$

Using above equation, equations (10) - (14) take the form

$$0=-\frac{d}{dz}p_b-Rm+RaT_b-\frac{Rs_1}{Le_1}C_b^{(1)}-\frac{Rs_2}{Le_2}C_b^{(2)}-Rn\phi_b, \tag{18}$$

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{Ln}\left(\frac{d\phi_b}{dz}.\frac{dT_b}{dz}\right)+\frac{N_A N_B}{Ln}\left(\frac{dT_b}{dz}\right)^2=0, \tag{19}$$

$$\frac{1}{Le_1}\frac{d^2C_b^{(1)}}{dz^2}=0, \tag{20}$$

$$\frac{1}{Le_2}\frac{d^2C_b^{(2)}}{dz^2}=0, \tag{21}$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0. \tag{22}$$

$$T_b(0)=1,\ C_b^{(1)}(0)=1,\ C_b^{(2)}(0)=1,\ \phi_b(0)=0,\ T_b(1)=0,\ C_b^{(1)}(1)=0,\ C_b^{(2)}(1)=0,\ \phi_b(0)=1. \quad (23)$$

In mostly cases of nanofluid layers, the value of $\frac{Ln}{\phi_1^*-\phi_0^*}$ is large of order 10^5-10^6[Buongiorno (2006)] and alsothenanoparticle fraction decrement $\left(\phi_1^*-\phi_0^*\right)$ is typically no smallerthan 10^{-3} implies that Ln is large, of order 10^2-10^3. It has also been observed that N_A will not be greater than 10. Using this approximation, the basicsolution can be written as follows;

$$T_b(z)=1-z,\ C_b^{(1)}(z)=1-z,\ C_b^{(2)}(z)=1-z,\ \phi_b(z)=z. \quad (24)$$

V. Perturbed State

If we impose the 'small perturbations' to the basic state of the nanofluid layer, the parameters become $\mathbf{q}(u,v,w)=\mathbf{q}(0,0,0)+\mathbf{q}(u',v',w')$, $p=p_b+p'$, $T=T_b+T'$, $C^{(1)}=C_b^{(1)}+C^{(1)\prime}$ and $C^{(2)}=C_b^{(2)}+C^{(2)\prime}$. Here prime denotes the quantities in a perturbed state.

We are using here linear stability theory and therefore we are neglecting all product and powers (higher than the first) of the increments. Now,the linearized perturbation equations are

$$\nabla.\mathbf{q}'=0, \quad (25)$$

$$\frac{Da}{\varepsilon\sigma\Pr}\frac{\partial\mathbf{q}'}{\partial t}=-\nabla p'+\tilde{D}a\nabla^2\mathbf{q}'-\mathbf{q}'+RaT'\hat{e}_z-\frac{Rs_1}{Le_1}C^{(1)\prime}\hat{e}_z-\frac{Rs_2}{Le_2}C^{(2)\prime}\hat{e}_z-Rn\phi'\hat{e}_z, \quad (26)$$

$$\frac{\partial T'}{\partial t}-w'=\nabla^2T'+\frac{N_B}{Ln}\left(\frac{\partial T'}{\partial z}-\frac{\partial\phi'}{\partial z}\right)-\frac{2N_AN_B}{Ln}\frac{\partial T'}{\partial z}, \quad (27)$$

$$\frac{1}{\sigma}\frac{\partial C^{(1)\prime}}{\partial t}-\frac{1}{\varepsilon}w'=\frac{1}{Le_1}\nabla^2C^{(1)\prime}, \quad (28)$$

$$\frac{1}{\sigma}\frac{\partial C^{(2)\prime}}{\partial t}-\frac{1}{\varepsilon}w'=\frac{1}{Le_2}\nabla^2C^{(2)\prime}, \quad (29)$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t}+\frac{1}{\varepsilon}w'=\frac{1}{Ln}\nabla^2\phi'+\frac{N_A}{Ln}\nabla^2T'. \quad (30)$$

The dimensionless boundary conditions are

$$w'=0,\ \frac{\partial w'}{\partial z}+\lambda_1\frac{\partial^2w'}{\partial z^2}=0,\ T'=0,\ C^{(1)\prime}=0,\ C^{(2)\prime}=0,\ \phi'=0 \text{ at } z=0 \quad (31)$$

and $$w'=0,\ \frac{\partial w'}{\partial z}-\lambda_2\frac{\partial^2w'}{\partial z^2}=0,\ T'=0,\ C^{(1)\prime}=0,\ C^{(2)\prime}=0,\ \phi'=0 \text{ at } z=1. \quad (32)$$

*It can be noted that the parameter Rm is just a measure of the basic static pressure gradient and is not involved in above equations. If we take a regular*binary fluid (not a nanofluid) the parameters Rn, N_A and N_B will be zero and second term in L.H.S in equation (30) is absent because $d\phi_b/dz=0$. The remaining equations are reduced to the familiar equations for the triple-diffusive Rayleigh-Benard problem.

Eliminating p' by operating a curl twice on (25), we obtain

$$\frac{Da}{\varepsilon\sigma\Pr}\frac{\partial}{\partial t}\left(\nabla^2w'\right)=\tilde{D}a\nabla^4w'-\nabla^2w'+Ra\nabla_H^2T'-\frac{Rs_1}{Le_1}\nabla_H^2C^{(1)\prime}-\frac{Rs_2}{Le_2}\nabla_H^2C^{(2)\prime}-Rn\nabla_H^2\phi'. \quad (33)$$

Here ∇_H^2 is the two-dimensional Laplacian operator in the horizontal plane. The differential equations (33), (27) - (30) and the boundary conditions (31) and (32) constitute a linear boundary value problem that can be solved using the method of normal modes. Now analyzing the perturbations into normal modes, we assume that the perturbation quantities are of the form

$$\left[w, T', C^{(1)\prime}, C^{(2)\prime}, \phi'\right] = \left[W(z), \Theta(z), \Gamma(z), X(z), \Phi(z)\right]\exp\{ik_x x + ik_y y + nt\}, \quad (34)$$

Where k_x and k_y are the wave numbers in x and y directions respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant. Using equation (34) in the equations (33) and (27)-(30), we get

$$\left[\frac{Da\, n}{\varepsilon\sigma \Pr} - \tilde{D}a\left(D^2 - k^2\right) + 1\right]\left(D^2 - k^2\right)W = -Rak^2\Theta + \frac{Rs_1}{Le_1}k^2\Gamma + \frac{Rs_2}{Le_2}k^2 X + Rnk^2\Phi\,, \quad (35)$$

$$W + \left(\left(D^2 - k^2\right) - n + \frac{N_B}{Ln}D - \frac{2N_A N_B}{Ln}D\right)\Theta - \frac{N_B}{Ln}D\Phi = 0\,, \quad (36)$$

$$\frac{W}{\varepsilon} + \left(\frac{1}{Le_1}\left(D^2 - k^2\right) - \frac{n}{\sigma}\right)\Gamma = 0\,, \quad (37)$$

$$\frac{W}{\varepsilon} + \left(\frac{1}{Le_2}\left(D^2 - k^2\right) - \frac{n}{\sigma}\right)X = 0\,, \quad (38)$$

$$\frac{W}{\varepsilon} - \frac{N_A}{Ln}\left(D^2 - k^2\right)\Theta + \left(\frac{n}{\sigma} - \frac{1}{Ln}\left(D^2 - k^2\right)\right)\Phi = 0\,, \quad (39)$$

We are considering the case of free boundaries forstability analysis. Therefore the boundary conditions are

$$W = 0\,,\ D^2W = 0\,,\ \Theta = 0\,,\ \Gamma = 0\,,\ X = 0\,,\ \Phi = 0 \text{ at } z = 0 \text{ and } z = 1\,. \quad (40)$$

Now, we assume the solutions of the equations (35) to (39) in the form

$$W = W_0\, Sin\pi z\,,\ \Theta = \Theta_0\, Sin\pi z\,,\ \Gamma = \Gamma_0\, Sin\pi z\,,\ X = X_0\, Sin\pi z \text{ and } \Phi = \Phi_0\, Sin\pi z\,, \quad (41)$$

because they satisfy the boundary conditions (40).

Substituting equation (41) in to equations (35) - (39), we have

$$\begin{bmatrix} \tilde{D}a\delta^4 + \frac{nDa}{\varepsilon\sigma\Pr}\delta^2 + \delta^2 & -Rak^2 & \frac{Rs_1}{Le_1}k^2 & \frac{Rs_2}{Le_2}k^2 & Rnk^2 \\ 1 & -\left(\delta^2 + n\right) & 0 & 0 & 0 \\ \frac{1}{\varepsilon} & 0 & -\left(\frac{1}{Le_1}\delta^2 + \frac{n}{\sigma}\right) & 0 & 0 \\ \frac{1}{\varepsilon} & 0 & 0 & -\left(\frac{1}{Le_2}\delta^2 + \frac{n}{\sigma}\right) & 0 \\ \frac{1}{\varepsilon} & \frac{N_A}{Ln}\delta^2 & 0 & 0 & \left(\frac{1}{Ln}\delta^2 + \frac{n}{\sigma}\right) \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ X_0 \\ \Phi_0 \end{bmatrix} = 0\,.$$

(42)

Here, $\delta^2 = \pi^2 + k^2$ is total wave number. The nontrivial solution of the above homogeneous equations requires that

$$Ra = \frac{\left(n+\delta^2\right)}{k^2}\left(\delta^2+\delta^4\tilde{D}a+\frac{n^2}{\Pr}+\frac{nDa\delta^2}{\varepsilon\sigma\Pr}\right)+\frac{\sigma\left(n+\delta^2\right)}{\varepsilon(nLe_1+\sigma\delta^2)}Rs_1+\frac{\sigma\left(n+\delta^2\right)}{\varepsilon\left(nLe_2+\sigma\delta^2\right)}Rs_2 - \frac{\sigma}{\varepsilon\left(nLn+\sigma\delta^2\right)}\left(nLn+\delta^2Ln+\delta^2N_A\varepsilon\right)Rn, \quad (43)$$

Now, we take $n = i\omega$ in equation (43), and we get

$$Ra = \Delta_1 + i\omega\Delta_2 . \quad (44)$$

Here,

$$\Delta_1 = \frac{\left(\delta^4\varepsilon\sigma\Pr+\delta^6\tilde{D}a\,\varepsilon\sigma\Pr-\delta^2\omega^2Da\right)}{k^2\varepsilon\sigma\Pr}+\frac{\sigma\left(\sigma\delta^4+\omega^2Le_1\right)}{\varepsilon\left(\sigma^2\delta^4+\omega^2Le_1^{\,2}\right)}Rs_1+\frac{\sigma\left(\sigma\delta^4+\omega^2Le_2\right)}{\varepsilon\left(\sigma^2\delta^4+\omega^2Le_2^{\,2}\right)}Rs_2 - \frac{\sigma\left(\sigma Ln\delta^4+\omega^2Ln^2+\sigma\varepsilon\delta^4N_A\right)Rn}{\varepsilon\left(\sigma^2\delta^4+\omega^2Ln^2\right)}, \quad (45)$$

and

$$\Delta_2 = \frac{\left(\delta^2\varepsilon\sigma\Pr+\delta^4\tilde{D}a\,\varepsilon\sigma\Pr+\delta^4Da\right)}{k^2\varepsilon\sigma\Pr}+\frac{\sigma\delta^2\left(\sigma-Le_1\right)}{\varepsilon\left(\sigma^2\delta^4+\omega^2Le_1^{\,2}\right)}Rs_1+\frac{\sigma\delta^2\left(\sigma-Le_2\right)}{\varepsilon\left(\sigma^2\delta^4+\omega^2Le_2^{\,2}\right)}Rs_2 - \frac{\sigma\delta^2Ln\left(\sigma-Ln-\varepsilon N_A\right)Rn}{\varepsilon\left(\sigma^2\delta^4+\omega^2Ln^2\right)}, \quad (46)$$

As the Rayleigh number Ra is a physical quantity and therefore, it must be real. Hence, it can be concluded from the equation(44) that either $\omega = 0$ (exchange of stabilities, steady-state) or $\Delta_2 = 0$ ($\omega \neq 0$, overstability or oscillatory onset).

VI. Stationary Convection

Steady onset corresponds to $\omega = 0$ and Rayleigh number is given by

$$Ra^{st} = \frac{\delta^4\left(1+\delta^2\tilde{D}a\right)}{k^2}+\frac{1}{\varepsilon}Rs_1+\frac{1}{\varepsilon}Rs_2-\frac{1}{\varepsilon}\left(Ln+\varepsilon N_A\right)Rn, \quad (47)$$

In the absence of concentrations (salts), equation (47) becomes

$$Ra^{st} = \frac{\delta^4\left(1+\delta^2\tilde{D}a\right)}{k^2}-\frac{1}{\varepsilon}\left(Ln+\varepsilon N_A\right)Rn, \quad (48)$$

This is same equation as obtained by Tzou [12] and Sheu[25].

The critical cell size at the onset of instability is obtained from the condition $\frac{\partial}{\partial k}Ra = 0$.

$$k_c = \frac{\pi}{\sqrt{2}}, \quad (49)$$

Thus, for steady onset, the corresponding critical thermal Rayleigh number i

VII. Oscillatory Convection

For oscillatory onset, $\Delta_2 = 0$ and $\omega \neq 0$, which gives

$$a_3(\omega^2)^3 + a_2(\omega^2)^2 + a_1(\omega^2) + a_0 = 0 . \quad (51)$$

Here, $a_3 = Le_1^{\,2}Le_2^{\,2}Ln^2\delta^2\left(\delta^2Da+\sigma\varepsilon\Pr\left(1+\delta^2\tilde{D}a\right)\right)$,

$$a_2=\left(\delta^2 Da+\varepsilon\sigma\Pr+\delta^2\varepsilon\sigma\tilde{D}a\Pr\right)\left(Le_1^{\,2}Le_2^{\,2}\delta^6\sigma^2+Le_1^{\,2}Ln^2\delta^6\sigma^2+Le_2^{\,2}Ln^2\delta^6\sigma^2\right)$$
$$+k^2\delta^4\sigma^2Le_2^{\,2}Ln^2\Pr\left(\sigma-Le_1\right)Rs_1+k^2\delta^4\sigma^2Le_1^{\,2}Ln^2\Pr\left(\sigma-Le_2\right)Rs_1$$
$$-k^2\delta^2Le_1^{\,2}Le_2^{\,2}Ln\sigma^2\Pr\left(\sigma-Ln-\varepsilon N_A\right)Rn$$

$$a_1=\left(\delta^2 Da+\varepsilon\sigma\Pr+\delta^2\varepsilon\sigma\tilde{D}a\Pr\right)\left(\delta^{10}\sigma^4Le_1^{\,2}+\delta^{10}\sigma^4Le_2^{\,2}+\delta^{10}\sigma^4Ln^2\right)$$
$$+\left(\sigma-Le_1\right)\left(k^2\delta^6Le_2^{\,2}\sigma^4\Pr Rs_1+k^2\delta^6\sigma^4Ln^2\Pr Rs_1\right)$$
$$+\left(\sigma-Le_2\right)\left(k^2\delta^6\sigma^4Le_1^{\,2}\Pr Rs_2+k^2\delta^6\sigma^4Ln^2\Pr Rs_2\right)$$
$$-\left(\sigma-Ln-\varepsilon N_A\right)\left(k^2\delta^6\sigma^4Le_1^{\,2}Ln\Pr+k^2\delta^6\sigma^4Le_2^{\,2}Ln\Pr\right)Rn$$

$$a_0=\left(\delta^2 Da+\varepsilon\sigma\Pr+\delta^2\varepsilon\sigma\tilde{D}a\Pr\right)\delta^{14}\sigma^6+k^2\delta^{10}\sigma^6\Pr Rs_1\left(\sigma-Le_1\right)$$
$$+k^2\delta^{10}\sigma^6\Pr Rs_2\left(\sigma-Le_2\right)-k^2\delta^{10}\sigma^6Le_1^{\,2}Ln\Pr\left(\sigma-Ln-\varepsilon N_A\right)Rn.$$

As we know, Ln is of order 10^2-10^3, $1<N_A<10$ and therefore $Ln+\varepsilon N_A-\sigma$ is always positive. If Le_1 and Le_2 are greater than 1, equation (51) does not admit the positive value of ω^2. In other words, we can say Oscillatory convection is possible only when $Le_1, Le_2>\sigma$.

Thus from Eq. (37) and (38), oscillatory Rayleigh number is given by

$$Ra^{osc}=\frac{\left(\delta^4\varepsilon\sigma\Pr+\delta^6\tilde{D}a\,\varepsilon\sigma\Pr-\delta^2\omega^2Da\right)}{k^2\varepsilon\sigma\Pr}+\frac{\sigma\left(\sigma\delta^4+\omega^2Le_1\right)}{\varepsilon\left(\sigma^2\delta^4+\omega^2Le_1^{\,2}\right)}Rs_1+\frac{\sigma\left(\sigma\delta^4+\omega^2Le_2\right)}{\varepsilon\left(\sigma^2\delta^4+\omega^2Le_2^{\,2}\right)}Rs_2-\frac{\sigma\left(\sigma Ln\delta^4+\omega^2Ln^2+\sigma\varepsilon\delta^4N_A\right)Rn}{\varepsilon\left(\sigma^2\delta^4+\omega^2Ln^2\right)},\quad(52)$$

Here, where ω^2 is given by equation (51). If no positive value of ω^2 exist, then oscillatory convection is not possible. This result is a good agreement with the result obtained by Sheu[25].

VIII. Case of Overstability

Here, we check the possibility of occurrence of overstability. Since, we wish to find the Rayleigh number for the onset of instability via a state of pure oscillation, for which it suffices to find the conditions for which equation (51) will have the solution with real values of ω. The three values of ω^2 (ω being real) are positive.

The product of root is $-(a_0/a_3)$, and this is to be positive. However, from Eq. (51) it is clear that b_3 is always positive and b_0 is positive if

$$Rn>0,\sigma>Le_1,\sigma>Le_2 \text{ and } Ln+\varepsilon N_A>\sigma. \quad (53)$$

Hence inequalities (53) are sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

IX. Results and Discussions

In this section, we describe our results numerically. Stationary thermal Rayleigh number is given by equation (47), and the expression of the oscillatory thermal Rayleigh number is obtained analytically using equation (52) where ω^2 is given by equation (51). From equation (47) and (52), it is clear that stationary Rayleigh number and oscillatory Rayleigh number do not depend on N_B because the effect of N_B in equation (36) is cancelled due to the integration of orthogonal functions. **The impact of Brownian motion and thermophoresis in the thermal energy equation of instability does not appear. The**

Brownian motion and thermophoresis directly contribute to the equation expressing the conservation of nanoparticles to produce their effects. In this way, the temperature and nanoparticle density is coupled in a particular way in which the instability is almost purely a phenomenon due to buoyancy coupled with the conservation of nanoparticle motion. Therefore, it is worth discussing the limiting case, $N_A = 0$, which indicates the absence of athermophoretic effect.

Figure 1 shows the neutral curves for different values of modified Darcy numbers. It has been observed that stationary Rayleigh number decreases with increasing the modified Darcy number. Thus, modified Darcy number destabilizes the stationary convection.

Figure 2 shows the neutral curves for different values of solute Rayleigh numbers. It shows that stationary Rayleigh number increases with increase the solute Rayleigh number. Thus, solute Rayleigh number stabilizes the stationary convection.

Figure 3 shows the neutral curves for different values of analogous solute Rayleigh numbers. It shows that stationary Rayleigh number increases with increase the solute Rayleigh number. Thus, solute Rayleigh number stabilizes the stationary convection.

Figure 4 shows the neutral curves for different values of nanofluid lewis number. It has been found that stationary Rayleigh number decreases with increasing the nanofluid lewis number. Thus, nanofluid lewis number destabilizes the stationary convection.

Figure 5 shows the neutral curves for different values of porosity. It has been observed that stationary Rayleigh number decreases with increase the porosity. Thus, porosity destabilizes the stationary convection.

Figure 6 shows the neutral curves for different values of concentration Rayleigh number. It has been observed that stationary Rayleigh number decreases with increase the concentration Rayleigh number. Thus, concentration Rayleigh number destabilizes the stationary convection.

For the bottom heavy distribution of the nanoparticles, is negative while is positive for top-heavy distribution. It is observed that stationary convection is possible for both bottom-heavy and top-heavy nanoparticles distribution and stationary Rayleigh number is smaller for top-heavy than that of bottom-heavy distribution of nanoparticles [25].
Figure 7 shows the neutral curves for different values of modified diffusivity ratio. It has been observed that stationary Rayleigh number decreases with increase the modified diffusivity ratio. Thus, the modified diffusivity ratio destabilizes the stationary convection.

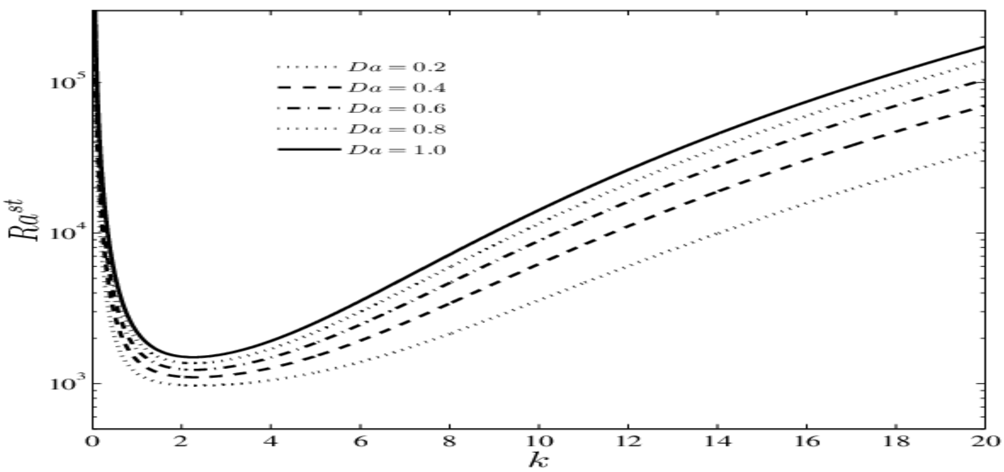

Fig. 1. Neutral stability curve for Rayleigh number for different values of modified Darcy number

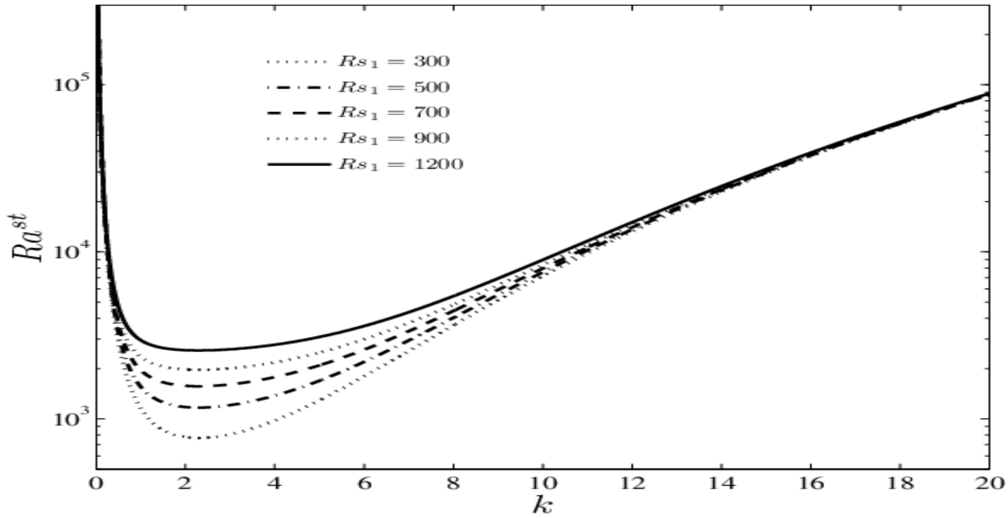

Fig. 2. Neutral stability curve for Rayleigh number for different values of solute Rayleigh number.

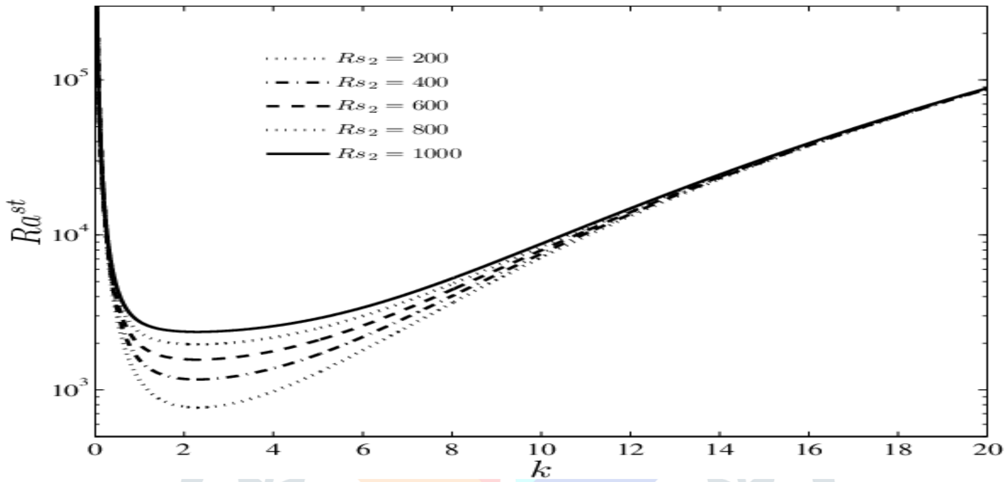

Fig. 3.Neutral stability curve for Rayleigh number for different values of analogous solute Rayleigh number

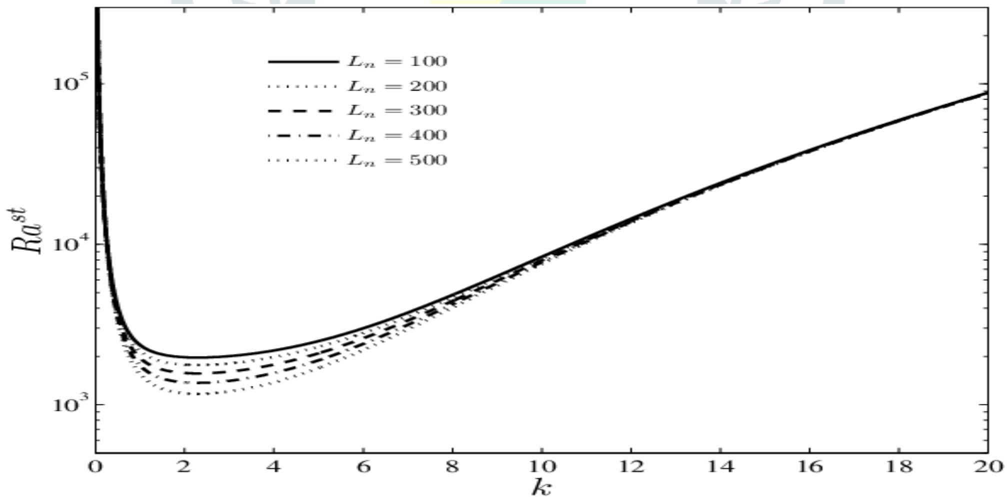

Fig. 4.Neutral stability curve for Rayleigh number for different values nanofluid Lewis number

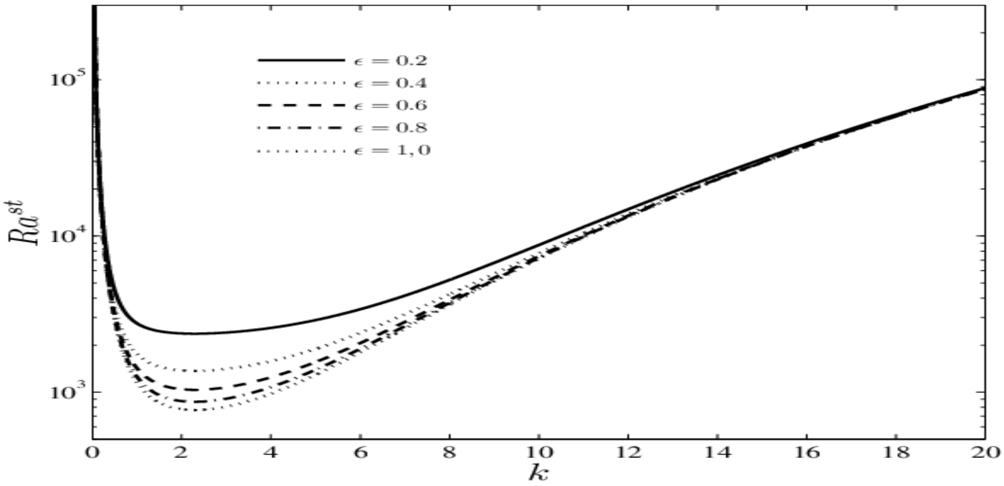

Fig. 5.Neutral stability curve for Rayleigh number for different values of porosity

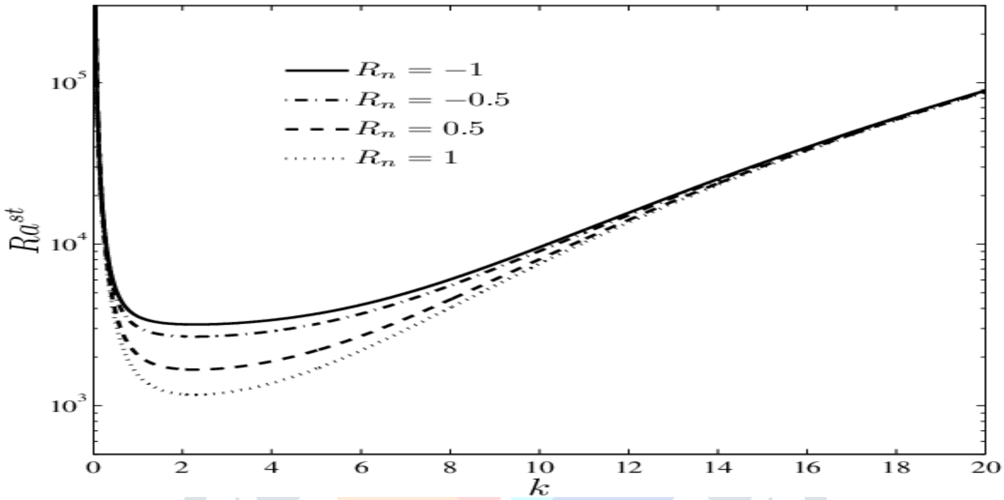

Fig. 6. Neutral stability curve for Rayleigh number for different values of concentration Rayleigh number

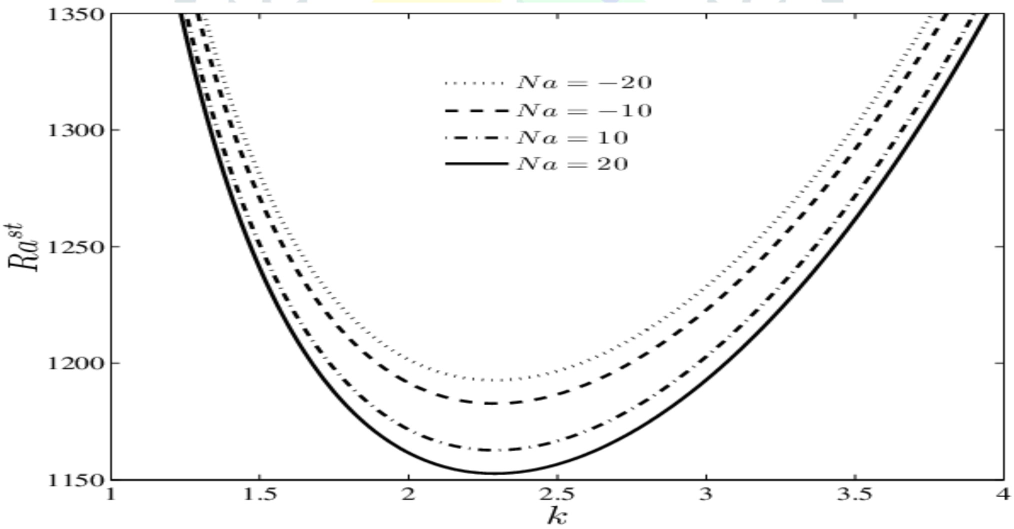

Fig. 7.Neutral stability curve for Rayleigh number for different values of modified diffusivity ratio

X. Conclusions

Linear stability analysis of triple-diffusive convection in a nanofluid layer is investigated. The problem is analyzed for free-free boundaries by employing the normal mode technique. The main conclusions are:

(i) The Oscillatory convection is not possible if both Lewis number and analogous Lewis number ≤ 1.

(ii) The critical cell size is not a function of any thermophysical properties of nanofluid.

(iii) The critical value of the Rayleigh number is independent of modified particle-density increment N_B.

(iv) Instability is unaffected by Brownian motion and thermophoresis. It is purely phenomenon due to the coupling of buoyancy and conservation of nanoparticles.

(v) Solute Rayleigh number and analogous solute Rayleigh number stabilize the stationary convection.

(vi) Lewis number and modified diffusivity ratio stabilize the stationary convection.

(vii) Concentration Rayleigh number destabilizes the stationary convection. It is also found that stationary convection is possible for both bottom-heavy and top heavy distribution of nanoparticles.

(viii) The sufficient conditions for the non-existence of overstability are $1 > Le_1, 1 > Le_2$ and $Ln + N_A > 1$.

References

1. Choi, S. Enhancing thermal conductivity of fluids with nanoparticles. In:Siginer D.A., Wang, H.P. (eds.) Developments and Applications ofNon-Newtonian Flows, ASME FED- Vol. 231/ MD-Vol. 66, New York,1995, pp. 99-105.
2. H. Masuda, A. Ebata, K. Teramea, N. Hishinuma, Altering the thermalconductivity and viscosity of liquid by dispersing ultra-fine particles, NetsuBussei 4 (4) (1993) 227-233.
3. B.C. Pak, Y. Cho, Hydrodynamic and heat transfer study of dispersed fluidswith submicron metallic oxide particles, Exp. Heat Transfer 11 (1998)151-170.
4. J.A. Eastman, S.U.S. Choi, S. Li, W. Yu, L.J. Thompson, Anomalously increasedeffective thermal conductivity of ethylene glycol-based nanofluids containingcopper nanoparticles, Appl. Phys. Lett. 78 (6) (2001) 718-720.
5. Y. Xuan, Q. Li, Investigation on convective heat transfer and flow features ofnanofluids, J. Heat Transfer 125 (2003) 151-155.
6. S. Das, Temperature dependence of thermal conductivity enhancement fornanofluids, J. Heat Transfer 125 (2003) 567-574.
7. J. Kim, Y.T. Kang, C.K. Choi, Analysis of convective instability and heat transfercharacteristics of nanofluids, Phys. Fluid 16 (7) (2004) 2395e2401.
8. H.A. Minsta, G. Roy, C.T. Nguyen, D. Doucet, New temperature dependentthermal conductivity data for water-based nanofluids, Int. J. Therm. Sci. 48(2009) 363-371.
9. K.S. Hwang, S.P. Jang, S.U.S. Choi, Flow and convective heat transfer characteristicsof water-based Al2O3 nanofluids in fully developed laminar flowregime, Int. J. Heat Mass Transf. 52 (2009) 193-199.
10. E. Abu-Nada, Rayleigh-Bénard convection in nanofluids: effect of temperaturedependent properties, Int. J. Therm. Sci. 50 (2011) 1720-1730.
11. BuongiornoJ.: Convective transport in nanofluids, ASME Journal of Heat Transfer 128, 240-250(2006).
12. D.Y. Tzou, Thermal instability of nanofluids in natural convection, Int. J. HeatMass Transfer 51 (2008) 2967-2979.
13. D.A. Nield, A.V. Kuznetsov, The onset of convection in a horizontal nanofluidlayer of finite depth, Eur. J. Mech. B Fluids 29 (2010) 217-223.
14. D.A. Nield, A.V. Kuznetsov, The onset of double-diffusive convection ina nanofluid layer, Int. J. Heat Fluid Flow 32 (2011) 771-776.
15. C.M. Vest, V.S. Arpaci, Overstability of a viscoelastic fluid layer heatedfrom below, J. Fluid Mech. 36 (1969) 613-623.
16. M. Sokolov, R.I. Tanner, Convective stability of a general viscoelasticfluid heated from below, Phys. Fluids 15 (1972) 534-539.
17. S. Rosenblat, Thermal convection in a viscoelastic liquid, J.Non-Newtonian Fluid Mech. 21 (1986) 201-223.
18. J. Martinez-Mardones, C. Perez-Garcia, Linear instability in viscoelasticfluid convection J. Phys. Condens. Matter 2 (1990) 1281-1290.
19. R.G. Larson, Instabilities in viscoelastic flows, Rheol. Acta 31 (1992)213-221.
20. J. Martinez-Mardones, R. Tiemann, D. Walgraef, Rayleigh-Benardconvection in binary viscoelastic fluid, Physica A 283 (2000) 233–236.
21. J. Martinez-Mardones, R. Tiemann, D. Walgraef, Amplitude equation forstationary convection in a binary viscoelastic fluid, Physica A 327 (2003)29–33.
22. D. Laroze, J. Martinez-Mardones, J. Bragard, Thermal convection in arotating binary viscoelastic liquid mixture, Eur. Phys. J. Spec. Top. 146(2007) 291–300.
23. D. Laroze, J. Martinez-Mardones, J. Bragardc, C. Peirez-Garcia, Realisticrotating convection in a DNA suspension, Physica A 385 (2007)433–438.
24. D. Yadav, G.S. Agrawal, R. Bhargava, Thermal instability of rotating nanofluid layer, International journal of Engineering Sciences, 49 (2011) 1171-1184.
25. L. J. Sheu, Linear stability of convection in a viscoelastic nanofluid layer, World academy of science, Engineering and Technology, 58, (2011) 289-295.
26. R. Chand, G. C. Rana, On the onset of thermal convection in rotating nanofluid layer saturating a Darcy-Brinkman porous medium, Int. J. Heat Mass Transf. 55, (2012) 5417-5424.
27. U. Gupta, J. Ahuja, R.K. Wanchoo, Magneto convection in a nanofluid layer, Int. J. Heat Mass Transf. 64, (2013) 1163-1171.
28. S. Rionero, Triple diffusive convection in porous media, Acta Mech. 224 (2013) 447-458.
29. Z. H. Khan, W. A. Khan, I. Pop, Triple diffusive free convection along a horizontal plate in porous media saturated by a nanofluid with convective boundary condition, Int. J. Heat Mass Transf. 66, (2013) 603-612.