A Note on Group Rings which Are F-Rings

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The author in [1] calls a ring R to be a F-ring if there is a finite set X of non zero elements in R such that

$aR \cap X \neq \phi$ for any non-zero a in R. If in addition X is contained in the centre of R; R is called an FZ-ring. In this note we obtain conditions under which a group ring is a F-ring. For more about F-rings please refer [1]. Throughout this paper RG (or FG) denotes the group ring of the group G over the ring R(or the field F).

Example 1: let $Z_2 = (0,1)$ be the field of characteristic 2 and $G = \langle g/g^2 = 1\rangle$. Then the group ring Z_2G is a F-ring. For take $x = \{1+ g\}$ Z_2G. Clearly a. $Z_2G \cap X \neq \phi$ for any non –zero a in R. Trivially Z_2G is a FZ-ring.

Example 2: let $G = \langle g/g^3 = 1\rangle$ and $Z_2= (0,1)$. Then Z_2G is a F-ring (FZ-ring) with $X = \{1+g, 1+g^2, g+g^2, 1+g+g^2\}$.

Example 3: let $Z_2 = (0,1)$ and

$S_3 = \{ 1 = p_0 = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$ $p_1 = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$ $P_2 = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$ $p_3 = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$ $p_4 = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$

$P_5 = \begin{pmatrix}1 & 2 & 3\end{pmatrix}$. $\}$

Be the symmetrical group of order 3.Z_2S_3 is a F-ring but Z_2S_3 is not a FZ-ring. For take X ={elements taken two time (i.e. $p_i + p_j$), element taken four at a time (i.e. $p_i + p_j + p_k + p_r$) and $(1 + p_1 + p_2 + p_3 + p_4 + p_5)$}. Clearly $\alpha Z_2S_3 \cap X \neq \phi$ for any non-zero α in Z_2S_3. Since S_3 is non commutative and X is not in the center of Z_2S_3 ; Z_2S_3 is not a FZ- ring.

THEOREM 1: Let $Z_2 = (0,1)$ be the field of characteristics two and S_n be the symmetric group of degree n. Then the group ring Z_2S_n is a F-ring.

PROOF: Take

$$\left\{ \quad = \quad = \sum_{\substack{\in \\ || \\ =}} \quad \leq \ \leq ! \right\}$$

Clearly for every non zero a $\in Z2Sn; aZ2Sn \cap X \neq \phi$. Hence Z2Sn is a F-ring.

THEOREM 2: let $Z_2 = (0,1)$ and G be any finite group . Then the group ring Z_2G is a F-ring.

PROOF: For take

$$\left\{ \quad = \quad = \sum / \in \quad | \quad | = , \quad \leq \quad \leq \right\}$$

if the order of G is even; adjoin to X the element $\acute{\alpha} = \sum$; ∈ h | ά| =order of G if order of G is odd. Clearly aZ2G∩ ≠ ∈ 2G.

REMARK: If order of G is infinite or G is torsion free Z_2G need not be a, F-ring.

THEOREM 3. Let G be a finite group and K be a field of characteristic zero. Then the group ring KG is an F-ring.

PROOF: Take X= { =∑ | ∈ ≠ , ≤ ≤ , || =∑ = }

Clearly X is a finite subset of KG with 0 KG∩ ≠ for any non zero a ∈.

REMARK: if the order of G is infinite and G is torsion free we cannot conclude KG to be a F-ring.

THEOREM 4: Let Zp = (o,1,….,p−1) be the field of characteristic p, p>2; p a prime and G be a finite group of order n. If p/n then Z_pG is a F-ring.

PROOF: Take X = { = ∑ | ∈ , ∈ ≠ ∑ = }

Clearly X is finite and a. Z_pG ∩ ≠ .

PROBLEM: if pxn we are not able conclude anything completely.

REFERENCE:

1. Chen, Jain Long, Zhao Yong Gan, A note on F-rings, J.Math. Res. Exp., 9, No. 1,317-318 (1989).