STUDY ON REED-MULLER CODES ACHIEVE CAPACITY USING ALGEBRA

[1]D Gomathi, [2]R Vanishree ,[3]CDeepika [1]Research scholar, Department of Mathematics, SPIHER , Avadi, Chennai-54[2]AssistantProfessor, Department of Mathematics, SPIHER, Avadi, Chennai-54. [3]Assistant Professor, Department of Mathematics,Pachayappas Women College kancheepuram.

ABSTRACT

An important consequence of this result is that a sequence of Reed-Muller codes with converging rate achieves capacity and increasing block length. This case has been suggested previously in the literature, but it has only been proven for cases where the limiting code rate is 0 or 1. The technique applies to any sequence of linear codes where the block lengths are strictly increasing, the code rates converge, and each code of permutation group is doubly transitive.

Keywords

Encoding algorithms,Monotone Boolean functions, Reed-Muller codes.

INTRODUCTION

Overview

In the introduction of channel capacities by Shannon in his seminal paper [1] theorists have been interested extremely by the idea of constructing codes that achieving capacity (Ex., under optimal decoding). Ideally, one would also like these codes to have low-complexity encoding/decoding algorithms, algebraic or and deterministic constructions geometric structure.

The polar codes [2] were the first codes proven to achieve capacity with low-complexity encoding and decoding algorithms. The polar codes receive some structure from the Hadamardmatrixin addition (Hadamard, is a square matrix a rows are mutually orthogonal and whose entries are either +1 or −1)and also have a deterministic construction.

This article considers the performance of structured and deterministic binary linear codes transmitted over the BECunder bitwise maximum-a-posteriori (MAP) decoding. In particular, our primary technical results was “*the sequence of linear codes achieves capacity on a memory less erasure channel under bit-MAP decoding if its block lengths are strictly increasing, its code rates converge to some* $r\epsilon(0, 1)$*, and the permutation group1 of each code is doubly transitive*”.

The discovery of polar codes, it was unclear whether or not codes with a simple deterministic structure could achieve capacity [3,4,5]. Even though polar codes derive from the Hadamard matrix as well as Reed-Muller codes, the ability of polar codes to achieve capacity appears unrelated to the inherent symmetry of this matrix. In contrast, the performance guarantees obtained here are a consequence only of linearity and the structure induced by the doubly-transitive permutation group.

Reed-Muller Codes

Reed-Muller codes were introduced by Muller[6].For integers v, n satisfying $0 \le v \le n$, a binary Reed-Muller codeRM (v, n) is a linear Code of length 2^n and dimension $\binom{n}{0}$+………+$\left(\binom{n}{v}\right)$. It is well known that the minimum distance of this code is 2^{n-v}.[7,8,9]Thus, it is impossible to simultaneously have a non-vanishing rate and a minimum distance that scales linearly with block length.

Reed-Muller codes to correct almost all erasure patterns up to the capacity limit. Muller codes remain an active area of research in theoretical computer science and coding theory. The early works [10.11.12] in culminated in obtaining asymptotically tight bounds for their weight distribution for the case of fixed order vand asymptotic n.[13]

PRELIMINARIES (Basic Setup and Notation)

Encoder

Encoder is a device to change something into a system for sending messages secretly, or to represent complicated information in a simple or short way for the purpose of standardization, compression or speed.

Decoder

A device capable of converting audio or video signals into a different from, for example from digital to analogue. Decoding is the reverse of encoding. It converts files to their original states and communication transmissions of encoded data

RthOrder Reed Mullercodes

the 0^{th} order Reed Muller code R(0,m) is defined to be the repetition code {0,1} of length 2^m, For any r>=2

The 1 th order Reed MullercodeR(1,m) are binary codes defined for all integers m>1,

i) R (1, 1) = {00,01,10,11}

ii) for m>1,

R(1,m) = {(u,u),(u,u+1):u *ϵ R(1,m-1)and 1= all 1 vector*}.

For example

R (1, 2) = {**00**00, **0101, 1010, 1111, 0011, 0110, 1001, 1100**}

A linear code is proper if no codeword position is 0 in all codeword. In the following, all codes are understood to be proper binary linear codes with minimum distance at least 2. Let C denote an (N,K) binary linear code with length N and dimension K.

The rate of this code is given by r∆ K/N.We say that a sequence a covers a sequence b, namely b, if a ≥ b, if ai ≥bi for all i. Denote [N] ∆{1,….,N} . A set A ⊂ [N] is said to cover a sequence a∈{0,1}^N .if the set of non-zero indices of a is a subset of A. Let π be a permutation on N elements, i.e., π: [N] → [N],a bisection. Let x be a vector with components indexed by [N]. The abusing notation, we will also let π (x) denote a length-N vector, say z, with components satisfying zπ(i) = xi.

Capacity –Achieving Codes

Suppose $\{C_n\}$ is a sequence of codes with rates $\{r_n\}$where $r_n \to r$ for $r \in (0, 1)$.

a) $\{C_n\}$ is said to be capacity achieving on the BEC under bit-MAP decoding, if for any $p \in [0, 1 - r)$, the average bit erasure probabilities satisfy lim n→ $\infty P_b^{(n)}(p)=0$.

b) {Cn} is said to be capacity achieving on the BEC under Block-MAP decoding, if for any p *ϵ[0,1-r))*, the average Block erasure probabilities satisfy lim n→ ∞ $P_B^{(n)}(p)=0$.

ACHIEVE CAPACITY

General Results

Let as measure µp on {0, 1} M such that

$$\mu_p(\Omega) = \sum_{\underline{a}\in\Omega} p^{|\underline{a}|}(1-p)^{M-|\underline{a}|}, \quad \text{for } \Omega \subseteq \{0,1\}^M.$$

One important result in the theory of Boolean functions is that symmetric monotone sets always exhibit a sharp transition [30], i.e., µp(Ω) transitions quickly 0 to 1 as discussion in this paper, we show that a sequence of binary linear codes achieves capacity if its block lengths are strictly in- creasing, its code rates converge to some r ∈ (0, 1), and Each code of the permutation group is doubly transitive. The straightforward approach leads to the analysis of functions that are neither Boolean nor monotonic.

Conclusion

We conclude that the theory, the results and the application obtained in this dissertation are derived and discussed by reed muller codes that increasing block length and converging rate achieves capacity the limiting code rate was 0 or 1.

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