OPTIMIZATION OF PRESTRESSED CONCRETE BEAM

***Shaikh Mubashshir Ahemad Shaikh Maheboob*[1]*, S.P. Nirkhe*[2]**

[1]P.G. Student (M.Tech- Structures), Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Dr. *Babasaheb Ambedkar Technological* University, Aurangabad, Maharashtra, 431005, India

[2]Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Dr. *Babasaheb Ambedkar Technological* University, Aurangabad, Maharashtra, 431005, India

Abstract

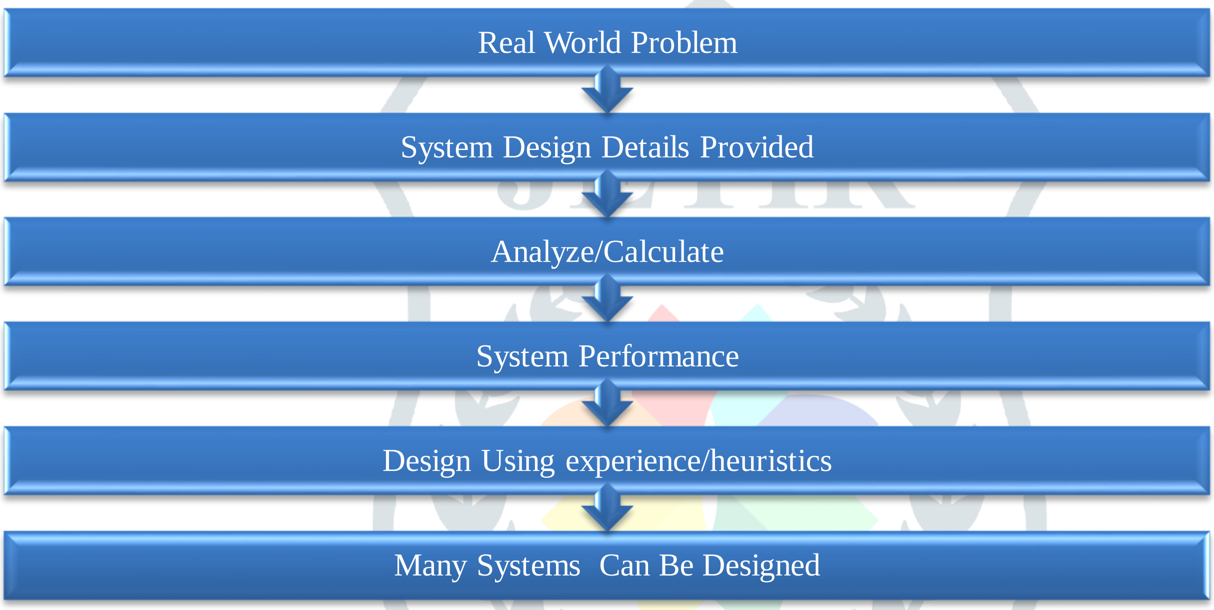
The primary and main objective of the structural designing is to achieve safety and economy, the conventional design methods achieve the safety factor but it is not that much easy to achieve both the objective at the same time so the optimizations techniques are developed by researchers to achieve the safety and economy. In most of the studies the optimization techniques are used to minimize the cost. In this paper the optimization technique is used to obtain the optimum weight of the prestressedconcrete beam by using genetic algorithm. I-section beam is used in this paper and design for optimum weight. Generally the idealized I-section used in previous studies due to simplicity of section in this study the general I-section beam is used with 8 design variables and 13 constraints. The computer program for genetic algorithm optimization method can assist the designer to achieve optimum values of design variables rapidly and easily.

Key words: *Prestressed concrete beam, Idealized I-section beam, Generalized I-section beam, Optimization, Genetic Algorithm, weight optimization and cost optimization*

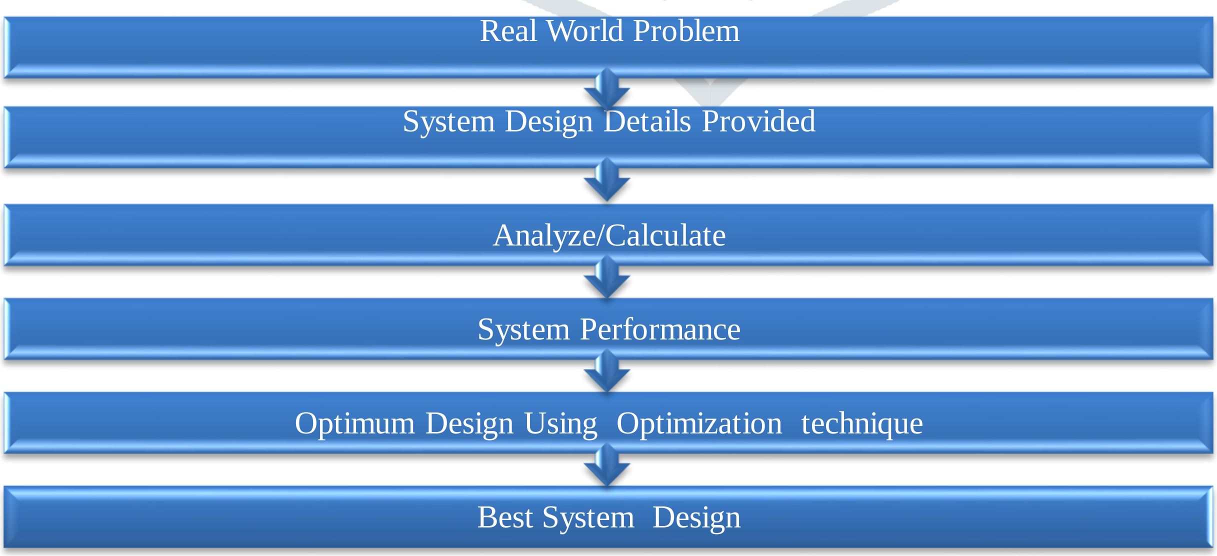
1. Introduction

Modern techniques are developed in structural engineering for analysis, design, fabrication, research and development. Economy is the main factor in structural designing but safety also plays the vital role in structural engineering. Design is the important field that has been developed and used for centuries. Automobiles, airplane, highways, railways, bridges, and other complex systems are excellent testimonials. In bridge construction prestressed concrete is used because prestressed members are light and best suited for architectural treatment. The treatment of prestressing eliminates cracking of concrete. Vibration and shock, impact and bearing capacity of structure to reversal stress are occurs due to cracks these cracks are removed by prestressing of concrete. Prestressing reduces maintenance cost also provides a smoother deck for high speed driving, this technique increases shear capacity of concrete. Conventional design method proposes a certain solution that related with mathematical analysis in order to verify that the problem requirements or specification are satisfied. But conventional methods are not that much efficient this method required time and it’s not feasible for complex problems. There is no formal way to reach best design in the sense of minimizing weight, cost and volume.

Optimum design method is alternate for conventional design method. Optimum structural design has importance and gives the motivation to all designers to achieve the optimal product in terms of weight, reliability, cost or the combination of these factors. Many of analytical studies of optimum design of prestress beam have been mentioned in literature (Kirsh 1972, Brown 1975, Jones 1985, Cohn and MacRae 1987, Fereig 1994). In these studies the traditional methods are used as optimization techniques. Modern optimization method and the conventional method of design are studied in this paper. In present study genetic algorithm optimization method is used to optimize the prestressed concrete beam problem. In this paper the, the proposed method is applied to example problem available in (Cagatay 1996) and (Taylor and Amirebrahimi 1987) and results are compared,results obtained by conventional method are also taken into account.



a) Conventional design process



b) Optimum design process

2. Optimization Method

Genetic Algorithm was developed by Holland in 1975; this method is based on principles inspired by natural evolution. The basic elements of natural genetics—reproduction, crossover, and mutation—are used in the genetic search procedure. GAs differs from the traditional methods of optimization. In genetic algorithm a population of points is used for starting the procedure. As we take design variables of n numbers then the population size is taken as 2n or 4n. The design variables in genetic algorithm are represented as a string in binary variables that correspond to chromosome in natural genetics. The string length can be made large enough to achieve any desired fitness of approximation and thus any desired accuracy can be achieved. The numerical value of objective function corresponds to concept of fitness in genetics. After trial solutions are selected, a new generation (a new set of string) is produced by selecting, using stochastic principles, the fittest parents to produce "children" from among the trial solutions. In each "child" crossover or exchange of portion of string of each of the two parents generates new solutions. Some random alteration of binary digits in a string reproduces the (advantageous and disadvantageous) effect of mutations. A schematic diagram of genetic algorithm used is as follows.

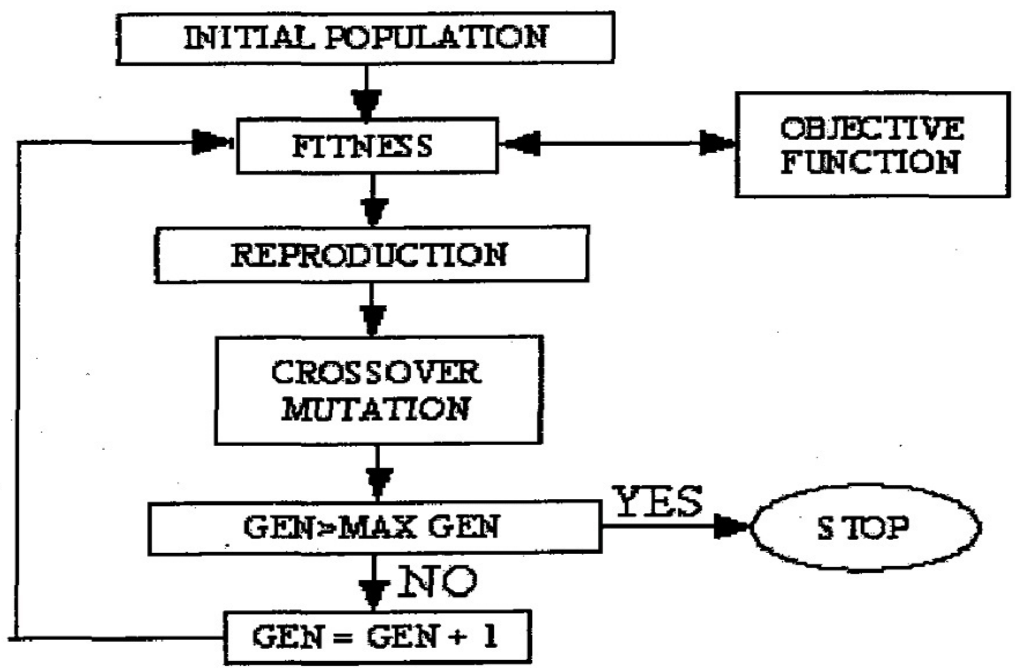


Fig.1 Outline of a genetic algorithm

In genetic algorithm the transition stages from one generation to next generation made of four components

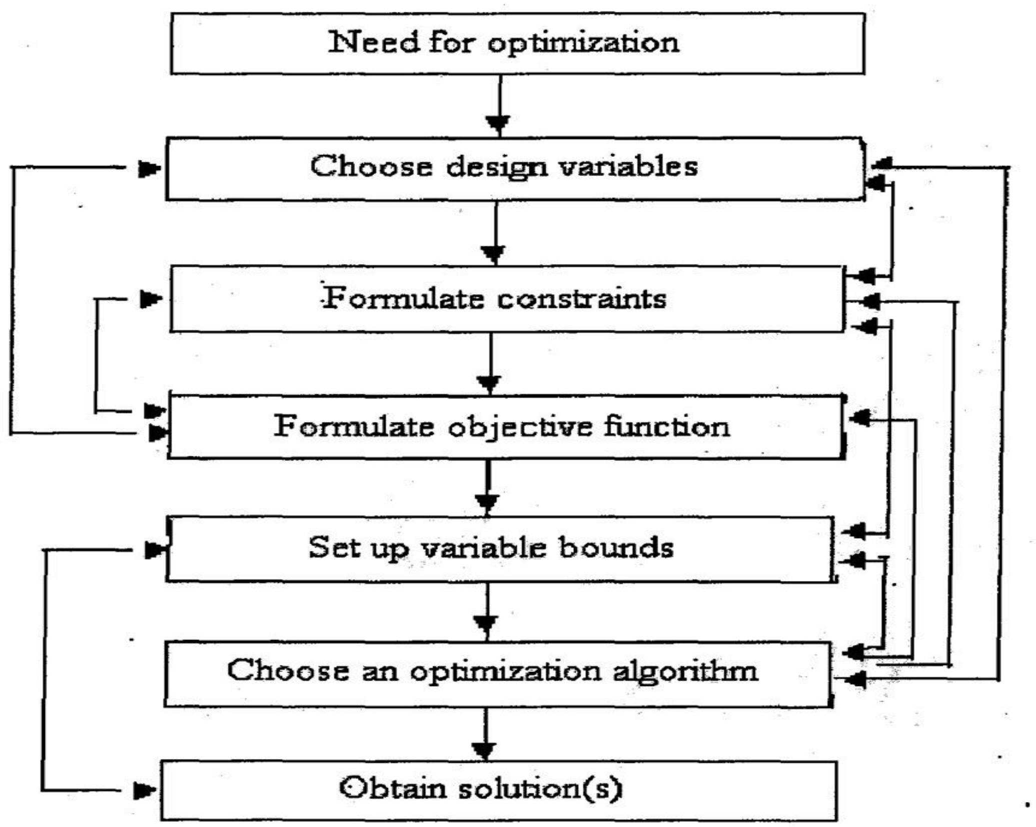
Selection: the individuals (strings) for reproduction are selects on basis their fitness (objective function value).

Crossover: two individual's genetic information merging together is nothing but the crossover. Proper selection of coding produces good children from two good parents.

Mutation: it is a genetic operator used to maintain genetic diversity from one generation of population of genetic algorithm chromosomes to the next.

Sampling: the procedure of computation of a new generation from the former one and its brood.

Steps involved in optimal design process are given as below flowchart



3. Problem formulation

The formulation of design problem requires design variables that describe the structure, and the constraints that must satisfied. The design variables, parameters and constraints should be expressed in suitable format for application optimization technique to find the optimum design of prestressed concrete beam.

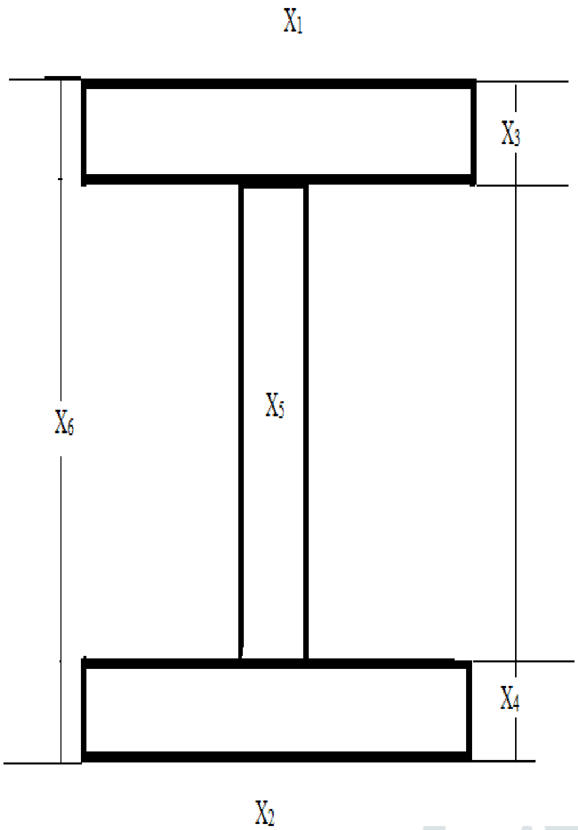


Fig.2 Idealized I-section

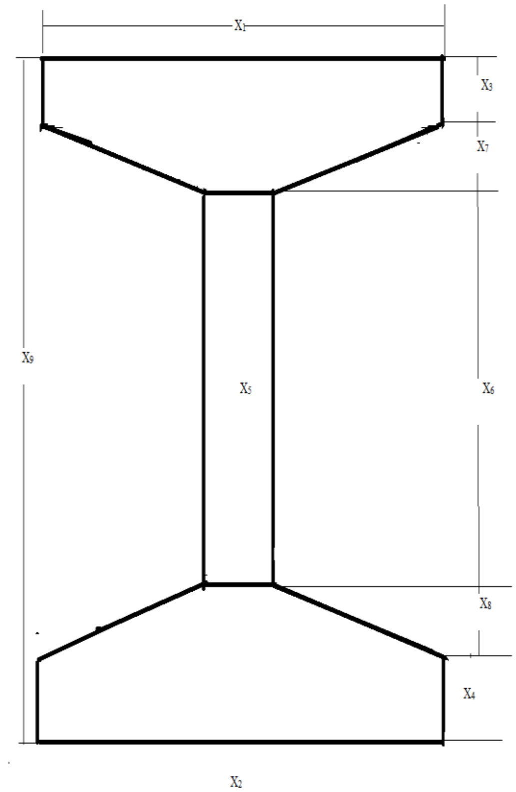


fig.3 General I-section

3.1 Design variables

Most of the research are done on optimization of prestressed beam which are deal with the idealized I section beam as shown fig. In this paper the method has been formulated for general I shaped cross-section beam and design variables are denote by $X_1$to X_8as shown in figure. Variable X_7 and X_8 are increased by 20% of flange depth since the slopping flanges are used [Krishna raju]. Additional variables are also considered which are eccentricity and prestressing force.

Table1. Design Variables

Design Variables	**Explicit Constraints**
Girder Depth (D) (mm)	$800 \leq D \leq 1000$
Width of top flange (btf) (mm)	$300 \leq btf \leq 800$
Thickness of top flange (ttf) (mm)	$75 \leq ttf \leq 300$
Width of bottom flange (bbf) (mm)	$300 \leq bbf \leq 800$
Thickness of bottom flange (tbf) (mm)	$75 \leq tbf \leq 300$
Width of web (bw) (mm)	$50 \leq bw \leq 300$

3.2 Objective Function

In this optimization study, objective function is the minimization of weight of prestressed girder by minimizing the cross-sectional area of girder.

Minimize $f(x) = A\rho L$

Subjected to $g(x) \leq 0,$

$X \geq 0$

Where f(x) is the objective function it means the self-weight of beam g(x) is constraint, L is the length of beam or girder A be the area of cross-section ρ be the density of concrete. The objective function is to be represented in the form of design variables. Aim of objective function is to minimize the cross-sectional area of prestressed concrete and find the minimum prestressing force for maximum eccentricity. The following constraints are considered.

1) Flexural stress constraints 2) Prestressing force and eccentricity
3) Cross-sectional dimension 4) Ultimate moment

1) Flexural stresses

There are two stages of loading the one is at transfer stage of prestressing force to the beam and is can be computed by following equations at top and bottom respectively.

$$f^{L} \leq f \leq f^{U}$$

f^{L} is allowable compressive stress (lower limit), $f^{U}$is the allowable tensile stress (upper limit) and f is actual stress in concrete

$$\frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z}$$

$$\frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

Where, P is the prestressing force A be the area of cross-section e be the eccentricity M be the ultimate moment and Z be the section modulus.

Flexural stresses at working stage are computed by following equations at top and bottom respectively.

$$\frac{P\alpha}{A} - \frac{Pe\alpha}{Z} + \frac{M}{Z}$$

$$\frac{P\alpha}{A} + \frac{Pe\alpha}{Z} - \frac{M}{Z}$$

Where α is the loss ratio.

2) Prestressing force and eccentricity

Prestress can lie between an upper and lower limit. Value of prestress within limit may be used safely used without exceeding permissible stresses at extreme fibers. The minimum prestressing force required will be obtained by maximum tensile prestress as shown by below equation 2.1 at the top fiber and the minimum compressive prestress, indicated by equation 2.2 corresponding to bottom fiber.

$$fsup \geq ftt - \frac{Mg}{Zt}$$

$$finf \geq \frac{ftw}{\alpha} + \frac{Mq + Mg}{\alpha Zb}$$

Where,

$$fsup = \frac{P}{A} - \frac{Pe}{Zt} \quad and \quad finf = \frac{P}{A} + \frac{Pe}{Zb}$$

We have the equation for the minimum prestressing force by eliminating e from the equations,

$$P = \frac{A(finf\ Zb + fsup\ Zt)}{(Zt + Zb)}$$

Similarly, eliminating P from the equations, we get corresponding eccentricity as follows

$$e = \frac{Zt\ Zb(finf - fsup)}{A(fsup\ Zt + finf\ Zb)}$$

3) Cross-sectional Dimensions

Cross-sectional dimensions cannot exceed aspect ratio the constraints for cross-sectional dimension as

$$(Xi)min \leq Xi \leq (Xi)max$$

(i = 1,2,3…..n) where n = number of variables

4) Ultimate Moment

The ultimate moment constraint for the section considered as

$$0 \leq M \leq Mu$$

Where, M = moment required Mu = Ultimate moment

4. Numerical example

The example represents the optimization of simply supported beam for least weight design of concrete is based on example first discussed by Khachaturain and Gurfinkel (1969), and later by Taylor and Amirebrahimi (1987), and then Ismail H. Cagatay, CengizDundar and OrhanAksogan (2000).

Example

A prestressed beam of simply supported of length 16460mm span subjected to live load of 23.34 kN/m assume the unit weight of concrete is 24kN/m^3, permissible stress for compression at transfer stage is 16.55 Mpa, at service stage 15.51 Mpa, and for tension at transfer -1.31 Mpa and at service stage -2.93 Mpa, the loss ratio is 0.85 characteristics strength of concrete is fc = 40 Mpa, fpu = 1862 Mpa clear cover= 50 mm. the overall depth of beam should not exceed 1000mm. find the minimum cross-section of beam.

This example is firstly discussed by Khachaturain and Gurfinkel (1969), and then Taylor and Amirebrahimi (1987). Value of permissible stresses are 16.55 in compressions and -1.31 in tension at transfer stage are used by Khachaturain and Gurfinkel (1969). 13.24 Mpa and -0.98 Mpa in compression and tension respectively at transfer stage are used by Taylor and Amirebrahimi (1987). Ismail H. Cagatay, CengizDundar and OrhanAksogan (2000) used constraint for section depth that does not exceed 914.4mm which is not considered by previous two authors.

This numerical procedure is converges to optimum solution in 50 iterations and the solution of Ismail H. Cagatay, CengizDundar and OrhanAksogan gives in 9 iterations so accuracy to get optimum solution will be less as compared to the present study.

Table.2 Variables, results

Variables	Initial values (mm)	Optimum Values (mm)	Maximum Values (mm)
X1	300	600	750
X2	300	600	750
X3	75	200	300
X4	75	200	300
X5	50	50	300
X6	950	1000	3000

Table.3 optimum results of stresses at transfer and service

Stage	Location	Stress (Mpa)	Permissible stress (Mpa)
Transfer	Top	6.209	16.55
	bottom	-1.747	-1.31
Working	Top	6.7336	15.51
	Bottom	-2.93	-2.93

Table.4 optimum prestressing force, eccentricity and cross-sectional area

Prestressing force P (kN)	Eccentricity e (mm)	Cross-sectional area A (mm^2)
605.61	976.732	270000

Table.5 Results by conventional method

Stage	Location	Stress (Mpa)	Permissible stress (Mpa)
Transfer	Top	4.526	16.55
	Bottom	-1.486	-1.31
Working	Top	5.508	15.51
	bottom	-2.92	-2.93

Table.6 Comparison of the results

		X1 (mm)	X2 (mm)	X3 (mm)	X4 (mm)	X5 (mm)	X6 (mm)	P (kN)	e (mm)	A (mm^2)
Khachaturain and Gurfinkel		457.2	439.4	137.2	152.4	137.2	624.8	1650	360.6	219999
Taylor and Amirch		558.8	457.2	101.6	177.8	101.6	660.4	1673	370.8	205160
Ismail H. Cagatay, CengizDundar and OrhanAksogan		552.3	463.7	111.5	137.5	101.6	664.6	1447	406.7	192876
Present study	Optimization	600	600	200	200	50	600	605.61	976.732	270000
	Conventional	590	590	290	290	230	650	747.384	1460.3	491700

As we compare above results the conventional method gives the larger cross-sectional area as compare to optimization method. The solutions obtained by Ismail H. Cagatay, CengizDundar and OrhanAksogan take nine iterations and the optimization by genetic algorithm take 50 iterations so the accuracy of present optimization technique will increase and get the most accurate values of design variables. In case of the prestressing force the prestressing force in present study is less than the previous studies and also less than the conventional method. The eccentricity of this study is maximum as compare to previous studies.

5. Conclusion

As we observe the results the cross-sectional area obtained by genetic algorithm is optimum and less than the conventional method by 43% and optimization technique required 50 iterations so it gives the better accuracy than the previous methods used. By obtaining optimum values of design variables we get the minimum weight and we achieve the objective function. The weight of the prestressed beam is get minimized so the cost of the section is also get minimizes and achieve the one more important objective of minimum cost.

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