A STUDY ON VARIOUS TYPE OF CORDIAL LABELING

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Abstract : Graph Theory deals with the study of problems involving discrete arrangement objects, where concern is not with the internal properties of the objects, but the relationship among them. A magic graph is a graph whose vertex are labeled and edge are labeled by integers, so that the sum over the edges incident with any vertex is the same, independent of the choice of vertex, or it is a graph that has such a labeling.

IndexTerms - **Cordial Label, Properties, Vertex, Graph**

I. INTRODUCTION

In recent years, due to the extension of the concepts and applications of the graph theory, many journals such as Journal of Graph Theory, Journal of Combinatorial Theory A & B, Discrete and Applied Mathematics, SIAM Journal of Discrete Mathematics, European Journal of Combinatorics, and Graphs and Combinatorics are being published to cover the advances made in this field.

The city of Konigsberg was located on the Pregel river in Prussia. The river di-vided the city into four separate landmasses, including the island of Kneiphopf. These four regions were linked by seven bridges as shown in the diagram. Res-idents of the city wondered if it were possible to leave home, cross each of the seven bridges exactly once, and return home. The Swiss mathematician Leon-hard Euler (1707-1783) thought about this problem and the method he used to solve it is considered by many to be the birth of graph theory.

II. PRELIMINARIES

All graphs considering in this thesis are simple and undirected graph.

Let G = (V, E) where V is the vertex set and E is the edge set. Unless otherwise mentioned the cardinality of the vertex set is denoted by n and that of the edges by m. Let us give the basic definitions which will be used in this paper
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Definition : 1.1

A graph G is a triple consisting of a vertex set V (G), an edge set E(G), and a relation that associates with each edge, two vertices called its endpoints.

Definition : 1.2

Graphs for which V (G) and E(G) are finites sets, then the graph is finites.

Definition : 1.3

The two endpoints of an edge are the same vertex. This is called a loop.

Definition : 1.4

More than one edge shares the same set of endpoints is called multiple edges.

Definition : 1.5

A simple graph is a graph having no loops or multiple edges. In this case, each edge e in E(G) can be specified by its endpoints u; v in V (G). Sometimes we write e = uv.

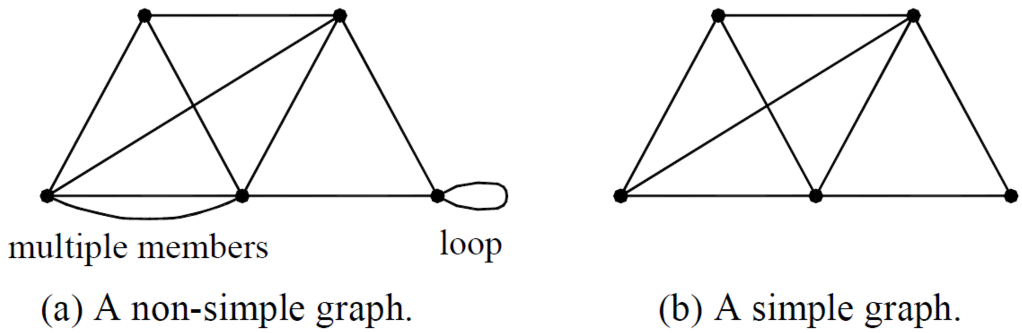

Figure : 1

Definition : 1.6

The degree of a vertex v in a graph G, denoted deg v, is the number of edges in G which have v as an endpoint.

Definition : 1.7

A path is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the ordering. A path which begins at vertex u and ends at vertex v is called a u, v-path.

Definition : 1.8

A cycle is a simple graph whose vertices can be cyclically ordered so that two vertices are adjacent if and only if they are consecutive in the cyclic ordering.

Definition : 1.9

A subgraph H of a graph G, is a graph such that V (H) $\subseteq$ V (G) and E(H) $\subseteq$ E(G) satisfying the property that for every e $\in$ E(H), where e has endpoints u, v $\in$ V(G) in the graph G, then u, v $\in$ V (H) and e has endpoints u, v in H, i.e. the edge relation in H is the same as in G.

Definition : 1.10

Graph H = (W, F) is a subgraph of graph G = (V, E) if W $\subseteq$ V and F $\subseteq$ E. (Since H is a graph, the edges in F have their endpoints in W.) H is an induced subgraph if F consists of all edges in E with endpoints in W. See figure 2. Whenever U $\subseteq$ V we denote the induced subgraph of G on vertices U as G[U].

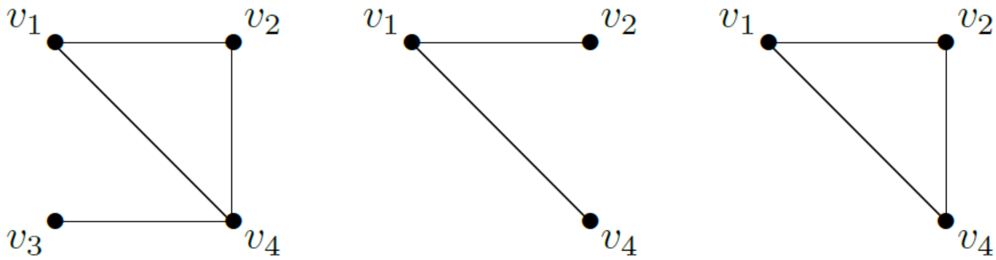

Figure 2 Left to right: a graph, a subgraph, an induced subgraph.

Definition 2.1

A Prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f:V(G) \rightarrow \{1,2,\ldots, |V(G)|\}$ such that each edge uv is assigned the label 1 if gcd $(f(u), f(v))$ =1 and 0 if gcd $(f(u), f(v))$ >1, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. A graph which admits prime cordial labeling is called prime cordial graph.

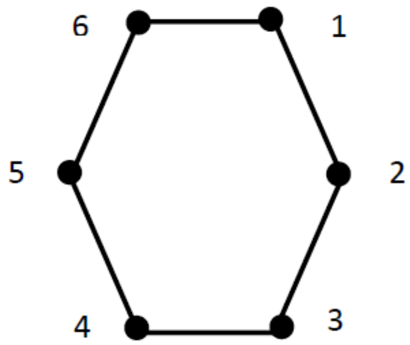

Figure 1 : Cycle G_6

Definition : 2.2

The union of two graphs $G_1 = (V_1, E_1)$ and $G_1 = (V_2, E_2)$, written $G_1 \cup G_2$, is the graph with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and the edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

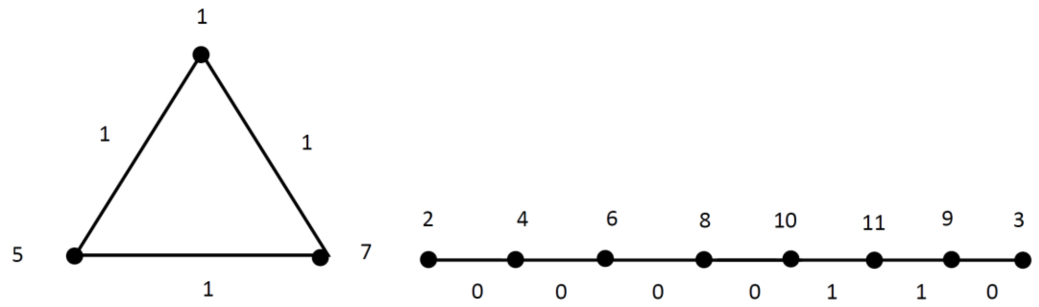

Figure 2 : Disconnected graph $C_3 \cup P_8$

Definition : 2.3

Fish graph is a graph with six vertices and seven edges.

Definition : 2.4

Butterfly graph is a graph with five vertices and six edges

Definition : 2.5

P_n is a path of length n – 1

Definition : 2.6

The corona = $G_1 \Theta G_2$ of two graph G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2, The Graph $P_n \Theta K_1$ is called a Comb

Definition : 2.7

(Pn^2) is a path of length n-1 of twice

Definition 2.8

Star of length one is joined with every vertex of a path P_n through an edge. It is denoted by $P_2 : S_n$

Definition 2.9

The join of G_1 and G_2 is the graph $G = G_1 + G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2 \cup \{UV : u \in V_1, v \in V_2\}$. The graph $P_n + K_1$ is called a Fan.

Theorem 2.10

$C_m \cup P_n$ is prime cordial if $m \leq 5$ is odd and $n \geq 6$.

Proof:

Let $G = (V, E, f)$ be a disconnected graph $C_m \cup P_n$ with order $p = m+n$ and size $q = m+n-1$. Here $u_1, u_2, .., u_m, v_1, v_2, \ldots, v_n$ are the vertices where $u_1, u_2, \ldots, u_m$ be the vertices of the cycle C_3 and $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n. Then the edges e_i, e_{ij}, are defined as $e_i = (u_i, u_{i+1})$ and $e_{ij} = (v_i, u_{i+1})$. Define the function $f : V(G) \rightarrow \{1, 2, .., m+n\}$ as follows,

Case (i) $C_3 \cup P_n$

If n is even,

$f(u_1) = 1$, $f(u_2) = 5$, $f(u_3) = 7$, $f(v_n) = 3$, $f(v_{n-1}) = 9$.

$f(v_i) = 2i$, $\quad$ $i = 1.2, \ldots, \dfrac{n+2}{2}$

$f(v_i) = f(v_{i-1}) + 2$, $i = n-2, n-3, \ldots, \dfrac{n+4}{2}$.

If n is odd,

$f(u_1) = 1, f(u_2) = 5, f(u_3) = 7, f(v_n) = 3, f(v_{n-1}) = 9$.

$f(v_i) = 2i$, $\quad$ $i = 1, 2 \ldots, \dfrac{n+3}{2}$.

$f(v_i) = (v_{i-1}) + 2$ $\quad$ $i = n-2, n-3, \ldots, \dfrac{n+5}{2}$

Case (ii) $G_5 \cup P_n$

If n is even.

$f(u_1) = 1$ $f(u_2) = 3$, $\quad$ $f(u_3) = 9$.

$f(u_{i+2}) = 2i+1$ $\quad$ $i = 2.3$.

$f(v_i) = 2_i$, $\quad i = 1, 2\ldots., \frac{n+4}{2}$.

$f(v_n) = 11$.

$f(v_i) = f(v_{i+1}) + 2$, $\quad i = n-1, n-2, \ldots, \frac{n+6}{2}$.

If n is odd.

$f(u_1) = 1$, $f(u_2) = 3$, $f(u_3) = 9$

$f(u_{i+2}) = 2i+1$, $\quad i = 2.3$.

$f(v_i) = 2i$, $\quad i = 1, 2, \ldots, \frac{n+5}{2}$.

$f(v_n) = 11$.

$f(v_i) = f(v_{i+1}) + 2$, $i = n-1, -2, \ldots, \frac{n+7}{2}$.

Then the above function f admits the prime cordial labeling. Hence $C_m \cup P_n$ are prime cordial labeling. The generalized graph of $Cm \cup Pn$ is shown in figure 26.

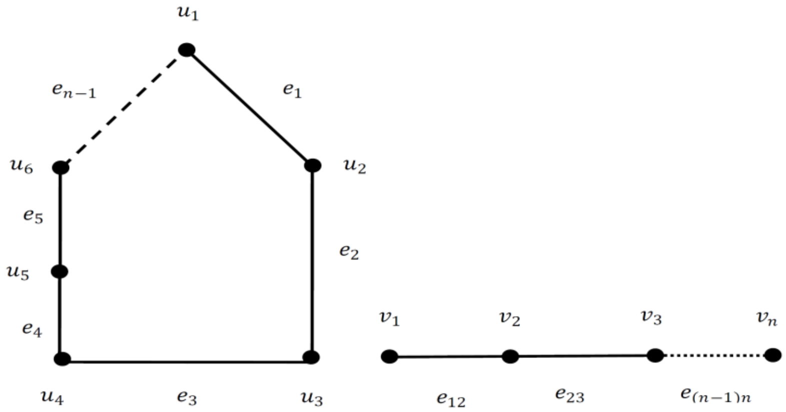

Figure 3: Disconnected graph $C_m \cup P_n$.

III. CONCLUSION

In graph theory, graph labeling plays a vital role by using graph labeling we can easily understand the graph. Most of the graph labeling problems have three ingredients.

1. A set of number S from which the labels are chosen
2. A rule that assigns a value to each edge
3. A condition that these values must satisfy

Labeled graphs are becoming an increasingly useful of mathematical model for a broad range of applications. Relatively prime numbers play a initial role in both analytic and algebraic number theory

In this dissertation we discuss about various types of labeling such as prime cordial labeling of wheel related graph families, relatively prime cordial graph, sum, mean, square divisor cordial labeling of graphs and difference cordial labeling of graphs.

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