AN INVENTORY POLICY FOR DETERIORATING ITEMS WITH MULTIVARIABLE DEMAND WITHPERMISSIABLE DELAY IN PAYMENT

[1]Govinda Dhaker [2]Jyoti S Raghav [3]Vipin Kumar
[1]Scholar, [2]Associate Professor, [3]Associate Professor
1, 2 Dept. of Mathematics, Mewar University, Chittorgarh,
3. Dept. of Mathematics, BKBIET, Pilani

Abstract : This study has been undertaken to investigate the determinants of stock returns in Karachi Stock Exchange (KSE) using two assets pricing models the classical Capital Asset Pricing Model and Arbitrage Pricing Theory model. To test the CAPM market return is used and macroeconomic variables are used to test the APT. The macroeconomic variables include inflation, oil prices, interest rate and exchange rate. For the very purpose monthly time series data has been arranged from Jan 2010 to Dec 2014. The analytical framework contains**.**

Keywords: Deterioration, Multivariable Demand, Partial Backlogging, Permissible delay in Payment

I. INTRODUCTION

Every businesses or organization based on demand. It directly affects the Economic Growth of any organization. The success of a business depends on the factor that how efficiently one can fulfil the demand of the customers. In the beginners, model demand was taken as constant, but practically this is not possible. Demand can of different types like short-term demand or long-term demand, Individual or organization demand, market dependent, dependent, independent demand, price dependent or stock dependent etc. Stock dependent demand mainly associated with the stock available in hand. It always affects the popularity or goodwill of a product. The best example can of this can be easily seen in the mall or supermarket. If the stock of any product is high in any supermarket that shows the popularity or high demand of it, on the revert the low level of the product shows the less demand of the particular product. Gupta &Vrat [1986] were the first one, who developed a model for stock dependent demand. This model was further modified by Mandal & Phaujdar [1989] by taking the stock dependent demand for deteriorating items. After these many researchers show their interest in stock dependent demand and gave their work in this. For example, Pal et. al. [1993], Sana &Chaudhary [2004], Giri [1996], Ray et. al. [1998], Ray & Choudhari [2009], Amutha&Chandrasekaran [2013] etc give their different inventory model for Stock dependent demand. Soni& shah [2008] gave their optimal ordering policy for stock dependent demand while changet. al. [2010] generates their optimal replenishment policies for the same. Sana[2012] developed inventorymodel stock dependent demand for perishable products. Gupta et. al.[] developed their model for non-instantaneous products. Vipin & Anupma [2014] developed an inventory model by using preservation technology for deteriorating items with parabolic holding cost. Gopal & Vipin [2017] developed a cost minimization policy for deteriorating in the Inflationary environment with partial backlogging.

In the classical time, the payment of the items was done exactly at the time of delivery or before it. But in the modern era, as the business is getting huge and complex, this practice is not actually possible. Nowadays, the retailer need not clear his dues at the time of delivery. Now Trade Credit is also known as permissible delay in payment, the practice followed by every business. In this, a grace period is provided by the supplier to his retailers to complete the payment. During this phase, the retailer need not pay any interest for the amount. S.K. Goyal [1985] developed an inventory model with the permissible delay in payment. S.P. Agarwal & C.K.Jaggi[1995] developed ordering policies for deteriorating items with the allowed delay in payment. That was further followed S.W. Shinn& Hwang [1996], Jamal, B.R. Sarkar, S. Wang [1997], J.T. Teng[2002], K.J.Chung& Hwang [2003] by developing different models with trade credit. During the last few years, the researchers developed the models with new ideas like Jaggi and Tiwari [2015] focused on price-dependent demand for non-instantaneous under permissible delay in payment. Das and Roy[2015] used the tread credit policy for multi warehouses with inflation. Bhunia et al. [2016] discussed the inventory model for two warehouses via particle swarm. Saxena and Singh [2016] developed a model to show the coordination between vendor and buyer for remanufacturing items. Tiwari et al. [2016] show the impact of tread credit for two warehouses for the non-instantaneous deteriorating items under inflation. Rastogi and Singh [2017] developed a model under tread credit with variable holding cost and cash discount policy. Jaggi et al.[2017] use the credit finance policy for two warehouses in there EOQ model for deteriorating items.

In the present paper, An EOQ model is developed with multivariable demand in which shortages are allowable and partially backlogged. The rate of deterioration is two-parameter Weibull distribution. Also, the holding cost is linear function of time. To increase the demand and profit permissible delay in payment are purposed in this model. Numerical examples are provided to illustrate the proposed model. The objective of this paper is to build an inventory model where the average profit will be maximum.

The rest of the paper is organized as follows: In section II the assumption and notations are described, which are used throughout this research paper In section III the mathematical formulation and solution are established to maximize the total average profit. Section IV provides the numerical example with connectivity demonstration by the diagram. The next section shows the sensitivity with respect to key parameters of the model.

II. ASSUMPTIONS AND NOTATIONS

The following notations and assumptions are used to develop the mathematical model.

1.1. Notations

i. A Ordering cost per order.

ii. C_1 Purchase cost per unit.

iii. C_2 Backordered cost per unit short per time unit.

iv. C_3 Cost of lost sales per unit.

v. T The length of the cycle time.

vi. IM Maximum inventory level during $[0,T]$.

vii. BM Maximum back ordered units during stock out period.

viii. $I = IM + BM$: Order quantity during a cycle of length T .

ix. $I_1(t)$ Level of positive inventory at the time t, $0 \le t \le t_1$

x. $I_2(t)$ Level of negative inventory at the time t, $t_1 \le t \le T$

xi. $P(t_1, s)$: Total profit per unit time.

1.2. Assumptions

i. The demand rate is a function of stock level and selling price considered as

$$D(t,s) = \begin{cases} a + bI(t) - s, & I(t) \ge 0 \\ a - s, & I(t) \le 0 \end{cases}$$, where $a > 0, 0 < b << 1, a > s$ and s is selling price

ii. Holding cost is assumed to be $h(t) = h_1 + h_2 t$, where $h_1 > 0$, and $h_2 > 0$ .which is time-dependent per unit per unit time.

iii. The rate of deterioration is a two-parameter Weibull distribution and is assumed to be $\theta(t) = \alpha\beta t^{\beta-1}$ where the scale parameter is $0 < \alpha < 1$ and the shape parameter $\beta > 0$.

iv. Instantaneous Replenishment rate.

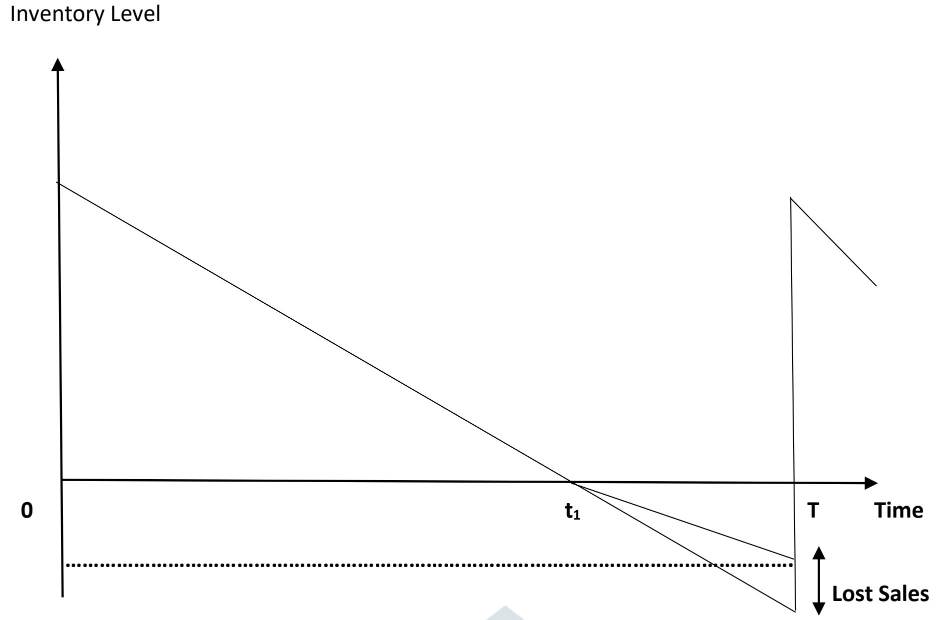
v. Lead time is zero.

vi. The planning horizon is infinite.

vii. For the negative inventory, the backlogging rate $B(t) = e^{-\delta(T-t)}; \delta > 0$ denotes the backlogging parameter and $t_1 \le t \le T$ which is waiting for time length dependent for the next replenishment.

III. MATHEMATICAL FORMULATION AND SOLUTION

Under the above assumption, the inventory system is developed as follows: the total cycle interval $[0,T]$ divided into two subintervals $[0,t_1]$ and $[t_1,T]$. In the first subinterval $[0,t_1]$ shows the decrement in inventory level due to combined effects of demand and deterioration and this decrement reaches to level zero at the time t_1. The second subinterval $[t_1,T]$ demonstrates the shortages which occur from or after time t_1 and due to partial backlogging some demand are lost as shown in fig. 1.



The differential equation (1) shows the state of inventory level at any time t during the subinterval $[0, t_1]$ and the differential equation (2) represent the state of inventory level at any time t during the second subinterval $[t_1, T]$

$$\frac{dI_1(t)}{dt} + \theta(t) I_1(t) = -(a + bI_1(t) - s), \qquad 0 \le t \le t_1 \tag{1}$$

With boundary condition $I_1(t_1) = 0$

$$\frac{dI_2(t)}{dt} = -(a-s)B(t), \qquad t_1 \le t \le T \tag{2}$$

With boundary condition $I_2(t_1) = 0$

The solution of (1) and (2) are given by

$$I_1(t) = (a-s)\left[(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) - bt(t_1 - t) - \alpha t^{\beta}(t_1 - t)\right] \tag{3}$$

$$I_2(t) = \frac{(a-s)}{\delta}\left[e^{-\delta(T-t_1)} - e^{-\delta(T-t)}\right] \tag{4}$$

The maximum positive inventory is

$$IM = I_1(0) = (a-s)\left[t_1 + \frac{b}{2}t_1^2 + \frac{\alpha}{\beta+1}t_1^{(\beta+1)}\right] \tag{5}$$

The maximum back orders

$$BM = -I_2(T) = \frac{(a-s)}{\delta}\left[1 - e^{-\delta(T-t_1)}\right] \tag{6}$$

Hence the order size during $[0, T]$ given by

$$I = (a-s)\left[t_1 + \frac{b}{2}t_1^2 + \frac{\alpha}{\beta+1}t_1^{(\beta+1)} + \frac{1}{\delta}\left[1 - e^{-\delta(T-t_1)}\right]\right] \tag{7}$$

The following cost is calculated for the total profit.

Ordering cost per cycle;

$$OC = A \tag{8}$$

Inventory holding cost per cycle is

$$HC = \int_0^{t_1} (h_1 + h_2 t) I_1(t)\, dt$$

$$=(a-s)\left[\begin{array}{l} h_1 t_1^2\left\{\frac{t_1^{\beta}\alpha\left((\beta+1)^2-\alpha\right)}{(\beta+1)^2(\beta+2)}+\frac{\left(3(\beta+1)+bt_1(2\beta-\alpha+2)\right)}{6(\beta+1)}\right\} \\ +h_2 t_1^3\left\{\frac{t_1^{\beta}\alpha\left((\beta+1)(\beta+2)-2\alpha\right)}{2(\beta+1)(\beta+2)(\beta+3)}+\frac{\left(4(\beta+1)+bt_1(3\beta-2\alpha+3)\right)}{24(\beta+1)}\right\}\end{array}\right] \quad (9)$$

Backordered cost per cycle is

$$BC=C_2\int_{t_1}^{T}-I_2(t)\,dt$$

$$=C_2\frac{(a-s)}{\delta}\left(-(T-t_1)e^{-\delta(t-t_1)}-1+\frac{1}{\delta}e^{-\delta(t-t_1)}\right) \quad (10)$$

Cost due to lost sales per cycle;

$$LSC=C_3\int_{t_1}^{T}\left[1-B(t)\right]D(t)\,dt$$

$$=\frac{C_3(a-s)}{\delta}\left((T-t_1-1)\delta+e^{-\delta(T-t_1)}\right) \quad (11)$$

Purchase cost per cycle is

$$PC=C_1\times I$$

$$=C_1\left[(a-s)\left[t_1+\frac{b}{2}t_1^2+\frac{\alpha}{\beta+1}t_1^{(\beta+1)}+\frac{1}{\delta}In\left[1+\delta(T-t_1)\right]\right]\right] \quad (12)$$

Sales revenue

$$SR=s\int_0^{t_1}\left(a+bI(t)-s\right)dt+s\int_{t_1}^{T}(a-s)\,dt=(a-s)s\left[T+b\left(\frac{1}{6}t_1^2(3+2bt_1)+\frac{\alpha}{\beta+1}\left(\frac{t_1^{\beta+2}\left((1+\beta)^2-\alpha\right)}{(\beta+1)(\beta+2)}-\frac{bt_1^3}{6}\right)\right)\right] \quad (13)$$

Case I $M\le t_1$, In this case, the supplier's offer a time period to the retailer to pay all dues which is on or before the inventory depleted completely.

Interest payable

The interest payable IP per cycle is given by in the time horizon $M<t\le t_1$

$$IP_1=pI_p\int_M^{t_1}I(t)\,dt$$

$$=pI_p(a-s)\left(\begin{array}{l}\frac{M^2}{2}+\frac{bM^3}{6}-Mt_1+\frac{v^2}{2}-\frac{1}{2}bMt_1^2+\frac{bt_1^3}{3} \\ -\frac{\alpha}{\beta+1}\left(\begin{array}{l} t_1^2\left(\frac{bt_1}{6}-\frac{t_1^{\beta}\left((\beta+1)^2-\alpha\right)}{(\beta+1)(\beta+2)}\right) \\ \frac{1}{6}M\left(bM(2M-3t_1)+6\left(t_1^{\beta+1}-\frac{M^{\beta}t_1\alpha}{\beta+1}+\frac{M^{1+\beta}(\alpha-1)}{\beta+2}\right)\right)\end{array}\right)\end{array}\right) \quad (14)$$

Interest earned

In addition, the interest earned per cycle IE_1 is given by in the time horizon $M<t\le t_1$

$$IE_1=sI_e\int_0^{t_1}D(t)\,t\,dt$$

$$=(a-s)st_1^2I_e\left(\frac{\left(12+4bt_1+3b^2t_1^2\right)}{24}-\frac{\alpha b^2t_1^2}{12(\beta+1)}-\frac{bt_1^{1+\beta}\alpha\left(2\alpha-(\beta+1)(\beta+2)\right)}{2(\beta+1)(\beta+2)(\beta+3)}\right) \quad (15)$$

Case II: $M \ge t_1$, In this case, the offered time period to the retailer to pay all dues which more than the inventory depleted completely.

Interest payable

In this case, the interest payable per cycle is zero, since the retailer has sold out the entire stock bought on credit from the supplier i.e., $IP_2 = 0$ (16)

Interest earned

In this case, the interest earned IE_2 per cycle is given by in the time horizon during the time period $\left[t_1, M\right]$

$$IE_2 = sI_e\left[\int_0^{t_1} D(t)tdt + (M - t_1)\int_0^{t_1} D(t)dt\right]$$

$$= sI_e(a-s)\left[\begin{array}{l} t_1^2\left(\dfrac{-bt_1^{1+\beta}\alpha(2\alpha-(\beta+1)(\beta+2))}{2(\beta+1)(\beta+2)(\beta+3)} + \dfrac{(12(\beta+1)+bt_1(4(\beta+1)+bt_1(3-2\alpha+3\beta)))}{24(\beta+1)}\right) \\ +(M-t_1)\left(t_1 + b\left(\dfrac{1}{6}t_1^2(3+2bt_1) + \dfrac{\alpha}{\beta+1}\left(-\dfrac{bt_1^3}{6} + \dfrac{t_1^{\beta+2}\left(-\alpha+(\beta+1)^2\right)}{(\beta+1)(\beta+2)}\right)\right)\right) \end{array}\right] \quad (17)$$

The total average profit of the system per unit time is given by

$$Max\left[TP(t_1,s)\right] = \begin{cases} TP_1(t_1,s), M \le t_1 \\ TP_2(t_1,s), M \ge t_1 \end{cases} \quad (18)$$

Where $TP_1 = \dfrac{1}{T}\left[SR - OC - HC - SC - LSC - PC - IP_1 + IE_1\right]$

And $TP_2 = \dfrac{1}{T}\left[SR - OC - HC - SC - LSC - PC - IP_2 + IE_2\right]$

Where $TP_1(p,t_1)$ and $TP_2(p,t_1)$ are discussed as follows

IV. SOLUTION PROCEDURE: We solve the above-mentioned problem using the following algorithm

Step 1. Input the value of all the required parameters of the proposed inventory model.

Step 2. In order to find the optimal value t_1^*, s^* to maximize the total average profit $TP_i\,(i=1,2)$

First, we, Differentiate $TP_i\,(i=1,2)$ and find out $\dfrac{\partial TP_i}{\partial t_1}\ \dfrac{\partial TP_i}{\partial s}\ \dfrac{\partial^2 TP_i}{\partial t_1^2}\ \dfrac{\partial^2 TP_i}{\partial s^2}\ \dfrac{\partial^2 TP_i}{\partial t_1 \partial s}$ where $i = 1,2$

The optimal value of t_1^*, s^* can be obtained by solving the following equations,

$\dfrac{\partial TP_i}{\partial t_1} = 0$ and $\dfrac{\partial TP_i}{\partial s} = 0$ where $i = 1,2$

Using the software Mathematica we get t_1^*, s^*

Evaluate $\dfrac{\partial^2 TP_i}{\partial t_1^2}\ \dfrac{\partial^2 TP_i}{\partial s^2}$ and $\dfrac{\partial^2 TP_i}{\partial t_1 \partial s}$ for the value of t_1^*, and s^*

The necessary and sufficient condition for maximization is

$$\left(\frac{\partial^2 TP_i}{\partial t_1^2}\right)\left(\frac{\partial^2 TP_i}{\partial s^2}\right) - \left(\frac{\partial^2 TP_i}{\partial t_1 \partial s}\right)^2 > 0 \text{ and } \frac{\partial^2 TC_i}{\partial t_1^2} < 0\ \frac{\partial^2 TC_i}{\partial s^2} < 0$$

Since the nature of the profit function is highly nonlinear, thus the concavity of the function is shown in the next section.

Step 3. Stop

V. NUMERICAL ILLUSTRATIONS The numerical examples given below cover all the two cases considered in the model

Example 1. Consider the following data to illustrates the inventory system $A = 600$, $a = 120$ $b = 0.02$ $C_1 = 20$ $C_2 = 30$ $C_3 = 50$ $\alpha = 0.02$ $\beta = 4$ $h_2 = 4$ $h_1 = 0.6$ $\delta = 0.6$ $T = 1$ $I_p = 0.2$ $I_e = 0.04$, $M = 0.6$ in appropriate units.

Then we get the optimum values of $t_1^* = 0.8604$ $s^* = 69.8501$, $Q^* = 952.988$ $TP^* = 1971.37$

Example 2. Consider the following data to illustrates the inventory system $A = 600$, $a = 120$ $b = 0.02$ $C_1 = 20$ $C_2 = 30$ $C_3 = 50$ $\alpha = 0.02$ $\beta = 4$ $h_1 = 0.6$ $h_2 = 4$ $\delta = 0.6$ $T = 1$ $I_p = 0.2$ $I_e = 0.04$, $M = 0.95$ in appropriate units.

Then we get the optimum values of $t_1^* = 0.8178$ $s^* = 69.7741$, $Q^* = 952.988$ $TP^* = 1984.68$

VI. CONCLUSION

In this paper, an EOQ model is developed for deterioration items with multivariable demand under the facility of delay in payment The two different cases has been discussed as Case-I deal with the credit period is less than or equal to the time where the inventory level depletes up to zero and case II deal with the credit period is greater than or equal to the time where the inventory level depletes up to zero. The holding cost is linear function of time and unsatisfied demand are backlogged considered in the model.The optimal solution in both the cases is illustrated with the help of numerical examples.

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