# A Study On Distinct Domination Parameters On Line Graph of Complete Graph 

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#### Abstract

In this paper, we have obtained the line graph of a complete graph $\mathrm{K}_{\mathrm{n}}$. It is denoted by $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$. In this we established bounds for distinct domination parameters such as domination number $\gamma(G)$, Inverse domination number $\gamma^{\prime}(G)$, 2- domination number $\gamma_{2}(G)$, Inverse 2- domination number $\gamma_{2}^{\prime}(G)$, Location 2- domination number $R_{D}^{2}(G)$, Total domination number $\gamma_{t}(G)$, Accurate domination number $\gamma_{a}(G)$, Efficient domination number $\gamma_{e}(G)$. Also studied some of the properties and illustrated with examples.


Keywords: Complete graph, Line graph, Domination number, Total domination number, Accurate domination number, Inverse domination number, 2- domination number, Inverse 2- domination number, Location 2- domination number, Efficient domination number.

## 1. INTRODUCTION

In this paper, we have taken the graph to be connected, undirected, finite and simple graph (2). The concept of line graph was invented by H.Whitney 1932. Line graph $L(G)$ of a complete graph $K_{n}$ denoted by $L\left(K_{n}\right)$ is a graph whose vertices of $L\left(K_{n}\right)$ are the edges of $K_{n}$ and two vertices of $L\left(K_{n}\right)$ are adjacent if the corresponding edges of $K_{n}$ are adjacent. In 1958, the concept of domination in graph was defined by Claude Berge and Ore. Here, we have applied different domination parameters in $L\left(\mathrm{~K}_{\mathrm{n}}\right)$. We proved some theorems and also discussed results. We begin with some basic definitions and notations.

## Definition 1.1: (1)

A graph in which any two distinct vertices are adjacent is called complete graph and it's denoted by $\mathrm{K}_{\mathrm{n}}$, where n is the order of the graph.
Definition 1.2: (3)
Line graph $L(G)$ of a graph $G$ is that the vertices of $L(G)$ are the edges of $G$ and two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent.

## Example:



G

fig. 1

## DOMINATION PARAMTERS:

## Definition 1.3: (6)

A non- empty subset D of V is called a dominating set of G if every vertex of D is adjacent to each vertex of V-D. The domination number $\gamma(\boldsymbol{G})$ of a graph G is the minimum cardinality of a dominating set of G .

## Definition 1.4: (10)

Let D be $\gamma$-set of a graph G. A dominating set of $D^{\prime} \subseteq \mathrm{V}$-D is called an inverse dominating set of G with respect to D . The smallest cardinality among all minimal dominating set in (V-D) is called inverse domination number and is denoted by $\gamma^{\prime}(G)$.

## Definition 1.5:

Let D be $\gamma$-set of a graph G. A dominating set of $D^{\prime} \subseteq$ V-D is called an inverse 2-dominating set of G with respect to D if V-D is adjacent to atleast two vertices of $D^{\prime}$. The smallest cardinality among all minimal dominating set in V-D is called inverse 2domination number and is denoted by $\gamma_{2}^{\prime}(G)$.
Definition 1.6: (7)
A non- empty subset $D$ of $V$ is called a 2-dominating set of a graph $G$ if for every $v \in V$ either $v \in D$ or $v$ is adjacent to atleast two vertices of D . The 2- dominating set is denoted as $\mathrm{D}_{2}$. The 2-domination number $\gamma_{2}(G)$ is the minimum cardinality of dominating set of G.
Definition 1.7: (7)
A non- empty subset $D$ of $V$ is location 2-dominating set of $G$ if $D$ is 2 - dominating set of $G$ and if for any two vertices $u, v \in V-D$ such that $\mathrm{N}(\mathrm{u}) \cap \mathrm{D} \neq \mathrm{N}(\mathrm{v}) \cap \mathrm{D}$. The minimum cardinality of location 2-dominating set is said to be the location 2-domination number and is denoted by $\boldsymbol{R}_{D}^{2}(G)=|D|$.
Definition 1.8: (9)
A dominating set $D$ of vertices of $G$ is called total dominating set if every vertex in $V(G)$ is adjacent to atleast one vertex in $D$. The total domination number of G denoted by $\mathcal{\gamma}_{t}(G)$ is the minimum cardinality of a total dominating set $\mathrm{D}^{\mathrm{t}}$ of G .
Definition 1.9: (4)
A dominating set D of a graph G is an accurate dominating set, if V - D has no dominating set of cardinality $|D|$. The accurate domination number $\gamma_{a}(G)$ is the minimum cardinality of an accurate dominating set.

## Definition 1.10: (5)

A dominating set D of G is an efficient dominating set if every vertex in V-D is adjacent to exactly one vertex in D . The efficient domination number $\gamma_{e}(G)$ is the minimum cardinality of an efficient dominating set.

## 2. LINE GRAPH OF A COMPLETE GRAPH

## Theorem 2.1

If $L\left(K_{n}\right)$ is a line graph of a complete graph $K_{n}$ of order $n \geq 3$ and $n \in Z^{+}$, then
$\left|E\left(L\left(K_{n}\right)\right)\right|=\frac{n(n-1)(n-2)}{2}$ where $\left|E\left(L\left(K_{n}\right)\right)\right|$ is the number of edges of line graph of a complete graph.

## Proof

Let $K_{n}$ be any complete graph of $n$ vertices and $L\left(K_{n}\right)$ be the line graph of a complete graph $K_{n}$ where $n$ is the number of vertices (order) of any complete graph $\mathrm{K}_{\mathrm{n}}$ and $\left|E\left(L\left(K_{n}\right)\right)\right|$ is the number of edges (size) of $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$. The order of $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ is given by the number of edges in $K_{n}$
For $n=2, L\left(K_{2}\right)$ is a null graph.
For $n=3, L\left(K_{3}\right)$ is of order 3 and size is given by $\left|E\left(L\left(K_{3}\right)\right)\right|=\frac{3(3-1)(3-2)}{2}=3$
For $\mathrm{n}=4, \mathrm{~L}\left(\mathrm{~K}_{4}\right)$ is of order 6 and size is given by $\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{4}\right)\right)\right|=\frac{4(4-1)(4-2)}{2}=12$
For $n=5, L\left(K_{5}\right)$ is of order 10 and size is given by $\left|E\left(L\left(K_{5}\right)\right)\right|=\frac{5(5-1)(5-2)}{2}=30$
For $n=6, L\left(K_{6}\right)$ is of order 15 and size is given by $\left|E\left(L\left(K_{6}\right)\right)\right|=\frac{6(6-1)(6-2)}{2}=60$
For $\mathrm{n}=7, \mathrm{~L}\left(\mathrm{~K}_{7}\right)$ is of order 21 and size is given by $\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)\right|=\frac{7(7-1)(7-2)}{2}=105$
In general, for any line graph of a complete graph of order $\frac{n(n-1)}{2}$, the number of edges is given by $\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)\right|=\frac{n(n-1)(n-2)}{2}$ for $n \geq 3$

## Example 2.1.1

For $\mathrm{n}=5$, then $\mathrm{L}\left(\mathrm{K}_{5}\right)$ is described below,

fig.2: $\mathrm{L}\left(\mathrm{K}_{5}\right)$

The order of $\mathrm{L}\left(\mathrm{K}_{5}\right)=\left|V\left(L\left(K_{5}\right)\right)\right|$ is given by the number of edges in $\mathrm{K}_{5}$.
ie) $\frac{n(n-1)}{2}=\frac{5(5-1)}{2}=10$. Therefore $\left|V\left(L\left(K_{5}\right)\right)\right|=10$
The size of $\mathrm{L}\left(\mathrm{K}_{5}\right)=\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)\right|$ is the number of edges of $\mathrm{L}\left(\mathrm{K}_{5}\right)$
ie) $\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)\right|=\frac{5(5-1)(5-2)}{2}=30$

For $\mathrm{n}=6, \mathrm{~L}\left(\mathrm{~K}_{6}\right)$ is described below,

fig. 3: $L\left(K_{6}\right)$

The order of $\mathrm{L}\left(\mathrm{K}_{6}\right)=\left|V\left(L\left(K_{6}\right)\right)\right|$, is given by the number of edges in $\mathrm{K}_{6}$.
ie) $\frac{n(n-1)}{2}=\frac{6(6-1)}{2}=15$. Therefore, $\left|V\left(L\left(K_{6}\right)\right)\right|=15$
The size of $\mathrm{L}\left(\mathrm{K}_{6}\right)=\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)\right|$ is the number of edges of $\mathrm{L}\left(\mathrm{K}_{6}\right)$
ie) $\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)\right|=\frac{6(6-1)(6-2)}{2}=60$

For $\mathrm{n}=7$ then $\mathrm{L}\left(\mathrm{K}_{7}\right)$ is described below,

fig.4: $L\left(K_{7}\right)$

The order of $L\left(\mathrm{~K}_{7}\right)=\left|V\left(L\left(K_{7}\right)\right)\right|$, is given by the number of edges in $\mathrm{K}_{7}$.
ie) $\frac{n(n-1)}{2}=\frac{7(7-1)}{2}=21$. Therefore, $\left|V\left(L\left(K_{7}\right)\right)\right|=21$
The size of $L\left(K_{7}\right)=\left|E\left(L\left(K_{7}\right)\right)\right|$ is the number of edges of $L\left(K_{7}\right)$
ie) $\left|\mathrm{E}\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)\right|=\frac{7(7-1)(7-2)}{2}=105$
Note: $\left[\frac{p}{q}\right]$ is the integral part of $\frac{p}{q}, \forall \mathrm{p}, \mathrm{q} \in \mathrm{Z}^{+}$

## Results:

1) The vertex set of $\mathrm{K}_{\mathrm{n}}$ is the edge set in $\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)$ and hence $\left|V\left(L\left(K_{n}\right)\right)\right|=\frac{n(n-1)}{2}$
2) For complete graph $K_{1}$, no line graph exist.
3) Line graph of $K_{2}$ is a null graph.
4) Line graph of $K_{3}$ graph is a $K_{3}$ graph. (ie) $L\left(K_{3}\right)=K_{3}$
5) For all $n>3$, line graph of complete graph is not complete.
6) Since the every line graph of complete graph has the same degree, hence $\left(L\left(K_{n}\right)\right)$ is a regular graph for non- negative $n \geq 3$

## 3. DISTINCT DOMINATION PARAMETERS:

In this section, we discussed some of the domination parameters in line graph of a complete graph and proved some theorems and results.

## Theorem 3.1

Let $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ be any line graph of a complete graph and $\gamma\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)$ the domination number of $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ and $\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)$ is the inverse domination number then,

$$
\gamma\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=\left[\frac{n}{2}\right]=\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right) \text { for } \mathrm{n} \geq 3 \& \mathrm{n} \in \mathrm{Z}^{+}
$$

## Proof

Given that, $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ is a line graph of $\mathrm{K}_{\mathrm{n}}$ and hence by definition of line graph we have $\frac{n(n-1)}{2}$ vertices and $\frac{n(n-1)(n-2)}{2}$ edges.
To prove: $\gamma\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right) \forall \mathrm{n} \geq 3$,
We prove this by induction method,
By the definition of domination number, we know that $\gamma\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)$ is the domination number of $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$,
For $n=3$, we have, $V\left(L\left(K_{3}\right)\right)=\left\{v_{1}, v_{2}, v_{3}\right\}$ (ie) $L\left(K_{3}\right)$ has order 3 and size of $L\left(K_{3}\right)$ is also 3 or $L\left(K_{3}\right)$ has the same order and size which is equal to 3 . The minimum dominating set of $\mathrm{L}\left(\mathrm{K}_{3}\right)$ is
$\mathrm{D}=\left\{\mathrm{v}_{1}\right\}$, hence $\gamma\left(\mathrm{L}\left(\mathrm{K}_{3}\right)\right)=\left[\frac{3}{2}\right]=1$
For $n=4$, we have $V\left(L\left(K_{4}\right)\right)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{6}\right\}$ is $L\left(K_{4}\right)$ of order 6 and size 12.
The minimum dominating set of $L\left(K_{4}\right)$ is $D=\left\{v_{1}, v_{6}\right\}$, hence $\gamma\left(L\left(K_{4}\right)\right)=\left[\frac{4}{2}\right]=2$
For $\mathrm{n}=5$, The order and size of $\mathrm{L}\left(\mathrm{K}_{5}\right)$ are 10 and 30 respectively
We have $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{10}\right\}$
By the definition of dominating set, we have $\mathrm{D}=\left\{\mathrm{v}_{1}, \mathrm{v}_{8}\right\}$, hence $\gamma\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)=\left[\frac{5}{2}\right]=2$
Similarly, for $n=6$, we get the line graph of order 15 and size 60 we have, $V\left(L\left(K_{6}\right)\right)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{15}\right\}$
The minimum dominating set is $\mathrm{D}=\left\{\mathrm{v}_{3}, \mathrm{v}_{6}, \mathrm{v}_{13}\right\}$, hence $\gamma\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)=\left[\frac{6}{2}\right]=3$
For $n=7, L\left(K_{7}\right)$ contains 21 vertices and 105 edges. Let, $V\left(L\left(K_{7}\right)\right)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{21}\right\}$
The minimum dominating set is $\mathrm{D}=\left\{\mathrm{v}_{1}, \mathrm{v}_{15}, \mathrm{v}_{18}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)$ hence $\gamma\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)=\left[\frac{7}{2}\right]=3$
Proceeding in this way for all $n \geq 3$, we have

$$
\begin{equation*}
\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=\left[\frac{n}{2}\right] \longrightarrow \tag{1}
\end{equation*}
$$

Next we determine inverse domination number of $L\left(K_{n}\right)$.
By the definition of inverse domination number, let $D^{\prime}$ be the inverse dominating set and $\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)$ is the inverse domination number of $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$
For $n=3$, we have $V\left(L\left(K_{3}\right)\right)=\left\{v_{1}, v_{2}, v_{3}\right\}$
Since $\mathrm{D}=\left\{\mathrm{v}_{3}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{3}\right)\right)$ is the dominating set of $\mathrm{L}\left(\mathrm{K}_{3}\right)$ where $D^{\prime}=\left\{\mathrm{v}_{1}\right\} \subseteq \mathrm{V}$-D is the inverse dominating set of $\mathrm{L}\left(\mathrm{K}_{3}\right)$ and hence by definition of inverse domination number,

$$
\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{3}\right)\right)==\left[\frac{3}{2}\right]=1
$$

For $\mathrm{n}=4$, we have $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{4}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{6}\right\}$
The dominating set is $\mathrm{D}=\left\{\mathrm{v}_{1}, \mathrm{v}_{6}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{4}\right)\right)$
Since $D^{\prime}=\left\{\mathrm{v}_{2}, \mathrm{v}_{5}\right\} \subseteq \mathrm{V}$-D is the inverse dominating set of $\mathrm{L}\left(\mathrm{K}_{4}\right)$ and hence by definition of inverse domination,

$$
\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{4}\right)\right)=\left[\frac{4}{2}\right]=2
$$

For $\mathrm{n}=5$, we have $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{15}\right\}$
The dominating set is $D=\left\{\mathrm{v}_{1}, \mathrm{v}_{8}\right\}$. Since $D^{\prime}=\left\{\mathrm{v}_{4}, \mathrm{v}_{5}\right\} \subseteq \mathrm{V}-\mathrm{D}$ is the inverse dominating set of $\mathrm{L}\left(\mathrm{K}_{5}\right)$ and hence by definition of inverse domination,

$$
\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{5}\right)\right)=\left[\frac{5}{2}\right]=2
$$

For $\mathrm{n}=6$, we have $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{15}\right\}$
The dominating set is $\mathrm{D}=\left\{\mathrm{v}_{3}, \mathrm{v}_{6}, \mathrm{v}_{13}\right\}$. Since $D^{\prime}=\left\{\mathrm{v}_{1}, \mathrm{v}_{12}, \mathrm{v}_{14}\right\} \subseteq \mathrm{V}$-D is the inverse dominating set of $\mathrm{L}\left(\mathrm{K}_{6}\right)$ and hence by definition of inverse domination we have,

$$
\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{6}\right)\right)=\left[\frac{6}{2}\right]=3
$$

For $\mathrm{n}=7$, we have $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{21}\right\}$
The dominating set is $\mathrm{D}=\left\{\mathrm{v}_{1}, \mathrm{v}_{15}, \mathrm{v}_{18}\right\}$. Since $D^{\prime}=\left\{\mathrm{v}_{2}, \mathrm{v}_{4}, \mathrm{v}_{7}\right\} \subseteq \mathrm{V}-\mathrm{D}$ is the inverse dominating set of $\mathrm{L}\left(\mathrm{K}_{7}\right)$ and hence by definition of inverse domination,

$$
\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{7}\right)\right)=\left[\frac{7}{2}\right]=3
$$

In general, for all $\mathrm{n} \geq 3$ the graph $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ has $\frac{n(n-1)}{2}$ vertices and $\frac{n(n-1)(n-2)}{2}$ edges. The vertex set of $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ is given by $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots \ldots, \mathrm{v}_{\mathrm{k}}\right\}$ and by the definition of inverse domination we have,

$$
\begin{aligned}
& \gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=\left[\frac{n}{2}\right], \mathrm{n} \geq 3 \\
& \text { e that } \gamma\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=\left[\frac{n}{2}\right]=\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right) \forall \mathrm{n} \geq 3
\end{aligned}
$$

## Result

The domination number and inverse domination number are equal for line graph of a complete graph of order $\mathrm{n} \geq 3$.
Example 3.1.1 Consider the line graph of a complete graph $\mathrm{K}_{6}$

fig.5: $\mathrm{L}\left(\mathrm{K}_{6}\right)$

For $\mathrm{n}=6$, we have $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{15}\right\}$
The dominating set is $\mathrm{D}=\left\{\mathrm{v}_{3}, \mathrm{v}_{6}, \mathrm{v}_{13}\right\}$, hence $\gamma\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)=\left[\frac{6}{2}\right]=3$
Let $\mathrm{V}-\mathrm{D}=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \mathrm{v}_{4}, \mathrm{v}_{7}, \mathrm{v}_{9}, \mathrm{v}_{10}, \mathrm{v}_{11}, \mathrm{v}_{12}, \mathrm{v}_{14}, \mathrm{v}_{14}, \mathrm{v}_{15}\right\}$
The inverse dominating set is $D^{\prime}=\left\{\mathrm{v}_{1}, \mathrm{v}_{12}, \mathrm{v}_{14}\right\} \subseteq \mathrm{V}-\mathrm{D}$, hence $\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)=\left[\frac{6}{2}\right]=3$

Therefore,

$$
\gamma\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=\left[\frac{n}{2}\right]=\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right) \forall \mathrm{n} \geq 3
$$

## Example 3.1.2

Consider the line graph of a complete graph $\mathrm{K}_{7}$

fig.6: $L\left(K_{7}\right)$

For $\mathrm{n}=7$, we have $\mathrm{V}=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots, \mathrm{v}_{21}\right\}$
The dominating set is $\mathrm{D}=\left\{\mathrm{v}_{1}, \mathrm{v}_{15}, \mathrm{v}_{18}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)$ and hence $\gamma\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)=\left[\frac{7}{2}\right]=3$
The inverse dominating set is $D^{\prime}=\left\{\mathrm{v}_{2}, \mathrm{v}_{4}, \mathrm{v}_{7}\right\} \subseteq$ V-D, hence $\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{K}_{7}\right)\right)=\left[\frac{7}{2}\right]=3$
Therefore, $\left.\gamma \mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)=\left[\frac{n}{2}\right]=\gamma^{\prime}\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right) \forall \mathrm{n} \geq 3$

## Theorem 3.2:

Let $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ be the line graph of a complete graph, then 2- domination number $\gamma_{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)$ and location 2-domination number $\boldsymbol{R}_{D}^{2}$ $\left(\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)\right)$ are given by,

$$
\gamma_{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=(2 \mathrm{n}-5)=\boldsymbol{R}_{D}^{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right), \forall \mathrm{n}>3
$$

## Proof:

Let $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ be the line graph of a complete graph. This theorem can be proved by the method of induction
When $n=4$, we've $L\left(K_{4}\right)$ graph with 6 vertices and 12 edges. Let $D_{2}$ be the 2 - domination set of $L\left(K_{5}\right)$. We define $D_{2}=\left\{v_{1}, v_{3}\right.$, $\left.\mathrm{v}_{5}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{4}\right)\right.$ ). Since each vertex of $\mathrm{D}_{2}$ has two neighbourhood (or) adjacent to atleast two vertices in V-D,
so $\gamma_{2}\left(\mathrm{~L}\left(\mathrm{~K}_{4}\right)\right)=2(4)-5=3$ and the definition of location 2- domination number we have to show that, $\mathrm{N}(\mathrm{u}) \cap \mathrm{D}_{2} \neq \mathrm{N}(\mathrm{v}) \cap \mathrm{D}_{2}$
Since, $\mathrm{V}-\mathrm{D}_{2}=\left\{\mathrm{v}_{2}, \mathrm{v}_{4}, \mathrm{v}_{6}\right\}$
Take, $u=v_{2}, v=v_{4}$
$\mathrm{N}\left(\mathrm{v}_{2}\right) \cap \mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{4}, \mathrm{v}_{6}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}\right\} \longrightarrow$ (1)
$\mathrm{N}\left(\mathrm{v}_{4}\right) \cap \mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{5}, \mathrm{v}_{6}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{5}\right\}$
$\mathrm{N}\left(\mathrm{v}_{2}\right) \cap \mathrm{D}_{2} \neq \mathrm{N}\left(\mathrm{v}_{4}\right) \cap \mathrm{D}_{2}$, by (1) and (2)
Therefore, $\boldsymbol{R}_{D}^{2}\left(L\left(K_{4}\right)\right)=\left|D_{2}\right|=3\left(\because \mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}\right\}\right)$ ie. $\boldsymbol{R}_{D}^{2}\left(L\left(K_{4}\right)\right)=2(4)-5=3$
When $n=5$, we've $L\left(K_{5}\right)$ graph with 10 vertices and 30 edges. Let $D_{2}$ be the 2 - domination set of $L\left(K_{5}\right)$. We define $D_{2}=\left\{v_{1}, v_{3}\right.$, $\left.\mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)$. Since each vertex of $\mathrm{D}_{2}$ has two neighbourhood (or) adjacent to atleast two vertices in V - $\mathrm{D}_{2}$, so $\gamma_{2}$
$\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)=2(5)-5=5$.
By the definition of location 2- domination number we have to show that, $N(u) \cap D_{2} \neq N(v) \cap D_{2}$
Since, $V-D_{2}=\left\{v_{2}, v_{4}, v_{5}, v_{6}, v_{10}\right\}$. Take, $u=v_{2}, v=v_{4}$ and by the definition of location 2- domination number we have
$N\left(v_{2}\right) \cap D_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{4}, \mathrm{v}_{5}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\}$
$N\left(v_{5}\right) \cap D_{2}=\left\{\mathrm{v}_{2}, \mathrm{v}_{1}, \mathrm{v}_{10}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\} \longrightarrow$ (4)
$\mathrm{N}\left(\mathrm{v}_{2}\right) \cap \mathrm{D}_{2} \neq \mathrm{N}\left(\mathrm{v}_{5}\right) \cap \mathrm{D}_{2}$, by (3) and (4)
Therefore, $\boldsymbol{R}_{D}^{2}\left(L\left(K_{5}\right)\right)=\left|D_{2}\right|=5\left(\because \mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\}\right)$ ie. $\boldsymbol{R}_{D}^{2}\left(L\left(K_{5}\right)\right)=2(5)$ - $5=5$
Similarly for $\mathrm{n}=6$, we've $\mathrm{L}\left(\mathrm{K}_{6}\right)$ graph with 15 vertices and 60 edges. Let $\mathrm{D}_{2}$ be the 2-domination set of $\mathrm{L}\left(\mathrm{K}_{6}\right)$. We define $\mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}, \mathrm{v}_{9}, \mathrm{v}_{10}, \mathrm{v}_{12}, \mathrm{v}_{14}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)$. Since each vertex of $\mathrm{D}_{2}$ has two neighbourhood (or) adjacent to atleast two vertices
in $V-D_{2}$, so $\gamma_{2}\left(L\left(K_{6}\right)\right)=2(6)-5=7$.
Since, $\mathrm{V}^{2} \mathrm{D}_{2}=\left\{\mathrm{v}_{2}, \mathrm{v}_{4}, \mathrm{v}_{6}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{11}, \mathrm{v}_{13}, \mathrm{v}_{15}\right\}$.
Take, $u=v_{7}, v=v_{15}$
$\mathrm{N}\left(\mathrm{v}_{7}\right) \cap \mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{4}, \mathrm{v}_{6}, \mathrm{v}_{8}, \mathrm{v}_{10}, \mathrm{v}_{12}, \mathrm{v}_{13}, \mathrm{v}_{15}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}, \mathrm{v}_{9}, \mathrm{v}_{10}, \mathrm{v}_{12}, \mathrm{v}_{14}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{10}, \mathrm{v}_{12}\right\}$
$N\left(v_{15}\right) \cap D_{2}=\left\{\mathrm{v}_{14}, \mathrm{v}_{13}, \mathrm{v}_{12}, \mathrm{v}_{9}, \mathrm{v}_{7}, \mathrm{v}_{6}, \mathrm{v}_{4}, \mathrm{v}_{2}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{5}, \mathrm{v}_{9}, \mathrm{v}_{10}, \mathrm{v}_{12}, \mathrm{v}_{14}\right\}=\left\{\mathrm{v}_{14}, \mathrm{v}_{12}, \mathrm{v}_{9}\right\}$
$\mathrm{N}\left(\mathrm{v}_{7}\right) \cap \mathrm{D}_{2} \neq \mathrm{N}\left(\mathrm{v}_{15}\right) \cap \mathrm{D}_{2}$, by (5) and (6)
Therefore, $\boldsymbol{R}_{D}^{2}\left(L\left(K_{6}\right)\right)=\left|D_{2}\right|=5$

$$
\text { ie. } \boldsymbol{R}_{D}^{2}\left(L\left(K_{6}\right)\right)=2(6)-5=7
$$

In general we conclude that we have, $\gamma_{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=(2 \mathrm{n}-5)=\boldsymbol{R}_{D}^{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right) \forall \mathrm{n}>3$

## Example 3.2.1

Consider the line graph of $\mathrm{K}_{5}$ of order 10 and size 30 .

fig. 7: $\left.L\left(K_{5}\right)\right)$
For $\mathrm{n}=5$,the 2-domination set is $\mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\}$

$$
\gamma_{2}\left(\mathrm{~L}\left(\mathrm{~K}_{5}\right)\right)=5
$$

Location 2- domination number:
$\mathrm{V}-\mathrm{D}_{2}=\left\{\mathrm{v}_{2}, \mathrm{v}_{4}, \mathrm{v}_{5}, \mathrm{v}_{6}, \mathrm{v}_{10}\right\} \subseteq \mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)$
Let $u=v_{2}, v=v_{5} \in V-D_{2}$
$\mathrm{N}\left(\mathrm{v}_{2}\right) \cap \mathrm{D}_{2}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{4}, \mathrm{v}_{5}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\}$
$N\left(v_{5}\right) \cap D_{2}=\left\{\mathrm{v}_{2}, \mathrm{v}_{1}, \mathrm{v}_{10}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\} \cap\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{7}, \mathrm{v}_{8}, \mathrm{v}_{9}\right\}=\left\{\mathrm{v}_{1}, \mathrm{v}_{7}, \mathrm{v}_{9}\right\}$
$\mathrm{N}\left(\mathrm{v}_{2}\right) \cap \mathrm{D}_{2} \neq \mathrm{N}\left(\mathrm{v}_{5}\right) \cap \mathrm{D}_{2}$
$\boldsymbol{R}_{D}^{2}\left(L\left(K_{5}\right)\right)=\left|D_{2}\right|=5$. Hence, $\gamma_{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=(2 \mathrm{n}-5)=\boldsymbol{R}_{D}^{2}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right), \forall \mathrm{n}>3$

## Theorem 3.3:

For any line graph of a complete graph $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$, the total domination number
$\gamma_{t}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)$ is equal to $\mathrm{n}-2$ for $\mathrm{n} \geq 4 \& \mathrm{n} \in \mathrm{Z}^{+}$
ie) $\gamma_{t}\left(L\left(K_{n}\right)\right)=(n-2)$, for $n \geq 4 \& n \in Z^{+}$

## Proof:

We proved this theorem by induction method
When $\mathrm{n}=4$, we have the line graph $\mathrm{L}\left(\mathrm{K}_{4}\right)$ is of order 6 and size 12
Let $D^{t}=\left\{\mathrm{v}_{1}, \mathrm{v}_{5}\right\}$ be the dominating set of $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{4}\right)\right)$. Since each vertex of $\mathrm{D}^{\mathrm{t}}$ dominates every vertices of $\mathrm{V}\left(\left(\mathrm{K}_{4}\right)\right)=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}, \ldots\right.$, ${ }^{v_{6}}$ ) and $\left|D^{t}\right|=2$
$\gamma_{t}\left(L\left(K_{4}\right)\right)=4-2=2$
When $\mathrm{n}=5$, we have the line $\operatorname{graph} \mathrm{L}\left(\mathrm{K}_{5}\right)$ of order 10 and size 30 .
Let $\mathrm{D}^{\mathrm{t}}=\left\{\mathrm{v}_{1}, \mathrm{v}_{3}, \mathrm{v}_{10}\right\}$ be the dominating set of $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)$. Since each vertex of $\mathrm{D}^{\mathrm{t}}$ dominates every vertex of $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{5}\right)\right)$
and $\left|D^{t}\right|=3$. By the definition of total domination, we have
$\gamma_{t}\left(L\left(K_{5}\right)\right)=5-2=3$
In the same way for $\mathrm{n}=6$, we have the line graph $\mathrm{L}\left(\mathrm{K}_{6}\right)$ of order 15 and size 60 .
Let $D^{t}=\left\{\mathrm{v}_{1}, \mathrm{v}_{6}, \mathrm{v}_{12}, \mathrm{v}_{15}\right\}$ be the dominating set of $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)$. Since each vertex of $\mathrm{D}^{t}$ dominates every vertex of $\mathrm{V}\left(\mathrm{L}\left(\mathrm{K}_{6}\right)\right)$ and $\left|D^{t}\right|=4$. By the definition of total domination, we have
$\gamma_{t}\left(L\left(K_{6}\right)\right)=6-2=4$
In general, for line graph of a complete graph $\mathrm{K}_{\mathrm{n}}$, we have the total domination $\left|D^{t}\right|=(\mathrm{n}-2)$
ie) $\gamma_{t}\left(\mathrm{~L}\left(\mathrm{~K}_{\mathrm{n}}\right)\right)=(\mathrm{n}-2)$, for $\mathrm{n} \geq 4 \& \mathrm{n} \in \mathrm{Z}^{+}$

## Observations

i) For any line graph of a complete graph $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ the accurate domination number, $\gamma_{a}\left(L\left(K_{n}\right)\right)$ does not exists since V-D has no dominating set of cardinality $|D|$.
ii) The efficient domination number $\gamma_{e}(G)$ does not exists in $\mathrm{L}\left(\mathrm{K}_{\mathrm{n}}\right)$ since no vertex in V -D is adjacent to exactly one vertex in D.
iii) Inverse 2-domination number $\gamma_{2}^{\prime}(G)$ is not possible since no vertex in V-D is not adjacent to atleast two vertices of $D^{\prime}$.

## 4. Conclusion

In this paper we've obtained line graph of a complete graph $\mathrm{L}(\mathrm{Kn})$ and established exact bounds for distinct domination parameters such as domination number, Inverse domination number, 2- domination number, Inverse 2- domination number, Location 2- domination number, Total domination number and found that accurate domination number, inverse 2 - domination number and efficient domination number does not exist. Also, some of its properties are studied and is illustrated with examples. This work can be extended by establishing various parameters using colouring, labelling. etc

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