SILKY NORMAL SPACES IN TOPOLOGICAL SPACES

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Abstract. The aim of this paper is to introduce and study a new class of normal spaces, called silky-normal spaces. Interrelation among some existing variants of normality is discussed and characterizations of these variants are obtained. We also proved that silk normality is a topological property and it is a hereditary property with respect to closed domain subspace. The decomposition of normality in terms of silky normality and some factorizations of normality in presence of some lower separation axioms are given.

Key words and phrases: π-closed and δ-closed sets; silky normal, quasi normal, softly normal, softly regular and almost regular spaces.
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1. Introduction

The notion of quasi normality as a weak form of normality was introduced by Zaitsev [**25**]. The concept of almost normality was introduced by Singal and Arya [**17**]. The notion of seminormal spaces were introduced by Vigilino [**24**]. The notion of mild normality was introduced by Shchepin [**20**] and Singal and Singal [**18**] independently. Lal and Rahman [**14**] further investigated quasi normal spaces and mildly normal spaces. In [**14**], they obtained several improvements of results due to Singal and Singal [**18**]. The concept of nearly normal space as a weak form of normal space was introduced by Mukherjee and Debray [**15**]. Dontchev and Noiri [**6**] introduced the notion of πg-closed sets. By using πg-closed sets, they obtained a new characterization of quasi normal spaces and use it to obtain some preservation theorems for quasi normal spaces. Arhangel'skii and Ludwig [**2**] introduced two new classes of normal spaces are called, α-normal spaces and β-normal spaces and obtained their characterizations. Kohli and Das [**9**] introduced the concepts of some new classes of normal spaces are called θ-normal, functionally θ-normal (briefly fθ-normal), weakly θ-normal (briefly wθ-normal) and weakly functionally θ-normal (briefly wfθ-normal) spaces and obtained their characterizations. π-normal topological spaces were introduced by Kalantan [**8**]. Das [**3**] introduced the concepts of some new classes of normal spaces are called Δ-normal, weakly Δ-normal (briefly wΔ-normal), and weakly functionally Δ-normal (briefly wfΔ-normal) spaces and obtained a relation with other weaker versions of normal spaces. Some variants of normality which lies between normal and mildly normal spaces are considered and interrelation among these variants of normality is established by Das [**5**]. Sharma and Kumar [**19**] introduced a new class of normal spaces is called, softly normal spaces in topological spaces and obtained some characterizations of softly normal spaces. Das and Bhat [**4**] discussed the interrelation among some existing variants of normality and obtained characterizations of these variants. Decomposition of normality in terms of near normality and some factorizations of normality in presence of some lower separation axioms are given. Kumar and Sharma [**13**] introduced two new concepts of separation axioms namely, softly regular spaces and partly regular spaces in topological spaces and obtained their characterizations with other separation axioms.

2. Preliminaries

Throughout this paper, spaces (X, ℑ), (Y, σ), and (Z, γ) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by cl(A) and int(A) respectively. A subset A is said to be **regular open** (resp. **regular closed**) if A = int(cl(A)) (resp. A = cl(int(A)). The finite union of regular open sets is said to be **π-open**. The complement of a π-open set is said to be **π-closed.** A point x is said to be **θ-limit point** [23] of A if closure of every neighbourhood containing x intersects A. A set **θ-cl(A)** is the **θ-closure** of A which contains all θ-limit points of A. A set A is **θ-closed** if A = θ-cl(A). The complement of a θ-closed set is said to be **θ-open** A subset A is said to be **δ-open** set if it is the union of regular open sets. The complement of a δ-open set is said to be **δ-closed** (or A subset A is said to be δ-closed if A = δ-cl(A), where δ-cl(A) = {x ∈ X : int(cl(U)) ∩ A ≠ ∅, U ∈ ℑ and x ∈ U} and the complement of a δ-closed set is said to be **δ-open**).
2.1 Remark. For a subset of a space, we have following implications:

$$\text{regular open} \Rightarrow \pi\text{-open} \Rightarrow \delta\text{-open} \Rightarrow \text{open}$$

Where none of the implications is reversible as shown by [**6**].

2.2 Definition. A space (X, ℑ) is said to be **softly regular** [**13**] if for every π-closed set A and a point x ∉ A, there exist open sets U and V such that x ∈ U, A ⊂ V, and U ∩ V = ∅.

2.3 Definition. A space (X, ℑ) is said to be **partly regular [13]** if for every point x and every π-open set U containing x, there is an open set V such that x ∈ V ⊂ cl(V) ⊂ U.

2.4 Definition. A space (X, ℑ) is said to be **almost regular** [**21**] if for every regularly closed set A and a point x ∉ A, there exist open sets U and V such that x ∈ U, A ⊂ V, and U ∩ V = ∅.

2.5 Definition. A space (X, ℑ) is said to be **weakly regular [21]** if for every point x and every regularly open set U containing x, there is an open set V such that x ∈ V ⊂ cl(V) ⊂ U.

By the definitions stated above, we have the following diagram:

regularity ⇒ soft-regularity ⇒ partly-regularity

⇓ ⇓

almost -regularity ⇒ weakly-regularity

Where none of the implications is reversible as shown by [**13**].

3. Silky normal spaces

3.1 Definition. A topological space X is said to be

1. **silky normal** if for every pair of disjoint subsets A and B of X, one of which is π-closed and the other is δ-closed, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

2. quasi normal [**25**] if for every pair of disjoint π-closed subsets A and B of X, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

3. mildly normal [**18, 20**] if for every pair of disjoint regularly closed subsets A and B of X, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

4. almost normal [**17**] if for every pair of disjoint sets A and B of X, one of which is closed and the other is regularly closed, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

5. π-normal [**8**] if for any two disjoint closed subsets A and B of X, one of which is π-closed, there exist two disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

6. **softly normal [19]** if for any two disjoint subsets A and B of X, one of which is π-closed and the other is regularly closed, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

7. **nearly normal** [**15**] if for any pair of nonempty disjoint sets A and B of X, one of which is δ-closed and the other is regularly closed, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

8. **α-normal** [**2**] if for any two disjoint closed subsets A, B of X, there exist disjoint open subsets U, V of X such that A ∩ U is dense in A and B ∩ V is dense in B.

9. **β-normal** [**2**] if for any two disjoint closed subsets A, B of X, there exist disjoint open subsets U, V of X such that cl(A ∩ U) = A and cl(B ∩ V) = B.

10. **θ-normal** [**9**] if for every pair of disjoint closed sets one of which is θ-closed are contained in disjoint open sets.

11. **weakly θ-normal** (briefly **wθ-normal [9]**) if for every pair of disjoint θ-closed sets are contained in disjoint open sets.

12. **functionally θ-normal** (briefly **fθ-normal [9]**) if for every pair of disjoint closed sets A and B one of which is θ-closed, there exists a continuous function f : X → [0, 1] such that f(A) = 0 and f(B) = 1.

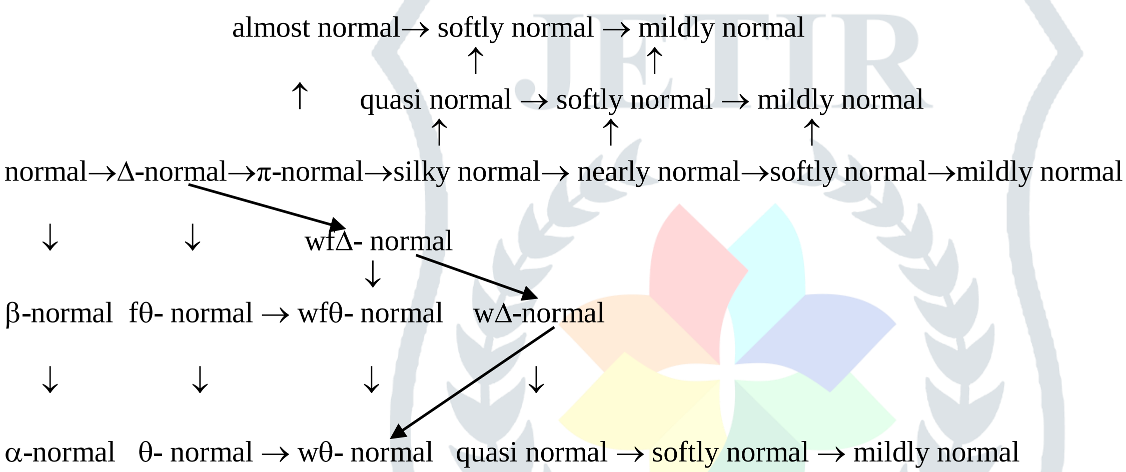
13. **weakly functionally θ-normal** (briefly **wfθ-normal [9]**) if for every pair of disjoint θ-closed sets A and B, there exists a continuous function f : X → [0, 1] such that f(A) = 0 and f(B) = 1.

14. **Δ-normal** [3] if for every pair of disjoint closed sets one of which is δ-closed are contained in disjoint open sets.

15. **weakly Δ-normal** (briefly **wΔ-normal [3]**) if for every pair of disjoint δ-closed sets are contained in disjoint open sets.

16. **weakly functionally Δ-normal** (briefly **wfΔ-normal [3]**) if for every pair of disjoint δ-closed sets A and B, there exists a continuous function f : X → [0, 1] such that f(A) = 0 and f(B) = 1.

The following implications are obvious from the definitions and above observations.



Where none of the implications is reversible as can be seen from the following examples and theorems:

3.2 Theorem. Every π-normal space is silky normal.
Proof. Since every δ-closed set is closed the proof is obvious.

3.3 Theorem. Every silky normal space is nearly normal.
Proof. Since every regular-closed set is π-closed the proof is obvious.

3.4 Theorem. Every silky normal space is softly normal.
Proof. The proof directly follows from the fact, that, every regular closed set is δ-closed.

3.5 Theorem. Every silky normal space is mildly normal.
Proof. The proof directly follows from the fact, that, every regular closed set is π-closed and every regular closed set is δ-closed.

3.6 Example. Let X = {a, b, c, d} and ℑ = {∅, {b}, {d}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X}. Then the space X is Δ-normal as well as π-normal but it is not normal.

3.7 Example. Let X = {a, b, c, d} and ℑ = {∅, {b}, {c}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, X}. Then the space X is nearly normal as well as softly normal but it is not almost normal because there does not exist disjoint open sets that separate disjoint closed set {d} and regular closed set {a, b} respectively.

3.8 Example. Let X = {a, b, c, d} and $\Im$ = {∅, {b}, {c}, {a, b}, {b, c}, {a, b, c}, {b, c, d}, X}. Then the space X is θ-normal but not Δ-normal, since the closed set {a} and the δ-closed set {c, d} can not be separated by disjoint open sets.

3.9 Example. Let X = {a, b, c} and $\Im$ = {∅, {c}, {b, c}, {a, c}, X}. Then the space X is almost normal as well as softly normal.

3.10 Theorem. For a space X, the following are equivalent:
(a) X is silky-normal.
(b) For every π-closed set A and every δ-open set B with A ⊂ B, there exists an open set U such that A ⊂ U ⊂ cl(U) ⊂ B.
(c) For every δ-closed set A and every π-open set B with A ⊂ B, there exists an open set U such that A ⊂ U ⊂ cl(U) ⊂ B.
(d) For every pair consisting of disjoint sets A and B, one of which is π-closed and the other is δ-closed, there exist open sets U and V such that A ⊂ U, B ⊂ V and cl(U) ∩ cl(V) = ∅.
Proof.
(a) ⇒ (b). Assume (a). Let A be any π-closed set and B be any δ-open set such that A ⊂ B. Then A ∩ (X – B) = ∅, where (X – B) is δ-closed. Then there exist disjoint open sets U and V such that A ⊂ U and (X – B) ⊂ V. Since U ∩ V= ∅, then Cl(U) ∩ V = ∅. Thus cl(U) ⊂ (X – V) ⊆ (X – (X – B)) = B. Therefore, A ⊂ U ⊂ cl(U) ⊂ B.

(b) ⇒ (c). Assume (b). Let A be any δ-closed set and B be any π-open set such that A ⊂ B. Then, (X – B) ⊂ (X – A), where (X – B) is π-closed and (X – A) is δ-open. Thus, by (b), there exists an open set W such that (X – B) ⊂ W ⊂ cl(W) ⊂ (X – A). Thus A ⊂ (X – cl(W)) ⊂ (X – W) ⊂ B. So, let U = (X – cl(W)), which is open and since W ⊂ cl(W), then (X – cl(W)) ⊂ (X – W). Thus U ⊂ (X – W), hence cl(U) ⊂ cl(X – W) = (X – W) ⊂ B.

(c) ⇒ (d). Assume (c). Let A be any δ-closed set and B be any π-closed set with A ∩ B = ∅. Then A ⊂ (X – B), where (X – B) is π-open. By (c), there exists an open set U such that A ⊂ U ⊂ cl(U) ⊂ (X – B). Now, cl(U) is closed. Applying (c) again we get an open set W such that A ⊂ U ⊂ cl(U) ⊂ W ⊂ cl(W) ⊂ (X – B). Let V = (X – cl(W)), then V is open set and B ⊂ V. We have (X – cl(W)) ⊂ (X – W), hence V ⊂ (X – W), thus cl(V) ⊂ Cl(X – W) = (X – W). So, we have cl(U) ⊂ W and cl(V) ⊂ (X – W). Therefore cl(U) ∩ cl(V) = ∅

(d) ⇒ (a) is obvious.

Urysohn's type lemma holds for most of the generalized notions of normality. Whereas Urysohn's type lemma does not hold for the class of θ-normal and weak θ-normal spaces defined by Kohli and Das [**9**]. Similarly, Δ-normality defined by Das [**3**] satisfies Urysohn's type lemma in contrary to weak Δ-normality. Using **Theorem 3.10,** it is easy to show the following theorem, which is a Urysohn's Lemma version for silk normality. A proof can be established by a similar way of the normal case.

3.11 Theorem. A space X is silky-normal if and only if for every pair of disjoint sets A and B, one of which is π-closed and the other is δ-closed, there exists a continuous function f on X into [0, 1], with its usual topology, such that f(A) = {0} and f(B) = {1}.

Urysohn's type lemma holds for silky normal spaces which is useful in the sequel to obtain an example of a mildly normal space which in not silky normal.

3.12 Example. A space X which is mildly normal but not silky normal.
Let X be the simplified Arens square in which S be the unit square and X = int(S) ∪ {(0, 0), (1, 0)}. Define the topology on X as defined in [**22**] by taking usual Euclidean neighbourhood for every point in int(S). The points (0, 0) and (1, 0) have neighbourhoods of the form U_n and V_n respectively, where

$$U_n = \{(0, 0)\} \cup \{(x, y) : 0 < x < \frac{1}{2}, 0 < y < \frac{1}{n}\}$$

and

$$V_n = \{(1, 0)\} \cup \{(x, y) : \frac{1}{2} < x < 1, 0 < y < \frac{1}{n}\}.$$

Since every pair of disjoint regularly closed sets in X are separated by disjoint open sets, then the space X is mildly normal. But the δ-closed set A = {(0, 0)} disjoint from π-closed set B = $\{(x, y) : 0 < x \le \frac{1}{2}, 0 < y \le \frac{1}{2}\}$can not be functionally separated. Thus by **Theorem 3.11**, the space X is not silky normal.

The following theorem gives the characterization of silky normal spaces in term of θ-open sets. Since θ-closure of a set defined by Velicko may not be θ-closed [**7**],θ-closure operator is not Kuratowski closure operator. In [**16**], M. Mrsevic and D. Andrijevic observed that θ-closure operator is Cech closure operator. Kohli and Das [**11**] obtained uθ-closure operator which is a Kuratowski closure operator.

3.13 Definition. Let X be a topological space and let $A \subset X$. A point $x \in X$ is called a **uθ-limit point** [**10**] of A if every θ-open set U containing x intersects A. Let $A_{u\theta}$ denotes the set of all uθ-limit points of A.

3.14 Lemma [11]. The correspondence $A \rightarrow A_{u\theta}$ is a Kuratowski closure operator.

It turns out that the set $A_{u\theta}$ is the smallest θ-closed set containing A. The above discussed uθ-closure operator is the closure operator in the θ-topology [$\mathfrak{I}_\theta$ for the space (X, $\mathfrak{I}$)]

3.15 Theorem. For a topological space X, the following statements are equivalent:
(a) X is silky normal.
(b) For every pair of disjoint sets A and B out of which A is δ-cosed and B is π-closed, there exist disjoint θ-open sets U and V such that $A \subset U$ and $B \subset V$
(c) For every δ-closed set A and every π-open set U with $A \subset U$, there exists a θ-open set V such that $A \subset V \subset V_{u\theta} \subset U$.
(d) For every π-closed set A and every δ-open set U with $A \subset U$, there exists a θ-open set V such that $A \subset V \subset V_{u\theta} \subset U$.
(e) For every pair of disjoint of disjoint sets A and B, one of which is δ-closed and the other is π-closed, there exist θ-open sets U and V such that $A \subset U$, $B \subset V$ and $U_{u\theta} \cap V_{u\theta} = \varnothing$.

Proof.
(a) ⇒ (b). Let X be a silky normal space and let A be any δ-closed set disjoint from the π-closed set B. Thus by **Theorem 3.11**, there exists a continuous function $f : X \rightarrow [0, 1]$ such that f(A) = 0 and (B) = 1. Since the inverse image of a θ-open set is θ-open under a continuous mapping, $f^{-1}([0, \frac{1}{2}))$ and $f^{-1}((\frac{1}{2}, 1])$ are two disjoint θ-open sets containing A and B respectively.
(b) ⇒ (c). Let A be any δ-closed set contained in a π-open set U. Here A is a δ-closed set disjoint from the π-closed set B = X – U. So, by (b), there exist disjoint θ-open sets V and W such that $A \subset V$ and $B \subset W$. Thus $A \subset V \subset X - W \subset U$. As X – W is θ-closed and $V_{u\theta}$ is the smallest θ-closed set containing V, $A \subset V \subset V_{u\theta} \subset U$.

Similarly, the implications (c) ⇒ (d) and (d) ⇒ (e) can be proved and the proof of (e) ⇒ (a) is obvious as every θ-open set is open.

Recall a covering $\mu = \{U_\alpha : \alpha \in \Lambda\}$ of a topological space X is said to be a shrinking of X if there exists a cover $\nu = \{V_\alpha : \alpha \in \Lambda\}$ of X such that $cl(V_\alpha) \subset U_\alpha$ for each $\alpha \in \Lambda$.

3.16 Theorem [15]. If X is a nearly normal space then every point finite regularly open cover of X is shrinkable.

3.17 Corollary. In a silky normal space X then every point finite regularly open cover of X is shrinkable.

It is easy to see that the inverse image of a δ-closed set under an open continuous function is δ-closed and the inverse image of a π-closed set under an open continuous function π-closed. We will use that in the next theorem.

3.18 Theorem. Let X is a silky normal space and $f : X \rightarrow Y$ is an open continuous injective function. Then f(X) is a silky normal space.

Proof. Let A be any π-closed subset in f(X) and let B be any δ-closed subset in f(X) such that $A \cap B = \varnothing$. Then $f^{-1}(A)$ is a π-closed set in X, which is disjoint from the δ-closed set $f^{-1}(B)$. Since X is silky normal, there are two disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is one-one and open, result follows.

3.19 Corollary. Silk normality is a topological property.

3.20 Theorem. A closed domain subspace of a silky normal space is silky normal.

Since every closed and open (clopen) subset is a closed domain, then we have the following corollary.

3.21 Corollary. Silk normality is a hereditary with respect to clopen subspaces.

4. Decompositions of normal spaces

4.1 Definition. A space X is said to be **silky regular** if for every π-closed set A and each open set U containing A there exists a δ-open set V such that $A \subset V \subset U$.

4.2 Definition. A space X is said to be **nearly regular** [**4**] if for every regularly closed set A and each open set U containing A, there exists a δ-open set V such that $A \subset V \subset U$.

4.3 Definition. A space X is said to be **Δ-regular** [**3**] if for every closed set A and each open set U containing A, there exists a δ-open set V such that $A \subset V \subset U$.

4.4 Definition. A space X is said to be **seminormal** [**24**] if for every closed set A and each open set U containing A, there exists a regular open set V such that $A \subset V \subset U$.

4.5 Definition. A space X is said to be **weakly seminormal** [**5**] if for every closed set A and each open set U containing A, there exists a π-open set V such that $A \subset V \subset U$.

Every seminormal space is weakly seminormal normal but the converse need not be true.

4.6 Example. Let X = {a, b, c, d, e} and $\Im$ = {∅, {a}, {d}, {a, b}, {a, d}, {c, d}, {a, d, e}, {a, b, c, d}, X}. Then the space X is weakly seminormal because the closed set {b, c} contained in the open set {a, b, c, d}. But since there is no regularly open set contained in the open set {a, b, c, d} containing the closed set {b, c}, the space X is not seminormal. However, being union of regularly open sets {a, b} and {c, d}, the set {a, b, c, d} is itself π-open.

By the definitions stated above, we have the following diagram:

weakly seminormal

⇑

normal ⇒ seminormal ⇒ Δ-regular ⇒ silky regular ⇒ nearly regular

⇑

regular ⇒ semi-regular

Where none of the implications is reversible.

4.7 Example [4]. A nearly regular space which is not Δ-regular. Let X be an infinite set with co-finite topology in which the only regularly closed set is the whole space X. Thus the space is vacuously nearly regular but not Δ-regular as for a closed set A containing finite points and contained in an open set U there does not exist a δ-open set V satisfying $A \subset V \subset U$.

The following theorems provide few decompositions of normality in terms of other variants of normality which lies between normal and mildly normal spaces.

4.8 Theorem. In a weakly seminormal space, the following statements are equivalent:
(a) X is normal.
(b) X is Δ-normal.
(c) X is wfΔ-normal.
(d) X is wΔ-normal.
(e) X is π-normal.

(f) X is silky normal.
(g) X is quasi normal.
Proof. The proof of (a) ⇒ (b) ⇒ (e) ⇒ (f) ⇒ (g) and (a) ⇒ (b) ⇒ (c) ⇒ (d) ⇒ (g) are obvious from the definitions.

To prove (g) ⇒ (a), let X be a weakly seminormal quasi normal space. Let A and B be two disjoint closed sets in X. Then X – B is an open set containing the closed set A. Since is weakly seminormal, there exists a π-open set U such that A ⊂ U ⊂ X – B. Thus A is a closed set which is disjoint from the π-closed set X – U. So, X – A is an open set containing the π-closed set X – U. Again by weakly seminormality of X, there exists a π-open set W such that X – U ⊂ W ⊂ X – A. Thus X – W and X – U are disjoint π-closed sets containing A and B respectively. By quasi normality of X, there exist disjoint open sets P and Q such that X – W ⊂ P and X – U ⊂ Q. Thus P and Q are disjoint open sets containing A and B respectively.

4.9 Theorem. In a seminormal space, the following statements are equivalent:
(a) X is normal.
(b) X is Δ-normal.
(c) X is wfΔ-normal.
(d) X is wΔ-normal.
(e) X is π-normal.
(f) X is quasi normal.
(g) X is silky normal.
(h) X is nearly normal.
(i) X is softly normal.
(j) X is mildly normal.
Proof. The implications (a) ⇒ (b) ⇒ (c) ⇒ (d) ⇒ (f) ⇒ (i) ⇒ (j) are observed in [**3**] and [**8**]. In [**1**], it is shown that the implications (a) ⇒ (b) ⇒ (e) ⇒ (g) ⇒ (h) ⇒ (i) ⇒ (j) follows from [**3**] and [**8**]. The prove of the implication (i) ⇒ (a) is similar to the prove of **Theorem 4.8**.
4.12 Theorem. In a β-normal space, the following statements are equivalent:
(a) X is normal.
(b) X is Δ-normal.
(c) X is π-normal.
(d) X is silky normal.
(e) X is nearly normal.
(f) X is softly normal.
(g) X is wfΔ-normal.
(h) X is wΔ-normal.
(i) X is quasi normal.
(j) X is mildly normal.
Proof. The proof of (a) ⇒ (b) ⇒ (c) ⇒ (d) ⇒ (e) ⇒ (f) ⇒ (j) and (a) ⇒ (b) ⇒ (g) ⇒ (h) ⇒ (i) ⇒ (j) are obvious from the definitions. The above of (j) ⇒ (a) directly follows from the fact that a space is normal iff it is mildly normal and β-normal [**2**].

5. Silky normal spaces with other separation axioms

5.1 Definition. A space is called **paracompact** if every open covering of the space admits a locally finite open refinement.
5.2 Theorem. Every weakly regular paracompact space is silky normal.
Proof. Let X is a weakly regular, paracompact space and let A be a δ-closed subset of X. Let B be a π-closed set such that A ∩ B = ∅. Now, for each point x ∈ A, cl{x} is contained in A. Therefore, X – B is a π-open set containing cl{x}. Since X is weakly regular, there is an open set V_x containing x such that cl(V_x) ∩ B = ∅. The family of subsets {V_x : x ∈ A} ∪ {X – A} is an open covering of X and so there is a locally finite open refinement of it. Suppose that μ = {U_α} denotes the members of that family which have nonempty intersections with A. Let $U_1 = \cup_\alpha U_\alpha$, which is clearly an open set containing A. Let $U_2 = X - \cup_\alpha \text{cl}(U_\alpha)$ which is an open set, because {U_α} being locally finite, $\text{cl}(\cup_\alpha U_\alpha) = \cup_\alpha \text{cl}(U_\alpha)$. Also, $U_1 \cap U_2 = \varnothing$. Since μ is a refinement and each member of it intersects A, for each U_α there must exist an x ∈ A such that $U_\alpha \subset V_x$. Now, since cl(U_α) ⊂ cl(V_x) ⊂ X – B, B ⊂ X – V_x ⊂ X – U_α for every U_α ∈ μ. Hence, B ⊂ U_2 and so X is silky normal.
In view of the fact that every compact space is paracompact, every softly-regular space is weakly regular, every partly regular space is weakly regular and every almost regular space is weakly regular, we have following corollaries:

5.3 Corollary. Every partly regular paracompact space is silky normal.

5.4 Corollary. Every softly regular paracompact space is silky normal.

5.5 Corollary. Every almost regular paracompact space is silky normal.

We know that every paracompact Hausdorff space is almost regular.

5.6 Corollary. Every paracompact Hausdorff space is silky normal.

5.7 Corollary. Every weakly regular compact space is silky normal.

5.8 Corollary. Every partly regular compact space is silky normal.

5.9 Corollary. Every softly regular compact space is silky normal.

5.10 Corollary. Every almost regular compact space is silky normal.

We know that every almost normal T_1-space is almost regular.
5.11 Corollary. Every compact almost normal T_1-space is silky normal.

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