VARIABLE VISCOSITY EFFECTS ON UNSTEADY LAMINAR FLOW AND HEAT TRANSFER OVER A STRETCHING SHEET

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Abstract : The purpose of this paper is to present the effect of variable viscosity on unsteady laminar boundary layer flow and the heat transfer of a fluid over a stretching sheet that is studied numerical manner. It is expected that the unsteadiness is noticed by time dependent stretching velocity and a sudden growth in surface temperature. This is done by using similarity solutions of partial differential equations corresponding to the momentum and energy equations that are converted into highly non-linear ordinary differential equations. Numerical solutions of these equations are obtained with the help of an implicit finite difference scheme along with a quasi-linearization technique. It shows an increase in the unsteady parameter whenever skin friction and heat transfer coefficients increase, while fluid velocity and temperature keep decreasing with an increase in temperature dependent viscosity. Similar trends have been obtained for different Prandtl number for any fixed viscosity and unsteady parameter.

Key words - stretching sheet, unsteady flow, skin friction, heat transfer, variable viscosity.

I. Introduction

The analysis of the flow field in a boundary layer near the stretching sheet is an important part in fluid dynamics and also heat transfer occurring in a number of engineering processes such as extraction of plastic, rubber sheets, polymer processing and metallurgy. The quality of the final product depends on the rate of heat transfer on the stretching surface. The physical situation was discovered as a backward boundary layer problem by Crane [1] who studied the boundary-layer flow caused by a moving stretching surface maintaining a temperature i.e. a constant on the surface of the ambient fluid. He gave a resemblance solution for a closed analytical form of steady two dimensional incompressible boundary layer flows. The case study concerned the case of velocity linearly variation with distance chosen from a fixed point. Carragher and Crane [2] considered the influence of heat transfer on the flow over a stretching surface in the case of temperature difference arising between the surface and the ambient fluid. The numerical study of steady heat transfer over a stretching surface with a variable surface heat flux and uniform heat flux subjected to injection and suction is conducted by Elbashbeshy [3]. In this study he has considered the case of steady flow and heat transfer. However, the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate studied by Ioan Pop [4]. The influence of variable viscosity on the flow and heat transfer on a continuous stretching surface is carried by A. Hassanien [5]. The results further on the variable viscosity on flow and heat transfer to a continuous moving flat plate studied by Pantokratoras A [6]. The work extended the dimension of the problem of heat transfer due to stretching sheet on unsteady laminar mixed convection boundary layer flow and heat transfer due to a stretching vertical surface by Ishak *et al.*

[7]. The effect is included the flow characteristics are seen to change substantially when compared to constant viscosity assumption carried by Gary et al. [8] and Mehta and Sood [9]. The effects of variable viscosity and thermal conductivity on a thin film flow over a stretching sheet is studied by Yasir Khan et al. [10]. The unsteady boundary layer flow and heat transfer due to stretching sheet for the especial distribution of the stretching velocity and surface temperature investigated by Sharidan et al. [11].

All the above said studies confined their discussions to assuming uniformity of fluid viscosity. However, it is accepted that the physical properties of the fluid may change significantly with a change in temperature. The increase of temperature leads to an increase locally in the transport phenomena due to reducing the viscosity across the momentum boundary layer and then the rate of heat transfer at the wall is also affected. Therefore, to predict the flow behavior accurately, it is necessary to take into account the viscosity variation for incompressible fluids.

The present work deals with unsteady fluid flow and heat transfer over a stretching sheet with temperature dependent viscosity. Fluid viscosity is assumed to vary as inverse functions of temperature. Similarity variable and similarity solutions are obtained and using them, a third order and a second order ordinary differential equation corresponding to momentum equation is derived. These equations are solved numerically by an implicit finite difference scheme along with quasi-linearization technique. The effects of different parameters viz. unsteadiness, temperature-dependent fluid viscosity on velocity and temperature fields are investigated and analyzed with the help of their graphical representations.

II. MATHEMATICAL FORMULATION

Consider an unsteady two-dimensional boundary layer flow and heat transfer over a continuous stretching sheet embedded in a moving viscous, incompressible, fluid in the region $y > 0$, as shown in Fig.1

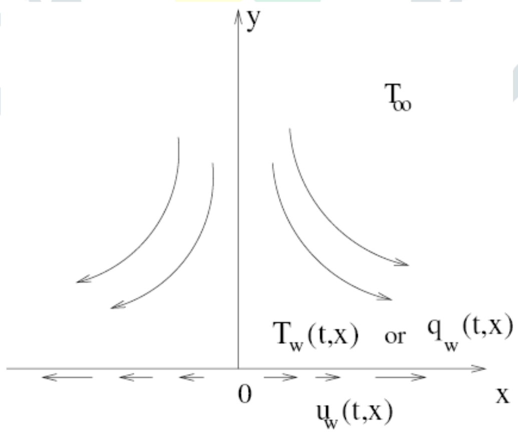

Fig.1 The Geometry and coordinate system.

Keeping the origin steady, two equal and opposite forces are suddenly applied along the x-axis, which result in stretching of the sheet that causes flow generation. At the same time, the wall temperature $T_w(t, x)$ of the sheet is suddenly raised from T_∞ to $T_w(t, x)$ ($> T_\infty$). As stated in the introduction, property variations with temperature are limited to density and viscosity. However, variations in the density are taken into account only in so far as its effect on the buoyancy term in the momentum equation is concerned.

The basic unsteady boundary layer equations governing momentum and heat transfer is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Subject to the initial conditions

$$\begin{gathered} t<0; u=v=0, T=T_\infty \quad for\ any\ x,y \\ t\geq 0; u=u_w(t,x),\ v=0 \\ T=\ T_w(t,x) \quad u\to 0,\ T\to\infty \text{ at } y\to\infty \end{gathered} \tag{4}$$

In the present investigation, a semi-empirical formula for the viscosity of the form

$$\frac{\mu}{\mu_\infty}=\frac{1}{1+\gamma(T-T_\infty)} \tag{5}$$

as developed by Ling and Dybbs [12] has been adopted, where μ_∞ is the viscosity of the ambient fluid. Here u and v are velocity components along x and y- directions, respectively, where t is the time, σ, ρ and ϑ denote respectively, electrical conductivity, density and kinematic viscosity. T is the temperature, α is the thermal diffusivity and k is the thermal conductivity. Here, we assume that, the velocity of the sheet is $u_w(t,x)$ and the sheet temperature $T_w(t,x)$ have the following form.

$$(t,x)\ =\frac{cx}{1-\gamma t}, \qquad T_w(t,x)=T_\infty+\frac{c}{2\vartheta x^2(1-\gamma t)^{3/2}} \tag{6}$$

Where c is the stretching rate being a positive constant and γ is a positive constant which measures the unsteadiness of heat transfer quantity. We introduce now the following new variables

$$\eta=\sqrt{\frac{c}{\vartheta(1-\gamma t)}}\,y, \qquad \psi=\sqrt{\frac{c\vartheta}{(1-\gamma t)}}\,xf(\eta)$$

$$T=T_\infty+\frac{c}{2\vartheta x^2(1-\gamma t)^{3/2}}\theta(\eta) \tag{7}$$

Where ψ is the stream function which is defined as

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x}, \quad \theta=\frac{T-T_\infty}{T_W-T_\infty}$$

Substituting the transformations given in (6) and (7) into Equations (1)-(3), we obtain the following ordinary differential equations

$$f'''-\left(\frac{\varepsilon}{1+\varepsilon\theta}\right)\theta' f''-(1+\varepsilon\theta)A\left(f'+\frac{1}{2}\eta f''\right)-(1+\varepsilon\theta)f'^2+(1+\varepsilon\theta)ff''=0 \tag{8}$$

$$\frac{1}{Pr}\theta''+f\theta'+2f'\theta-\frac{1}{2}A(3\theta+\eta\theta')=0 \tag{9}$$

Subject to the boundary conditions (4), which becomes,

$$\begin{gathered} f(0)=0,\ f'(0)=1,\ f'(\infty)=0 \\ \theta(0)=1 \quad \text{or} \quad \theta(\infty)=0 \end{gathered} \tag{10}$$

where $\varepsilon\ [=(T_w-T_\infty)\gamma]$ is termed as the viscosity variation parameter, that is positive for heated surface and negative for a cooled surface. Here η is taken as the transformed dimensionless independent variable, f is the dimensionless stream function and f' is the dimensionless velocity. Where Pr is the Prandtl number, θ is dimensional temperature, $A = \frac{\gamma}{c}$ is a non-dimensional constant which measures unsteadiness of the flow and heat transfer and the prime$(')$ denotes the differentiation with respect to the similarity variable η. The parameter of engineering interest is the skin friction coefficient (c_f) and heat transfer coefficient in terms of local Nusselt number(N_{ux}) is given by

$$c_f\sqrt{(R_{ex})} = \frac{\tau_w}{\rho(u_w)^2} = \frac{\mu(\frac{\partial u}{\partial y})_{y=0}}{\rho(u_w)^2} = \frac{f''(0)}{(1+\varepsilon\theta)} \tag{11}$$

where μ being the dynamic viscosity, τ_w is the skin friction and q_w is the heat transfer from the sheet.

$$\frac{N_{ux}}{\sqrt{R_{ex}}} = \frac{xq_w}{k(T_w - T_\infty)} = \frac{-xk(\frac{\partial u}{\partial y})_{y=0}}{k(T_w - T_\infty)} = -\theta'(0) \tag{12}$$

where $R_{ex} = \frac{u_w x}{\vartheta}$ is the local Reynolds number.

III. NUMERICAL SOLUTION

The system of coupled, nonlinear partial differential equations (8) and (9) along with the boundary conditions (10) using the relations (11) - (12) has been solved numerically employing an implicit finite difference scheme using quasilinearization technique. A unique feature of the quasilinear implicit finite difference scheme is quadratic convergence and monotonicity, which has been found superior to the built in iteration of the upwind technique or finite amplitude technique. The efficiency and accuracy of this method has been illustrated through its applications in many boundary value problems, mentioned in the book by Bellman and Kalaba [13].

Applying the quasilinearization technique to the non-linear coupled partial differential equations (8) and (9) having boundary conditions (10) are replaced by the following sequence of linear ordinary differential equations

$$^{(k+1)}f''' + ^{(k)}X_1\, ^{(k+1)}f'' + ^{(k)}X_2\, ^{(k+1)}f' + ^{(k)}X_3\, ^{(k+1)}\theta' + ^{(k)}X_4\, ^{(k+1)}\theta = ^{(k)}U_1 \tag{13}$$

$$^{(k+1)}\theta'' + ^{(k)}Y_1\, ^{(k+1)}\theta' + ^{(k)}Y_2\, ^{(k+1)}\theta + = ^{(k)}U_2 \tag{14}$$

The coefficient functions with iterative index k are known and the functions with iterative index (k + 1) are to be determined. The corresponding boundary conditions given by

$$^{(k+1)}f' = 0, \quad ^{(k+1)}\theta = 1 \quad at \quad \eta = 0$$
$$^{(k+1)}f' = 0, \quad ^{(k+1)}\theta = 0 \quad at \quad \eta = \infty \tag{15}$$

The coefficients of equations (13) and (14) are given by

$$^{(k)}X_1 = -\left(\frac{\varepsilon\theta\prime}{1+\varepsilon\theta}\right) - A(1+\varepsilon\theta)\left(\frac{\eta}{2}\right) + (1+\varepsilon\theta)f$$

$$^{(k)}X_2 = -A(1+\varepsilon\theta) - 2f'(1+\varepsilon\theta)$$

$$^{(k)}X_3 = -\left(\frac{\varepsilon f\prime\prime}{1+\varepsilon\theta}\right)$$

$$^{(k)}X_4 = \left(\frac{\varepsilon^2\theta\prime f\prime\prime}{1+\varepsilon\theta}\right) - \varepsilon A\left(f''\left(\frac{\eta}{2}\right) + f'\right) - \varepsilon f'^2 + \varepsilon f f''$$

$^{(k)}U_1 = -f'^2(1+\varepsilon\theta) - \left(\frac{\varepsilon f''\theta}{1+\varepsilon\theta}\right) + X_4\theta$

$^{(k)}Y_1 = fP_r - P_r A(\frac{\eta}{2})$

$^{(k)}Y_2 = 2f'P_r - (\frac{3}{2})P_r A$

$^{(k)}U_2 = 0$

Since the method is presented for ordinary differential equations by Inouye and Tate [14] and for partial differential equations recently reported by Srinivasa and Eswara [15, 16], its description omitted here for the sake of brevity. At each iteration step, the sequence of linear partial differential equations (13) and (14) along with the boundary conditions (15), were expressed in difference form using the central difference scheme in the η-direction. Thus at each step, the resulting equations were deduced to a system of linear algebraic equations with a block tri-diagonal matrix, which can be solved by Varga`s algorithm [17].

IV. RESULTS AND DISCUSSION

The transformed Equation consisting of (8) and (9) when subjected to the boundary condition (10) is solvable in a numerical manner by using a stable finite difference method in addition to quasilinearization technique. The results obtained are presented through the graphs for velocity and temperature profiles in different values of unsteadiness parameter (A) and Prandtl number (Pr) and is as shown in Figs.2 – 5 have been computed and these have been compared with the different values of variable viscosity. The computed results have been compared with those of Sharidan et al. [11], for both skin friction and heat transfer as is given coefficients in Table 1, Pr =0.01, 0.1 and 1.0. Our results are found to be in good agreement with those of [11] correct to four decimal places of accuracy.

Table 1. Comparison results for the values of the heat transfer $[\theta'(0)]$ and skin friction coefficient $[f''(0)]$ for various values of A and Pr [11].

$\varepsilon = 0$												
A	0.8				1.2				2			
	Present	Previous	Present	Previous	Present	Previous	Present	Previous	Present	Previous	Present	Previous
Pr	$\theta'(0)$	$\theta'(0)$	$-f''(0)$	$-f''(0)$	$\theta'(0)$	$\theta'(0)$	$-f''(0)$	$-f''(0)$	$\theta'(0)$	$\theta'(0)$	$-f''(0)$	$-f''(0)$
0.01	0.2091	0.2092	1.2591	1.2610	0.2175	0.2174	1.3747	1.3777	0.2338	0.2331	1.5816	1.5873
0.1	0.2630	0.2629	1.2591	1.2610	0.3305	0.3306	1.3747	1.3777	0.4457	0.4387	1.5816	1.5873
1	0.4722	0.4712	1.2591	1.2610	0.7890	0.7882	1.3747	1.3777	1.2445	1.2437	1.5816	1.5873

Table 2.Given below are the values obtained as results for the heat transfer $[\theta'(0)]$ and skin friction coefficient $[f''(0)]$ for various values of A and Pr with ε = 0.5

$\varepsilon = 0.5$				
A	0.8		1.2	
Pr	$\theta'(0)$	$-f''(0)$	$\theta'(0)$	$-f''(0)$
0.1	0.2812	1.5627	0.3452	1.7074
0.72	0.4788	1.5815	0.7284	1.7351

Table 2. gives the effect of skin friction and heat transfer coefficients for a fixed variable viscosity ($\varepsilon = 0.5$). In this we see an increase occurring for skin friction as well as heat transfer increase for various Prandtl number (Pr) with time parameter (A). The percentage of increased skin friction coefficient is 14.47 % and heat transfer is about 6.40 % for A = 0.8 to 1.2.

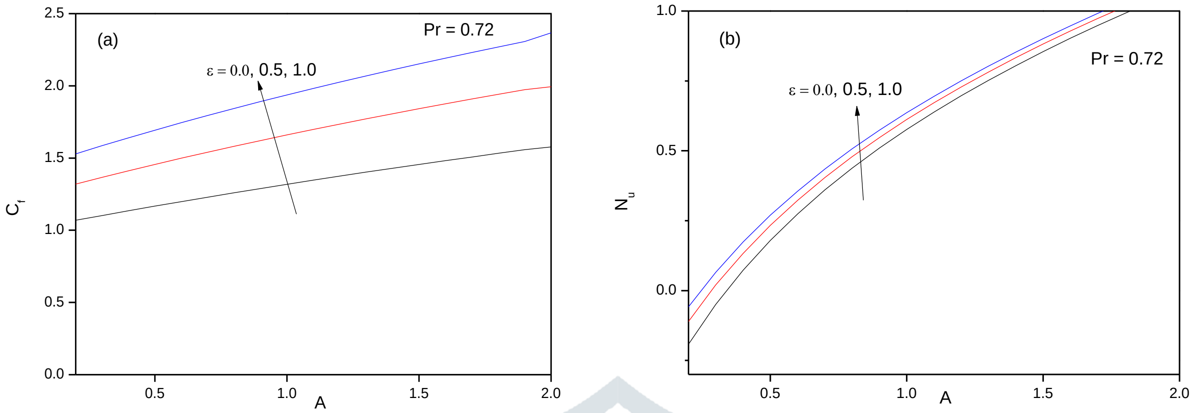

Fig. 2(a) Skin friction and (b) Heat transfer coefficient for different values of ε

The variable viscosity effect (ε) on skin friction $[c_f]$ and heat transfer $[N_u]$ coefficients for fixed Prandtl number ($Pr = 0.72$) is displayed in the Fig. 2(a) and 2(b). It is found that both $[c_f]$ and $[N_u]$ increase whenever there is an increase in the variable viscosity parameter. The percentage of increase in $[c_f]$ is 61.92% and $[N_u]$ is 5.99%.

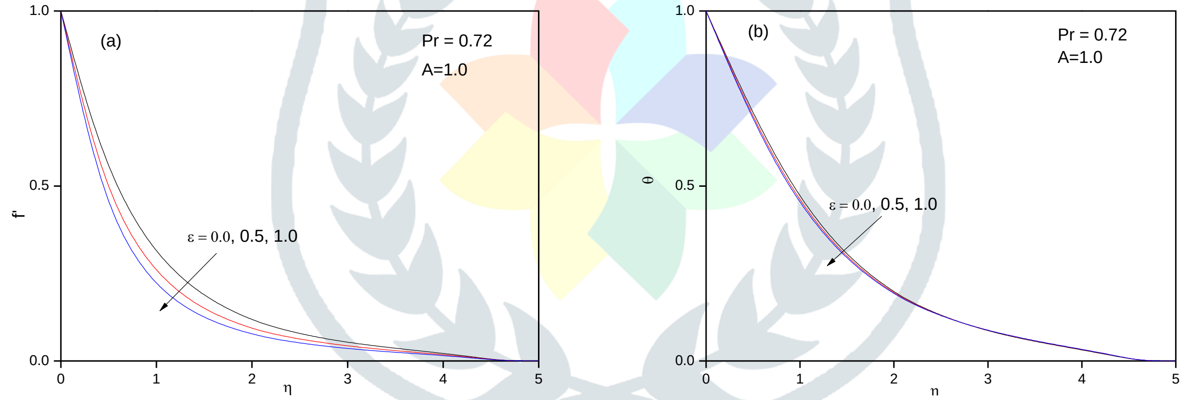

Fig. 3(a) The velocity and (b) temperature profiles for different values of ε with A = 1.0

Fig. 3 represents the impact of variable viscosity (ε) on the corresponding profiles of velocity and temperature profiles for a fixed Pr =0.72 and A = 1.0. These show that both velocity and temperature decreases with a rise of ε.

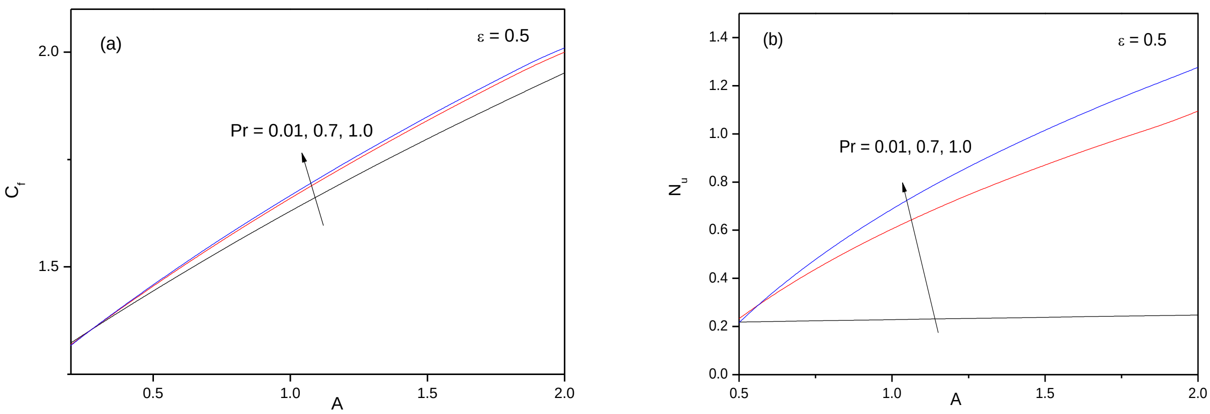

Fig. 4(a) Skin friction and (b) Heat transfer coefficient for different values of *Pr*

The impact of variable viscosity on skin friction $[c_f]$ and heat transfer $[N_u]$ coefficients for different Prandtl numbers (Pr = 0.01, 0.7, 1.0) when ε =0.5 is individually represented, in the Figs. 4(a) and 4(b). It is noticed that both $[c_f]$ and $[N_u]$ increase whenever there is a increase of Pr. The percentage of increase in $[c_f]$ is 3.65% and $[N_u]$ is 45.97%.

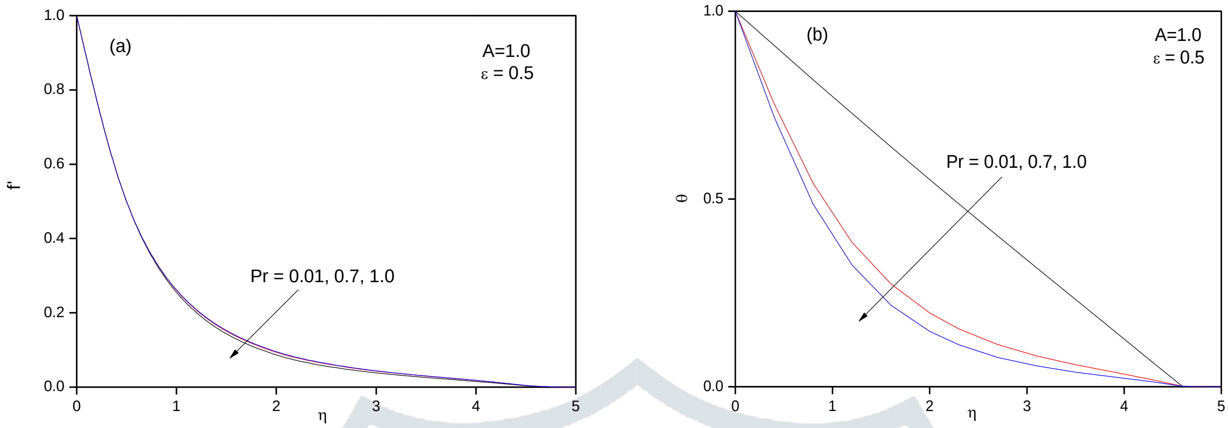

Fig. 5 (a) The velocity and (b) temperature profiles for different values of Pr with ε =0.5 and A=1.0

Fig.5 represents the variation of velocity and temperature for different values of the Prandtl number (*Pr*) using fixed parameter *A = 1.0* and ε =0.5. It shows that there is a decreases in the both the velocity and temperature as the Prandtl number increases for fixed value of η. The temperature decreases as the distance away from the sheet increases and it becomes almost zero at $\eta = 5$ which ends the boundary layer thickness. The temperature decreases with in the boundary layer for all values of Pr. This is consistent with the fact the boundary layer thickness decreases with an increase of η but negligible amount in velocity.

V. CONCLUSIONS

For different values of relevant physical parameters, the effect of unsteady laminar flow and heat transfer over a screeching sheet with temperature dependent variable viscosity, from the present investigation the following conclusions may be drawn.

i. Both the skin friction and heat transfer coefficient increases with the increase of variable viscosity parameter.
ii. For the fixed Prandtl number velocity and temperature profile decreases with the increase of variable viscosity.
iii. Increase in Prandtl number both the skin friction and heat transfer coefficient increases.
iv. The velocity and temperature decrease along the sheet with a corresponding increase of Prandtl number.

VI. ACKNOWLEDGEMENT

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