TWO-DIMENSIONAL LANGUAGE IN AUTOMATA

[1]M.MARIMUTHU, [2] Dr.M.KAVITHA

[1]Research scholar, Department of Mathematics, SPIHER, Avadi, Chennai-54,
[2]Assistant Professor, Department of Mathematics, SPIHER, Avadi, Chennai-54.

Abstract:

Two-dimensional languages can be recognized by tiling systems. A tiling system becomes an effective device for recognition when a scanning strategy on pictures is fixed. We define a Tiling Automaton as a tiling system together with a scanning strategy and a suitable data structure. In this framework it is possible to define determinism, nondeterminism and unambiguity. The class of languages accepted by tiling automata coincides with REC family. Tiling automata are able to simulate on-line tessellation automata. Then tiling automata are compared with the other known automata models for recognition of two-dimensional languages.

Keywords: Automata, Formal Languages, Two-dimensional languages, Equivalence theorem, Tiling system.

Introduction:

The families of two- dimensional languages defined on different approaches to recognize or generate pictures that were all generalizations from the one-dimensional languages theory. It turns out that equivalence theorems hold among these families and that such theorems are, in some sense, the analogous of fundamental equivalence theorems among the families of recognizable string languages.

1.Tiling systems and automata

Tiling systems for picture languages were defined generalizing to the two dimensional case a characterization of finite automata for strings in terms of local sets and projections. In K. Inoue and I. Takanami proved that tiling systems can be viewed as machine devices.

More specifically, a tiling system can simulate an on-line tessellation automaton and vice versa. This is the contents of the following theorem.

Theorem 1.1:

$$\mathscr{L}(2OTA) = \mathscr{L}(TS)$$

To make the proof of the theorem easier to read, *we split the theorem in two lemmas corresponding to the two inclusions in the theorem.*

Lemma 1.1.:

If a language is recognized by two-dimensional on-line tessellation automata then it is recognized by finite tiling systems
($\mathscr{L}(2OTA) \subseteq \mathscr{L}(TS)$).

Proof:

Let L $\subseteq \Sigma^{**}$ *be* a language recognized by a two-dimensional on-line tessellation automaton **A** = (*Σ*, *Q*, *I* , *F* , □). We have to show that there exists a tiling system T that recognizes L. Let T = (Σ, □, Θ, π) be a tiling system such that:

- □ = Σ ∪ {#} × Q;

- Θ = $\Theta_m \cup \Theta_t \cup \Theta_b \cup \Theta_l \cup \Theta_r \cup \Theta_{tl} \cup \Theta_{tr} \cup \Theta_{bl} \cup \Theta_{br}$ (where m,t,b,l,r stand for "middle", "top", "bottom","left","right", respectively) with:

$$\Theta_m = \left\{ \begin{array}{|c|c|} \hline (a,r) & (b,s) \\ \hline (c,t) & (c,t) \\ \hline \end{array} \;\middle|\; a,b,c,d \neq \# \text{ and } q \in \delta(s,t,d) \right\}$$

$$\Theta_t = \left\{ \begin{array}{|c|c|} \hline (\#,q_0) & (\#,q_0) \\ \hline (a,s) & (b,q) \\ \hline \end{array} \;\middle|\; a,b \neq 0 ,\, q_0 \in I \text{ and } q \in \delta(q_0,s,b) \right\}$$

$$\Theta_b = \left\{ \begin{array}{|c|c|} \hline (a,s) & (b,q) \\ \hline (\#,q_0) & (\#,q_0) \\ \hline \end{array} \;\middle|\; a,b \neq 0 ,\, q_0 \in I \right\}$$

$$\Theta_l = \left\{ \begin{array}{|c|c|} \hline (\#,q_0) & (a,s) \\ \hline (\#,q_0) & (b,q) \\ \hline \end{array} \;\middle|\; a,b \neq 0\, \# ,\, q_0 \in I \text{ and } q \in \delta(s,q_0,b) \right\}$$

$$\Theta r = \left\{ \begin{array}{|cc|} \hline (a,s) & (\#,q_0) \\ (b,q) & (\#,q_0) \\ \hline \end{array} \;\middle|\; a,b \neq 0\, \# \text{ and } q_0 \in I \right\};$$

$$\Theta tl = \left\{ \boxed{\begin{matrix} (\#, q_0) & (\#, q_0) \\ (\#, q_0) & (a, q) \end{matrix}} \mid a \neq \#, q_0 \in I \text{ and } q \in \delta(s, q_0, b) \right\};$$

$$\Theta tr = \left\{ \boxed{\begin{matrix} (\#, q_0) & (\#, q_0) \\ (a, q) & (\#, q_0) \end{matrix}} \mid a \neq \# \text{ and } q_0 \in I \right\};$$

$$\Theta bl = \left\{ \boxed{\begin{matrix} (\#, q_0) & (a, q) \\ (\#, q_0) & (\#, q_0) \end{matrix}} \mid a \neq \# \text{ and } q_0 \in I \right\};$$

$$\Theta br = \left\{ \boxed{\begin{matrix} (a, q_f) & (\#, q_0) \\ (\#, q_0) & (\#, q_0) \end{matrix}} \mid a \neq \# \text{ and } q_0 \in I, q_f \in F \right\};$$

-$\pi : (\Sigma \cup \{\#\}) \times Q \rightarrow \Sigma$ such that $\pi(a,q) = a$, for all $a \in \Sigma \cup \{\#\}$, $q \in Q$

Lemma 1.2.:

If a language is recognizable by finite tiling systems then it is recognizable by two-dimensional on-line tessellation automata

($\mathscr{L}(2OTA) \subseteq \mathscr{L}(TS)$).

2. Tiling systems and regular expression :

Let $L \subseteq \Sigma^{**}$ be a language defined by a finite tiling system. L can be defined as well by domino system, i.e.,L is a projection of a hv-local language K. We first , analyze the relations between hv-local Languages and languages denoted by regular expressions.

Theorem 2.1.:

The family of hv-local languages is included in the family $\mathscr{L}$(CFRE).

Proof:

If $K \subseteq \Gamma^{**}$ is hv-local,then there exists a finite set Δ of dominoes over $\Gamma \cup \{\#\}$ such that

$K=\{ p \in \Gamma^{**} \mid B_{1,2}(\hat{p}) \cup B_{2,1}(\hat{p}) \subseteq \Delta\}$

Let Δ_h denote the set of horizontal dominoes of Δ. Denote K_h by the hv-language obtained replacing Δ with Δ_h.Now,one can associate to the language K_h a local string language $S_h \subseteq \Gamma^*$ defined the set of dominoes Δ_h,considered as a subset of $(\Gamma \cup \{\#\})^2$.similarly,one can associate to the picture language K_v the local string language $S_v \subseteq \Gamma^*$,defined by the set of dominoes Δ_v considered as a subset of $(\Gamma \cup \{\#\})^2$. It is esay to verify that a picture p over Γ is in K If and only if each row of p is in S_h and each column of p is in S_v.This Corresponds to the following equality

$$K=S_h \oplus S_v$$

Since S_h is a local string language,it is also regular, i.e., there exists a regular expression α_h denoting S_h. Let β_h the regular expression obtained replacing in α_h the concatenation with operation ⦶ and the * operation with the operation *⦶then the language K corresponds to the following complementation-free regular expression:

$$K=(\beta_h)^{*\ominus} \cap (\beta_v)^{*\oslash}.$$

Using this result,one can obtain the following Kleene-like characterization for family $\mathscr{L}$(TS).

Theorem 2.2.:

$$\mathscr{L}(TS)= \mathscr{L}(PCFRE).$$

Proof:

We prove first inclusion $\mathscr{L}(TS) \subseteq \mathscr{L}(PCFRE)$.

We recall that ,for any languages $\mathscr{L} \subseteq \Sigma^{**}$ belonging to $\mathscr{L}$(TS),there exists an alphabet Γ,a projection π : □→ Σ and a hv-local languages K over Γ such that L=π(K). Then, by theorem K ϵ $\mathscr{L}$(CFRE) there fore K ϵ $\mathscr{L}$(PCFRE).

To prove the converse ,

we first remark that the atomic languages belong to $\mathscr{L}$(TS). More ever,since $\mathscr{L}$(TS) is closed under all operation in R_1 plus projection ,one concludes that $\mathscr{L}(PCFRE) \subseteq \mathscr{L}(TS)$.

The previous theorem can be restated also as follows : "Family $\mathscr{L}$(TS) coincides with the smallest family of languages that contains the atomic languages and is is closed under regular operations in R_1 plus projection". In other words, a language in $\mathscr{L}$(TS) can be expressed by a formula containing regular operation and projection.

In the course of the proofs of previous theorems we also proved the following result which is of independent interest, since it allows to define tiling recognizable picture languages in terms of recognizable string languages . This characterization will be use full for further generalization.

Theorem 2.3.:

A picture language L over an alphabet Σ is tiling recognizable if and only if there exist two recognizable string languages S_1 and S_2 over an alphabet □ and a projection π : □→ Σ such that $L=\pi(S_1 \oplus S_2)$

As a consequence of this theorem, a tiling recognizable language can be specified by a triple (A_1, A_2,π) ,where A_1 and A_2 are two finite automata and π is projection.

3. Comparing all families:

We summarize in the following theorem the equivalence results.

Theorem 3.1.:

Given two dimensional language L ,the following condition are equivalent.

(i) L is defined by a complementation -free regular expression with projection (Lϵ $\mathscr{L}$(PCFRE)).

(ii) L is recognized by an on- line tesselation automaton (Lϵ $\mathscr{L}$(2OTA)) (iii)L is recognized by finite tiling system (L∊ $\mathscr{L}$(TS)).

For the sequel ,the family of two dimensional languages defined in the thorem above will be denoted by REC and the elements of REC will be simply referred as recognizable two dimensional languages.

Remark that theorem indicates the "robustness" of this notation of "finite -state " recognizability for two - dimensionl languages . In fact,it can be defined in term of machine models ,regular expressions,logic formulaes, and tiling systems.

we summarize the inclusion relationships between the familes of two dimensional language. And defined in terms of different formal modelsfor recognised or generating languages

As far as machine models are concerned , we showed that $\mathscr{L}$(4DFA) is properly included in $\mathscr{L}$(4NFA) and that $\mathscr{L}$(2DOTA) is properly included in $\mathscr{L}$(2OTA).more ever, $\mathscr{L}$(2DOTA)is properly included in $\mathscr{L}$(2OTA)where as the family $\mathscr{L}$(2DOTA) is incomparable with $\mathscr{L}$(4DFA) and $\mathscr{L}$(4NFA).

In the connection with tiling systems it has been proved in that the family of locally testable languages LT is properly included in $\mathscr{L}$(4DFA) and we showed that the family of locally threshold testable languages LTT coincides with family $\mathscr{L}$(FO) of first order formulas definable languages.

Regarding grammer models ,we proved that $\mathscr{L}$(2RLG) is properly included in $\mathscr{L}$(DFA). More ever $\mathscr{L}$(2RLG) is not comparable with LOC and LT . indeed it easy to give examples of in $\mathscr{L}$(2RLG) that are not in LOC . On other hand, the fact that the family $\mathscr{L}$(2RLG)is closed under projection and that is properly concluded in REC,implies that LOC Cannot be included in $\mathscr{L}$(2RLG). We do not know whether there is a some inclusion relation between $\mathscr{L}$(2RLG) and $\mathscr{L}$(2DOTA).

Some interesting open problem in the theory arises for two dimensional languages defined by regular expressions. The following inclusions trivially hold .

$$\mathscr{L}(\text{SFRE}) \subseteq \mathscr{L}(\text{RE})$$

$$\mathscr{L}(\text{CFRE}) \subseteq \mathscr{L}(\text{RE})$$

It is easy to verify that the first inclusion is strict .we are not able to prove that also the second inclusion is strict. A related open problem is whether $\mathscr{L}$(SFRE) is included in $\mathscr{L}$(CFRE) .These inclusion holds and it is strict in the one dimensional case. we showed $\mathscr{L}$(CFRE) Is included in REC and such inclusion is strict . The open problem concerning regular expressions is whether $\mathscr{L}$(RE) is included in REC. We know that REC is not closed under complementation: on other hand do not have any example of languages in $\mathscr{L}$(RE) and not in REC.

Relation between larger families of regular expressions and recognizable picture languages are studied.

CONCLUSION

We define picture languages by means of regular expressions.The regular operations are introduced for set of pictures: row and column concatenations, row and column kleene closures and boolean operations. A regular expression is then a formula expressing how a specific picture language can be obtained from some elementary languages by regular operations. Different families of languages can be defined, depending on the choice of operations allowed to be used in the expression.

Automata , we introduce recognizability of pictures in terms of automata and two dimensional online tessellation automata and family of picture languages recognized by these models of automata have been discussed.

Grammars different systems to generate pictures using grammars have been explored. Tiling system, recognizability of a set of pictures in terms of tiling systems is introduced.

Equivalence theorem, we have shown that some of the different approaches are indeed equivalent and give rise to the same notion of finite-state recognizability for picture languages.

Reference:

1. G.Siromoney, R.Siromoney k,Kirthivasan. Picture langauge with array rewtitting rules . Info. and control
2. R.Siromoney, K.Subramaniyan,K.Rangarajan. parallel/sequential rectangular arrays with tables .Inter .Journ. Computer Math. 6,pp.143- 158.1977.
3. H.Straubing . Finite automata , Formal Logic Circute complexity. Bikhauser, Basel, 1994.
4. D.Giammarresi. Two dimensional langauges and recognizable functions. In Proc. Developments in Langauge Theory,Finland,1993.G.Rozebnerg and A.Salomma(Eds,).World Scientific publishing co.,Singapore,1994,pp.290-301
5. D.Glammarresi and A.Restivo. Recogizable picture langauge. International Journal Pattern Recignitition and Artificial Intelligence. Specil issue on paralel Image Processing. M.Nivat,A,.Saudi and P.S.P. Wang (Eds),pp 31-46,1992. also in Proc .First International Colloquium on parallel image processing ,1991. Same journal,Vol,6,No. 2&3 ,pp 241- 256,1992
6. W.Thomas. On Logic, Tilling, and Automata ,Language and Programming,pp.441-453. Lecture Notes in Computer Science 510, Springer-Verlag,Berlin, 1991.
7. S.Eilemnberg, Automata,Langauge ,Machine .Vol.A, Acadamic press,New york,1974.
8. H.D Ebbinghaus,J.Flum and W.Thomsan. Mathematical Logic,springerverlag, Berlin,1984.
9. R.Fagin. Monadic generalized spectra, Z.Math. Logic Grundl.Math.21(1975),pp.89-96
10. R.Fagin. L.Stockmeyer and M.Y.Vardi. on monadic NP vs monadic co-NP. In Pro.8th IEEE conf. on structure in complexity Theory,1993,pp.19-30.
11. k.S.Fu.Syntatic method in pattern regonation. Academic Press, New Yolk 1974.
12. D.Giammarresi.Two dimensional langauges and recognizable functions.In Proc. Developments in Langauge Theory,Finland,1993.G.Rozebnerg and A.Salomma(Eds,). World Scientific publishing co.,Singapore,1994,pp.290-301.
13. D.Glammarresi and A.Restivo. Recogizable picturelangauge.International Journal Pattern Recignitition and Artificial Intelligence. Specil issue on paralel Image Processing. M.Nivat,A,.Saudi and P.S.P. Wang (Eds),pp 31-46,1992. also in Proc .First International Colloquium on parallel image processing ,1991. Same journal,Vol,6,No. 2&3 ,pp 241-256,1992
14. Glammarresi and A.Restivo. Two Dimensional Finite srate recognizability.Fundamenta Informaticae. Special issue : Formal Langauge Theory,Vol.25, no 3,4, (1996),pp. 399-422

15. D.Glammarresi and A.Restivo. S..Seibert and W.Thomas. Monadic second order logic over rectangulsar pictures and recognizability by tiiling systems. Information and Computation , Vol.125, no, 1, pp.32-45,1996. An abstract appeared also in Proc. xi symposium on Theoritical Aspect of Computers science 775,pp.365-376,Spring-Verlag, Berlin, 1994.

16. A.Galon. Langages d'images Memde Licencee en Science Mathematiques realize soul la direction de V.Bruyere. Universie de Mons-Hainuat,1995.

17. J.E Hopcorft, and J.D. Ullman. Introduction to Automata Theory , Langauge and computation. Addison-Wesley, Reading, MA,1979.

18. K.Inoue and A. Nakamura . Some properties of Two Dimensional on- line tes tessellation accepttor and one -way parallel/sequenntial array acceptors. Tranaction of IECE of Japan, Vol.6,pp.475-476,, 1977.

19. K.Inoue and A.Nakamura. Two Dimensional finite automata and acceptable function . Intern. J. Comput-Math., Sec. A Vol.7,pp.207=213, 1979.

20. K.Inou and I.Takanami. A survey of Two Dimensional autmata theory. In Proc. 5th Int.Meeting of young Computer Scientists, J.Dasson and J.,Kelemen (Eds),pp.72-91. Lecture Notes in Computer science 381, Springer-Verlag,Berlin,1990

21. K.Inoue and I.Takanami. A characterization of recognizable picture langauges. In Prov. Second International Colloquium on paralle image processing , A. Nakamura et la. (Eds), Lecture nates in Computer science 654,Sprinr -Verlag, Berlin 1993. [23] kInoue , I. takanami and A.Nakamur. Anote on two dimensional finite automata. Information Processing Letters. Vol, 7: No 1,pp 49-52,1978.

22. E,B.Kinber. Three way autonsata on rectangualr taps overr aone letter alphabet. Information science , Vol, 35 pp. 61-77,1985.

23. M.Lattteux and D.Simot. Recognizable Picture Langaugr and Domino Tiling. Internal reports IT-94-264. Laboratirie d'Informatiqe Foundamentale de Lille.Universitie de Lille, France.

24. O.Matz . Classification of picture langauge with Rational Expressions, Grammars and Logic Formulas. Diplama thesis. Christian Alberchts Universitat institut fur Informatik and Prakische Mathematik Kiel, germany, 1996 (In Germany)

25. R. Mc. Naughton and S.Papert.counter free automata M.I.T.press,1971.

26. M.Minski and S.Papert. Perception M.I.T. Press, 1969.

27. M.nivat , A. Saoudi and V.R. Dare. Parallel generation of finite image International Journals of recognitition and Artificial Intelligence, Vol.3,No.3&4,pp.279-294,1989.

28. N.Nivat, A.Saoudi and V.R. dare. Parallel generation of finite images.International Journal of Computer Math, Vol.35,pp.25-42,1990

29. A.Rosenfield. Picture Langauges Academic Press,New York,1979.

30. R.Siromoney. On equal matrix liagauge. Info. and Control 14,pp.135- 151,1969.

31. R.Siromoney. Advances in array langaugrs. In Graph, Grammers and their Applications to Computer science , Enrig et al (Edr), pp.549-563. Lecture notes in Computer science 291, Springer-Verlag, Berlin, 1987.

32. R.Siromonry, and K.Kirthivasan. parallel context-free langauges. Info . and Control 24,pp.155-162, 1974.

33. G.Siromoney, R.Siromoney k,Kirthivasan. Picture langauge with array rewtitting rules . Info. and control

34. R.Siromoney, K.Subramaniyan,K.Rangarajan. parallel/sequential rectangular arrays with tables .Inter .Journ. Computer Math. 6,pp.143- 158.1977. [37] H.Straubing . Finite automata , Formal Logic Circute complexity. Bikhauser, Basel, 1994.

35. W.Thomas. On Logic, Tilling, and Automata ,Language and Programming,pp.441-453. Lecture Notes in Computer Science 510, Springer-Verlag,Berlin, 1991.