Estimation of Reliability Indices of Two-Component Identical Parallel System in the presence of CCS

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Abstract: Quality and Reliability play a dominant role in the present day where competitiveness is the main interest in the Globalization process. The word 'failure' is very important in the theory of reliability and it is defined as 'the extinction of the ability of an item to perform its intended function. Hence in this present research work an attempt is made to estimate the reliability measures of the two component parallel system using maximum likely hood estimation approach to evolve appropriate estimate for the reliability measures. The components failure time distribution is assumed to follow exponential law in the probability distribution function of the maximum likelihood estimates. To establish the results Monte Carlo simulation (N) was attempted to generate sample of size n = 5 (5) 30 with minimum of 10,000 (20,000) to 90,000 samples and mean square estimates are developed using computer package.

Keywords: Reliability, Availability, Identical Parallel system, Common Cause Failure, MLE.

1. INTRODUCTION:

The field of reliability analysis was first introduced in the early 1940's during world war - II. In recent years the field of reliability plays a critical role in systems design and development. The term 'Reliability' was defined by the Advisory Group on Reliability of Electronic Equipment (AGREE) as "the probability of a product performing its intended function satisfactorily under given conditions for a specified period of time".

This research paper accentuates the presence of most dominant causes of failures namely, Common Cause Shock (CCS) failures. The Common Cause Shock failure is defined as an event that must be a single extended cause which produces simultaneous failure of two or more components of the system which could result in damage of equipment and property.

Billinton and Allan [1983] stressed the role of Common Cause Shock (CCS) Failures and discussed the need for considering such failures in reliability analysis. The definition of CCS failures was given by the Task Force on IEEE APM sub-committee [106] in early 1980's. The definition of common cause failures would mean that an event is purely external cause which produces multiple failures instead of the internal system event causing the failure of one or more other components simultaneously.

Common Cause Shock Failures are classified into two categories depending on the intensity of the shock, one is Lethal Common Cause Shock Failure which is the occurrence of simultaneous outage of all components in the system and the other is Non-Lethal Common Cause Shock Failure which is the occurrence of random number of components to simultaneous outage, follows probability distribution, viz., Binomial distribution

Therefore, in the present research work, an attempt is made to find an approach of estimation method, which could establish a formal estimation procedure to estimate the reliability measures such as $R_s(t)$, Availability [$A_s(t)$, $A_s(\infty)$], MTBF, MTTF under the influence of common cause shock failures.

2. ESTIMATION OF RELIABILITY MEASURES OF TWO COMPONENT IDENTICAL SYSTEM: M L APPROACH

David & Epstein (1958) have remarked that exponential distribution plays an important role in life experiments, just as that normal distribution plays a role in agricultural experiments. Of course interest in estimation is very important for statistical inference of random phenomena more so with random phenomena say "life" or time to failure in reliability theory. The random phenomena is to be known completely to take managerial decision if we have the estimate of some quantities with which 'life' and reliability measures of the components / system are associated.

Therefore in this paper, system reliability measures like reliability function Rs(t), availability function As(t) were attempted to be estimated in the case of two component identical non-identical parallel system.

3. ASSUMPTIONS

1. The system has two components, which are stochastically independent.
2. The system is affected by individual as well as common cause failures.
3. The components in the system will fail singly at the constant rate λ_a and failure probability is P_1.
4. The components may fail due to common causes at the constant rate λ_c and with failure probability is P_2
5. Time occurrences of CCS failures and individual failures follow Exponential law.
6. The individual failures and CCS failures occurring independent of each other.
7. The failed components are serviced singly and service time follows exponential distribution with rate of service μ.

4. NOTATIONS

λ_a	:	Individual failure rate.
λ_c	:	Common cause failure rate.
μ	:	Service rate of individual components
$R_s(t)$	:	System Reliability function.

$A_s(t)$: System Availability function.
$R_{ps}(t)$: Reliability function of two component parallel system.
$\hat{R}_{ps}(t)$: MLE of time dependent Reliability function of parallel system.
$E_{ps}(T)$: Expected time of failure for parallel system
$A_{ps}(t)$: Time dependent Availability function of parallel system .
$\hat{A}_{ps}(t)$: MLE of time dependent availability function of parallel system .
$\hat{A}_{ps}(\infty)$: MLE estimate of steady state Availability function of parallel system.
$\bar{X}$: Sample mean of individual failures occurrence.
$\bar{Y}$: Sample mean of common cause failures occurrence.
$\bar{Z}$: Sample mean of service time of the components.
$\bar{X}^*$: Sample estimate of individual failures rate occurrence.
$\bar{Y}^*$: Sample estimate of common cause failures rate occurrence.
$\bar{Z}^*$: Sample estimate of service time of the components.
n : Sample size.
N : Number of simulated samples.

5. MODEL

Under the stated assumptions Markovian model is formulated to drive the Reliability function R(t) under the influence of individual as well as CCS and the Markovian graph is given in fig.1.
The quantities $\lambda_1, \lambda_2, \lambda_3, \mu_1 \& \mu_2$ are as follows $\lambda_1 = 2\lambda_a P_1$, $\lambda_2 = \lambda_a P_1$, $\lambda_3 = \lambda_c P_2$, $\mu_1 = \mu$, & $\mu_2 = 2\mu$.

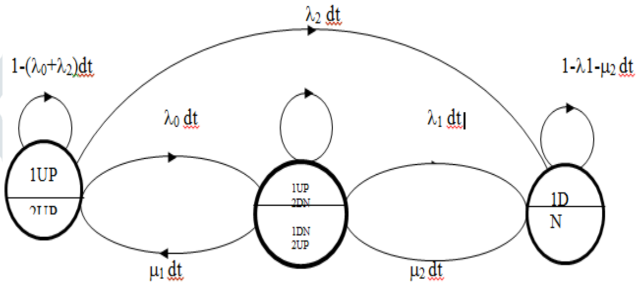

Fig.1: MARKOV GRAPH FOR TWO COMPONENT SYSTEM WITH INDIVIDUAL AND COMMON CAUSE FAILURES.

6. RELIABILITY FUNCTION OF TWO COMPONENT IDENTICAL SYSTEM

The expression of Reliability function for parallel system are derived and is given by

$$R_{ps}(t) = [(\gamma_1 + \mu + 3\lambda_a P_1) \exp(\gamma_1 t) - (\gamma_2 + \mu + 3\lambda_a P_1) \exp(\gamma_2 t)] / [\gamma_1 - \gamma_2]$$

Where

$$\gamma_1 = [-(\mu + 3\lambda_a P_1 + \lambda_c P_2) + \mathrm{SQRT}((\lambda_a P_1 + \lambda_c P_2 - \mu)^2 + 8\lambda_c \mu P_2)]/2$$
$$\gamma_2 = [-(\mu + 3\lambda_a P_1 + \lambda_c P_2) - \mathrm{SQRT}((\lambda_a P_1 + \lambda_c P_2 - \mu)^2 + 8\lambda_c \mu P_2)]/2$$

Where λ_a, λ_c & μ the individual failure rate, common cause failure rate and repair rates respectively.

7. ESTIMATION OF RELIABILITY INDICES – M L ESTIMATION APPROACH

The Maximum likelihood estimation approach for estimating Reliability function of two component parallel system, which is under the influence of Individual as well as common cause failures is studied.
Let $X_1, X_2, X_3 \ldots X_n$ be a sample of 'n` number of times between individual failures which will obey exponential law.
Let $Y_1, Y_2, Y_3 \ldots Y_n$ be a sample of 'n` number of times between common cause system failures assume to follow exponential law.
Let $Z_1, Z_2, Z_3 \ldots Z_n$ be a sample of 'n` number of times of repair of the components with exponential population law.
$\bar{X}^*$, $\bar{Y}^*$, & $\bar{Z}^*$ are the maximum likelihood estimates of individual failure rate λ_a , Common cause failures rate λ_c and repair rate μ of system respectively.
Where

$$\bar{X}^* = 1/\bar{X}, \quad \bar{Y}^* = 1/\bar{Y}, \quad \bar{Z}^* = 1/\bar{Z}$$
$$\bar{X} = \sum X_i / n, \quad \bar{Y} = \sum Y_i / n, \quad \bar{Z} = \sum Z_i / n \qquad \ldots (1)$$

are sample estimates of rate of the individual failure λ_a , rate of common cause failure λ_c and rate of repair μ of the components respectively.
The maximum likelihood estimate of time dependent Reliability function for parallel system is given by

$$\hat{R}_{ps}(t) = [(G_1 + \bar{Z}^* + 3\bar{X}^* P_1) \exp(G_1 t) - (G_2 + \bar{Z}^* + 3\bar{X}^* P_1) \exp(G_2 t)] / [G_1 - G_2] \qquad \ldots (2)$$

Where

$$G_1 = [-(3\bar{X}^* P_1 + \bar{Y}^* P_2 + \bar{Z}^*) + \mathrm{SQRT}((\bar{X}^* P_1 + \bar{Y}^* P_2 - \bar{Z}^*)^2 + 8\bar{Y}^* \bar{Z}^* P_2)]/2$$
$$G_2 = [-(3\bar{X}^* P_1 + \bar{Y}^* P_2 + \bar{Z}^*) - \mathrm{SQRT}((\bar{X}^* P_1 + \bar{Y}^* P_2 - \bar{Z}^*)^2 + 8\bar{Y}^* \bar{Z}^* P_2)]/2$$

Where $\bar{X}^*$, $\bar{Y}^*$, $\bar{Z}^*$ are sample estimates given in (1)

Time dependent expression of Availability function of parallel system is given by

$$A_{ps}(t) = [2\mu(\mu + 2\lambda_a P_1 + \lambda_c P_2)] / [2\mu^2 + 4\lambda_a P_1 \mu + 3\lambda_c P_2 \mu + 2\lambda_a^2 P_1^2 + \lambda_a \lambda_c P_1 P_2] + [H_1 \exp(\gamma_1 t) - H_2 \exp(\gamma_2 t)] / [\gamma_1 - \gamma_2]$$

Where

$H_1 = [\ \gamma_1^2 + 3\,\gamma_1\,(\mu + \lambda_a P_1) + 2\,\mu(\,2\,\lambda_a P_1 + \lambda_c P_2 + \mu\,)]\ /\ \gamma_1$

$H_2 = [\ \gamma_2^2 + 3\,\gamma_2\,(\mu + \lambda_a P_1) + 2\,\mu(\,2\,\lambda_a P_1 + \lambda_c P_2 + \mu\,)]\ /\ \gamma_2$

$\gamma_1 = [-(3\,\mu + 3\,\lambda_a P_1 + \lambda_c P_2\,) + SQRT((\,\mu + \lambda_a P_1 + \lambda_c P_2\,)^2 - 8\,\lambda_c\,\mu\,P_2\,)\,]\,/2$

$\gamma_2 = [\,-(3\,\mu + 3\,\lambda_a P_1 + \lambda_c P_2\,) - SQRT((\,\mu + \lambda_a P_1 + \lambda_c P_2\,)^2 - 8\,\lambda_c\,\mu\,P_2\,)\,]\,/\,2$

The maximum likelihood estimate of time dependent Availability function the of parallel system is given by

$$\hat{A}_{ps}(t) = [2\ \bar{Z}^*(\ \bar{Z}^*+2\ \bar{X}^*P_1+\ \bar{Y}^*P_2)]\ /\ [2\ \bar{Z}^{*2}+4\ \bar{X}^*P_1\ \bar{Z}^*+3\ \bar{Y}^*\ \bar{Z}^*+2\ \bar{X}^{*2}P_1^2+\ \bar{X}^*\ \bar{Y}^*P_1P_2] + [K_1\exp(G_1t)-K_2\exp(G_{2t})]\ /\ [G_1-G_2]$$

Where

$K_1 = [G_1^2 + 3\,G_1\,(\ \bar{Z}^*+\ \bar{X}^*\,P_1) + 2\ \ \bar{Z}^*(\,2\ \ \bar{X}^*\,P_1 +\ \ \bar{Y}^*\,P_2 +\ \ \bar{Z}^*\,)]\ /\ G_1$

$K_2 = [\,G_2^2 + 3\,G_2\,(\ \bar{Z}^*+\ \bar{X}^*\,P_1) + 2\ \ \bar{Z}^*(\,2\ \ \bar{X}^*\,P_1 +\ \ \bar{Y}^*\,P_2 +\ \ \bar{Z}^*\,)]\ /\ G_2$ (3.27)

$G_1 = [-(3\ \ \bar{Z}^*+3\ \ \bar{X}^*\,P_1+\ \bar{Y}^*\,P_2)+SQRT((\ \bar{Z}^*+\ \bar{X}^*\,P_1+\ \bar{Y}^*\,P_2)^2 - 8\ \bar{Y}^*\ \bar{Z}^*P_2)]\ /\ 2$

$G_2 = [-(3\ \bar{Z}^*+3\ \bar{X}^*P_1+\ \bar{Y}^*\,P_2)-SQRT((\ \bar{Z}^*+\ \bar{X}^*\,P_1+\ \bar{Y}^*P_2\,)^2 - 8\ \bar{Y}^*\ \ \bar{Z}^*\,P_2)]\ /\ 2$

$\bar{X}^*$, $\bar{Y}^*$ & $\bar{Z}^*$ are sample estimates of the individual failure rate λ_a, Common cause failure rate λ_c and repair rate μ respectively.

8. INTERVAL ESTIMATION- RELIABILITY, AVAILABILITY

Let us consider

$$\psi = \sqrt{n}\,[\,\hat{R}_s\,(t) - R_s\,(t)\,]\ /\ \sigma^2_\theta \sim N(0,1)$$
$$\psi = \sqrt{n}\,[(\,\hat{F}_s\,(T) - F_s\,(T)\,]\ /\ \sigma^2_\theta \sim N(0,1)$$
$$\psi = \sqrt{n}\,[\hat{A}_s\,(\infty) - A_s\,(\infty)\,]\ /\ \sigma^2_\theta \sim N(0,1)$$

From Slutsky's theorem, we have $P[-Z_{\alpha/2} \le \psi \le Z_{\alpha/2}] = 1-\alpha$

Where $Z_{\alpha/2}$ are the $\alpha/2$ percentiles points of normal distribution and are available from normal tables.

Hence $(1-\alpha)\%$ confidence interval for $R_p(t)$ is given by

$$R_{ps}(t)\ \pm\ Z_{\alpha/2}\ \sigma^2_{(Rsp(t))}\ /\ \sqrt{n}$$

$(1-\alpha)\%$ confidence interval for Availability function are given by

$$A_{ps}(\infty)\ \pm\ Z_{\alpha/2}\ \sigma^2_{(Asp(\infty))}/\sqrt{n}$$

However in the context of Reliability analysis, when fewer sample are available immediate usefulness of these estimates are not taken for granted. Therefore, in this paper it is further investigated with appropriate simulated samples empirically the suitability and accuracy of these estimates in the absence of analytical approach of the estimates since nature of estimates are not established so far.

9. MONTE CARLO SIMULATION AND VALIDITY

Having proposed maximum likelihood estimates of Reliability and availability functions of two component identical systems, the exact probability density function of these estimates are not known and not much literature is seen in this direction. Hence in the present work an attempt is made to develop empirical evidence of M L Estimation approach by Monte Carlo simulation procedure for validity of results.

By the property of the M L E, it is known that they are good only for large sample case, but in the context of reliability large sample are rare. Therefore in this research work, samples of sizes n = 5 (5) 30 is generated and in each case sample estimates were calculated and finally mean square error was computed for each sample sizes adequately with N=10,000 (20,000) 90,000, simulation were attempted using the C-program and MSE of the estimates reliability indices was tabulated for all samples of size n = 5 (5) 30.

For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interesting to note that for a sample size n = 5, M L estimate is still seem to be reasonably good giving near accurate estimate. This shows that ML approach and estimators are quite useful in estimating Reliability indices like $R_s(t)$, $A_s(t)$.

TABLE 1: Results of the simulations for Time dependent Reliability function for parallel system with λ_a= 0.002; λ_c = 0.02; μ.= 0.02; P_1 = 0.5; t =1.

Sample size n =5					Sample size n =10				
N	$R_{Ps}(t)$	$R`_{Ps}(t)$	M S E	CI (95%)	N	Rps(t)	$R`_{Ps}(t)$	M S E	CI (95%)
10000	0.990059	0.986189	0.000093	(0.548142, 1.000000)	10000	0.990059	0.988016	0.000024	(0.677577, 1.000000)
30000	0.990059	0.986304	0.000089	(0.548142, 1.000000)	30000	0.990059	0.98797	0.000025	(0.677577, 1.000000)
50000	0.990059	0.986195	0.000089	(0.548142, 1.000000)	50000	0.990059	0.987938	0.000025	(0.677577, 1.000000)
70000	0.990059	0.986241	0.000089	(0.548142, 1.000000)	70000	0.990059	0.987943	0.000026	(0.677577, 1.000000)
90000	0.990059	0.986279	0.000087	(0.548142, 1.000000)	90000	0.990059	0.988013	0.000025	(0.677577, 1.000000)

Sample size n =15					Sample size n =20				
N	$R_{Ps}(t)$	$R`_{Ps}(t)$	M S E	CI (95%)	N	Rps(t)	$R`_{Ps}(t)$	M S E	CI (95%)
10000	0.990059	0.988349	0.000015	(0.734918, 1.000000)	10000	0.990059	0.988682	0.00001	(0.769100, 1.000000)
30000	0.990059	0.98843	0.000015	(0.734918, 1.000000)	30000	0.990059	0.988639	0.00001	(0.769100, 1.000000)
50000	0.990059	0.988426	0.000014	(0.734918, 1.000000)	50000	0.990059	0.988634	0.00001	(0.769100, 1.000000)
70000	0.990059	0.988447	0.000014	(0.734918, 1.000000)	70000	0.990059	0.988703	0.00001	(0.769100, 1.000000)
90000	0.990059	0.988517	0.000014	(0.734918, 1.000000)	90000	0.990059	0.988781	0.00001	(0.769100, 1.000000)

Sample size n =25					Sample size n =30				
N	$R_{Ps}(t)$	$R`_{Ps}(t)$	M S E	CI (95%)	N	Rps(t)	$R`_{Ps}(t)$	M S E	CI (95%)
10000	0.990059	0.988794	0.000008	(0.792428, 1.000000)	10000	0.990059	0.988884	0.000006	(0.809647, 1. 000000)
10000	0.990059	0.988793	0.000008	(0.792428, 1.000000)	30000	0.990059	0.988876	0.000006	(0.809647, 1. 000000)
30000	0.990059	0.988791	0.000008	(0.792428, 1.000000)	50000	0.990059	0.988864	0.000006	(0.809647, 1. 000000)
50000	0.990059	0.988829	0.000008	(0.792428, 1.000000)	70000	0.990059	0.988913	0.000006	(0.809647, 1. 000000)
70000	0.990059	0.988932	0.000008	(0.792428, 1.000000)	90000	0.990059	0.989039	0.000006	(0.809647, 1. 000000)

10. CONCLUSIONS

We established the results that maximum likelihood approach and estimate for the Reliability measures are quite satisfactory because MSE`s obtained was observed to be too small i.e. as low as 0.0001 in most of the cases. Hence in the rest of the research work the above approach was used to evolve MLE`s for the reliability measures like Rs(t), As(t) MTBF, MTTF etc in the case of two component system with identical and non-identical solutions for parallel system.

From the results, it is seen that maximum likelihood approach used was found useful in the estimation process to find estimate for the reliability measures of the system, where small sample is essential point of interest in the case of reliability analysis. The estimates so derived using empirical procedure do posses the property that MSE in each case is well within the prescribed error, i.e. coincides even to the three decimal places are more. It is found that ML method of estimation is useful to estimate the reliability measures of two component system as mean square estimate evolved is reasonably small even for the small samples of size n = 5. Therefore the research result established that ML method of estimation approach is satisfactory to estimate reliability measures in the presence of individual as well as common cause shock failures.

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