OPTIMAL REPLENISHMENT POLICES FOR TIME DEPENDENT DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND

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Abstract : This paper deals with an economic production model for deteriorating items with quadratic, linear and constant demand rate with respect to the different time factor. A single item is produced and finite rate of production is proportional to the demand rate. The deterioration rate of the item is taken as linear, constant and fractional with respect to different time factor. Shortage is allowed in the stock and model is under the finite time horizon. This inventory model is solved under modern calculus techniques to evaluate optimum solution. Model is illustrated with a numerical example by taking various number of replenishment. Presented sensitivity analysis to the given numerical example and to draw graphs by using mathematica9.0. Objective of the paper is to find the optimal solution of production time and minimize the total inventory cost during the working time interval.

IndexTerms **- inventory, deterioration, time-dependent demand, shortage.**

I. INTRODUCTION

In recent years, many researchers have studied the problem of pricing and inventory control simultaneously for deteriorating items. As is apparent in the literature, deterioration is defined as lose of utility or lose of marginal value of a commodity that results in decreased usefulness. Most physical goods undergo decay or deterioration over time, examples being volatile liquids; bold bank etc. extra inventory desires additional space for storing that's expedited by a rented warehouse. Ignoring the consequences of your time worth of cash and inflation may yield dishonest results. The warehouse storage capability is outlined because the quantity of space for storing required accommodating the materials to be keep to fulfil a desired service level that specifies the degree of space for storing availableness. Stock things to be delivered precisely once required square measure impractical. Therefore, it's necessary to analyze the influence of warehouse capability in varied inventory policy issues. As pointed out by Levin, McLaughlin, Lamone, and Kottas (1972), ‘‘at times, the presence of inventory has a motivational effect on the people around it. It is the common belief that large piles of goods displayed in a supermarket will lead the customers to buy more’’. Desmet and Renaudin (1998) also noted that impulsive buying categories have higher space elasticises, which is consistent with the interpretation that space has a causal effect on sales and not the converse. Due to the facts, a number of authors have developed the EOQ models that focused on stock-dependent demand rate patterns. Normally, there are two types of stock-dependent demand patterns. Gupta and Vrta (1986) assumed that the demand rate was a function of initial stock level. Baker and Urban (1988a, 1988b) considered a power-form inventory-level-dependent demand rate, which would decline along with the stock level throughout the entire cycle. Datta and Pal (1990) modified the model of Baker and Urban (1988b) by assuming that the stock dependent demand rate was down to a given level of inventory, beyond which it is a constant. Goh (1994) relaxed the assumption of a constant holding cost in Baker and Urban (1988b). Later, Urban (1995) extended Datta and Pal’s (1990) model to allow shortages, where the unsatisfied demand is backlogged at a fixed fraction of the constant demand rate. Recently, Jaggi, Aggarwal, and Goel (2006) presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions by using the DCF approach over a finite planning horizon. Zhou, Min, and Goyal (2008) and Sajadieh, Thorstenson, and Jokar (2010) both studied the coordination of supply chain with power-form inventory-level-dependent demand. Bishi, Behera, and Sahu(2018) presented the deteriorating inventory model with linear, exponential demand with shortage. Bishi, Behera, and Sahu(2018) presented the An Inventory Production Policy for a Single Parameter Weibull Deteriorating Item With Exponential Demand and Shortages. Bishi, Behera, and Sahu(2018) presented the Two-Warehouse Inventory Model for Non-Instantaneous Deteriorating Items with Exponential Demand Rate .

This paper is the first paper to study both the preservation technology investment and pricing strategies of deteriorating seasonal products. In this paper, a model for deteriorating seasonal products is built; in which deterioration rate can be controlled by preservation technology investment. The decision variables are the market demand, the preservation technology Investment parameter and the ordering frequency. To get the optimal solution, an algorithm is designed. To foster additional managerial insights, we perform extensive sensitivity analyses and illustrate our results with a case study.

II. ASSUMPTIONS AND NOTATIONS

NOTATION: The notation in this paper is listed below.

Decision Variables.

Q_{max}: maximum inventory level.

α: Cost of preservation technology investment per unit Time.

P: production quantity.

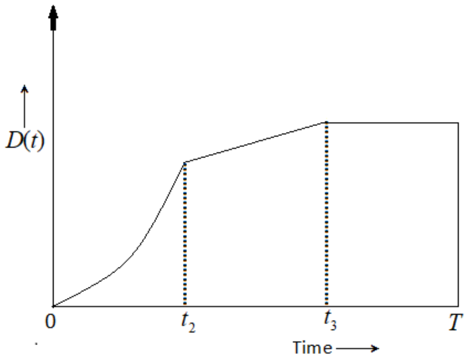
Constant Parameters:

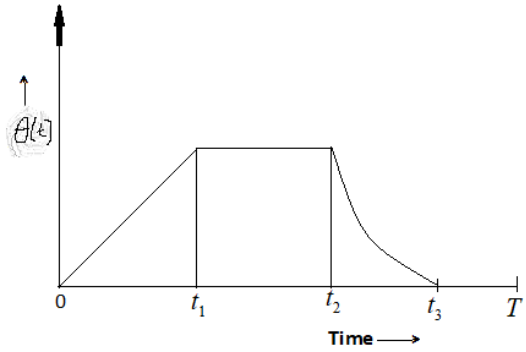
K: production cost.

h: Inventory holding cost per unit per time.

$I(t)$: Inventory level of a time point.
d: deterioration cost.
s: shortage cost.
O: Ordering cost per order.
D (t): Market demand, $D(t) = A + Bt + ct^2 \; ; \; 0 \le t \le t_2.$

$$= A + Bt \; ; \; t_2 \le t \le t_3.$$
$$= A \; ; \; t_3 \le t \le T.$$





$\theta(t)$: Deterioration rate, $\theta(t) = \alpha + \beta t; t \in (0, t_1)$

$$= \alpha; t \in (t_1, t_2)$$
$$= \frac{1}{\alpha + t}; t \in (t_2, t_3)$$
$$= 0; t \in (t_3, T)$$

TVC: Total inventory cost.

ASSUMPTIONS:

The model in this paper is built on the base of the following assumptions.
(1) Market demand is Quadratic, linear and constant related to time.
(2) Market demand only exists in a limited time horizon T.
(3) Demand can be backlogged.
(4) Ordering lead time is zero.
(5) Deteriorated products have no value, and there is no cost to dispose or store them.

III. MATHEMATICAL MODEL:

This study considers a single retailer's inventory policy in which the deterioration rate is affected by the preservation technology investment. The decision variables are the market demand, the ordering cost, and the preservation technology investment parameter. According to the assumption, the time length is equal in all the ordering periods. So, we only study the first period. In the first period, according to the modelling of trapezoidal type deteriorating inventory. The inventory level $I(t)$ can be depicted as Figure 1 and formulated as follows:

$$\frac{dI(t)}{dt} + \theta(t) I(t) = P - D(t), \quad 0 \le t \le t_1 . \tag{a}$$

$$\frac{dI(t)}{dt} + \theta(t) I(t) = -D(t), \quad t_1 \le t \le T . \tag{b}$$

Where $D(t) = A + Bt + ct^2 \; ; \; 0 \le t \le t_2.$

$$= A + Bt \; ; \; t_2 \le t \le t_3.$$
$$= A \; ; \; t_3 \le t \le T.$$

And $\theta(t) = \alpha + \beta t; t \in (0, t_1)$.

$$= \alpha; t \in (t_1, t_2).$$
$$= \frac{1}{\alpha + t}; t \in (t_2, t_3).$$
$$= 0; t \in (t_3, T).$$

With boundary condition $I(0) = 0 = I(t_3)$

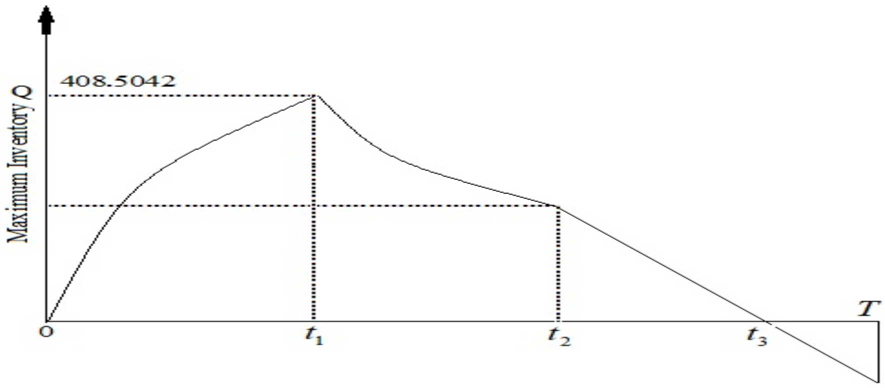


Fig: 1-Inventory graphical representation.

By solving with boundary condition $I(0)=0$ to the equation (1) we get equation (2)

$$\frac{dI(t)}{dt}+(\alpha+\beta t)I(t)=P-(A+Bt+Ct^2),\quad 0\le t\le t_1 \tag{1}$$

$$I(t)=\left[(P-A)t+\left(\frac{P\alpha}{2}-B-A\alpha\right)t^2+\left(\frac{P\beta}{6}-C-\alpha B-\frac{A\beta}{2}\right)t^3-\left(\alpha C+\frac{B\beta}{2}\right)\frac{t^4}{4}-\frac{\beta C}{10}t^5\right]\Big/\left(1+\alpha t+\frac{\beta t^2}{2}\right) \tag{2}$$

$$\frac{dI(t)}{dt}+\alpha I(t)=-(A+Bt+Ct^2),\quad t_1\le t\le t_2 \tag{3}$$

By solving the equation (2) we get equation (3)

$$I(t)=-\left[At+\frac{(A\alpha+B)t^2}{2}+\frac{(C+\alpha B)t^3}{3}+\frac{\alpha C}{4}t^4\right]\Big/(1+\alpha t) \tag{4}$$

$$\frac{dI(t)}{dt}+\frac{I(t)}{\alpha+t}=-(A+Bt),\quad t_2\le t\le t_3 \tag{5}$$

By solving with boundary condition $I(t_3)=0$ to the equation (3) we get equation (4)

$$I(t)=-\left[At+\frac{(B\alpha+A)t^2}{2}+\frac{Bt^3}{3}\right]\Big/(\alpha+t) \tag{6}$$

$$\frac{dI(t)}{dt}=-A,\quad t_3\le t\le T \tag{7}$$

By solving with boundary condition $I(t_3)=0$ to the equation (4) we get equation (5)

$$I(t)=-At \tag{8}$$

Production Cost $=K\left[\int_0^{t_1}I(t)\,dt\right]=-0.02t_1^4-0.468t_1^3+6532t_1^2+3255t_1$

Holding Cost $=h\int_0^{t_3}I(t)\,dt=h\left[\int_0^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt+\int_{t_2}^{t_3}I(t)dt\right]$

$$=h(0.011t_1^4-0.002t_1^3+165.88t_1^2+3259.845t_1-0.006t_2^4+0.002t_2^3-0.015t_2-0.117t_3^3-2.58t_3^2-4.83t_3)$$

Deterioration Cost $=d\int_0^{t_3}I(t)\,dt=d\left[\int_0^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt+\int_{t_2}^{t_3}I(t)dt\right]$

$$=\frac{1}{2}[0.011t_1^4-0.002t_1^3+165.88t_1^2+3259.845t_1-0.006t_2^4+0.002t_2^3-0.015t_2-0.117t_3^3-2.58t_3^2-4.83t_3]$$

Shortage Cost $=s\int_{t_3}^{T}I(t)dt=s(T^2-t_3^2)$

Ordering Cost $=O$

Total Inventory Cost $= PC + HC + DC + SC + OC$

$$= 100 + (-0.0215t_1^4 - 0.471t_1^3 + 6697.88t_1^2 + 6680.125t_1)$$
$$+ (-0.009t_2^4 - 0.006t_2^3 - 0.015t_2) + (-0.165t_3^3 - 4.87t_3^2 - 13.05t_3) + T^2$$

IV. NUMERICAL EXAMPLE

To evaluate the value of $t_1, t_2,$ t_3 and TVC this numerical example has taken and solved with the help of mathematica9.0 using the following parameters values:

Let $A = 10$, $B = 1, C = 0.1, \alpha = 1, \beta = 0.1, P = 10000/unit, K = 4, h = 1, \theta = 0.4, s = 2,$ $O = 100/unit$ then

$$I(t) = \left[\frac{990t + 489t^2 + 15.96t^3 - 0.0375t^4 - 0.001t^5}{1 + t + 0.05t^2} - \frac{(10t + 5.5t^2 + 0.37t^3 + 0.025t^4)}{(1+t)} - \frac{(10t + 5.5t^2 + 0.33t^3)}{(1+t)} - t \right]$$

$$I(t) = \left[-0.02t^3 - 0.35t^2 + 326.3t + 3255 + \frac{-2591.6t - 3255}{1 + t_1 - 0.05t^2} - \left(0.025t^3 + 0.345t^2 + 5.155t + 4.845 - \frac{4.845}{1+t}\right) - \left(0.33t^2 + 5.17t + 4.83 - \frac{4.83}{1+x}\right) - t \right]$$

$$\text{PC} = 4\int_0^{t_1} I(t)dt = -0.02t_1^4 - 0.468t_1^3 + 6532t_1^2 + 3255t_1$$

$$HC = 0.011t_1^4 - 0.002t_1^3 + 165.88t_1^2 + 3259.845t_1 - 0.006t_2^4 + 0.002t_2^3 - 0.015t_2 - 0.117t_3^3 - 2.58t_3^2 - 4.83t_3$$

$$DC = \frac{1}{2}[0.011t_1^4 - 0.002t_1^3 + 165.88t_1^2 + 3259.845t_1 - 0.006t_2^4 + 0.002t_2^3 - 0.015t_2 - 0.117t_3^3 - 2.58t_3^2 - 4.83t_3]$$

$$\text{SC} = 2\int_{t_3}^{T} I(t)\,dt = 2\left[\frac{t^2}{2}\right]_{t_3}^{T} = T^2 - t_3^2$$

$$\text{TVC} = 100 + (-0.0215t_1^4 - 0.471t_1^3 + 6697.88t_1^2 + 6680.125t_1) + (-0.009t_2^4 - 0.006t_2^3 - 0.015t_2) + (-0.165t_3^3 - 4.87t_3^2 - 13.05t_3) + T^2$$

$$\frac{\partial TVC}{\partial t_1} = -0.086t_1^3 - 1.413t_1^2 + 13395.76t_1 + 6680.125$$

$$\frac{\partial TVC}{\partial t_2} = -0.036t_2^3 - 0.018t_2^2 - 0.015$$

$$\frac{\partial TVC}{\partial t_3} = -0.495t_2^2 - 9.74t_3 - 13.05$$

By equating the above 3 equations to zero and solving for respective $t_i's$

and we get that

$t_1 = 0.4987$

$t_2 = 0.95595$

And

$t_3 = 0.20.93$

$$\left.\frac{\partial^2 TVC}{\partial t_1^2}\right]_{t_1 = 0.4987} = -0.258t_1^2 - 2.826t_1 + 13395.76 > 0$$

$$\left.\frac{\partial^2 TVC}{\partial t_2^2}\right]_{t_2=0.95595} = 0.108t_1^2 - 0.036 = 0.0627 > 0$$

$$\left.\frac{\partial^2 TVC}{\partial t_3^2}\right]_{t_3=20.93} = 1.485t_1^2 - 9.74 = 21.34 > 0$$

Since all $\frac{\partial^2 TVC}{\partial t_i^2} > 0 \ \forall \ t_i$

So TVC will attain minimum at t_i's is 1619.27 for all i.

Maximum inventory $Q_{\max} = I(t_1) = 408.5042$

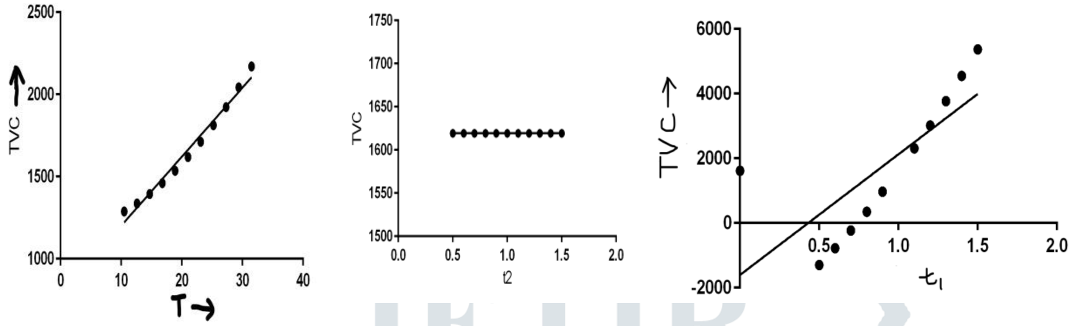


Figure 2 Relationship between parameters and TVC

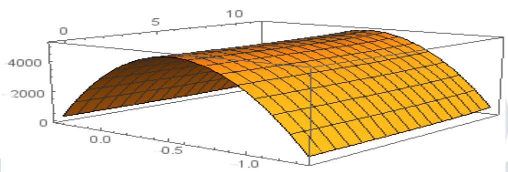


Figure 3. Change in TVC with respect to parameters.

V. SENSITIVITY ANALYSIS

We now study effects of changes in the values of system parameters α, β, A, B, C t_1, t_2, t_3 and T on the cycle time T ,the optimum inventory and minimum total relevant cost per unit time TVC . The sensitivity analysis is performed by changing the value of the parameters by increasing and decreasing 50%, 40%, 30%, 20%and 10% simultaneously, taking one parameter at a time and keeping the remains unchanged. The results are given in table 1, table 2, table 3 and table 4.

On the basis of the results, the following observations can be made:

1. TVC is highly insensitive to change in the parameter k & t_3 and slightly insensitive to change in parameter β & t_2.
2. TVC is highly sensitive to change in the parameter α & t_1 and slightly sensitive to change in TVC in parameter h & T .
3. With increasing in β & k TVC increases but by increasing in α & h TVC decreases.
4. Parameter A, B and C are infeasible towards solution.

Table 1 $(t_1 = 0.4987)$

α	Value	C Value	TVC	Changes
+50 %	2.31	0.7480	5366.67	3747.40
+40 %	1.81	0.6982	4551.54	2932.27
+30 %	1.33	0.6483	3768.11	2148.84
+20 %	0.86	0.5984	3018.02	1398.75
+10 %	0.42	0.5486	2306.91	687.640
+ 0 %	0	0.4987	1619.27	0.00000
-10 %	-0.40	0.4488	969.19	650.080
-20 %	-0.78	0.3989	352.47	1266.80
-30 %	-1.14	0.3491	-229.76	1849.03
-40 %	-1.48	0.2992	-779.83	2399.10

-50 %	-1.80	0.2494	-1295.54	2914.81

Table 2 $(t_2 = 0.95595)$

β	Value	TVC	Ratio	Changes
+50 %	1.43392	1619.22	0.9997	0.05
+40 %	1.33833	1619.23	0.9997	0.04
+30 %	1.24274	1619.24	0.9998	0.03
+20 %	1.14714	1619.25	0.9998	0.02
+10 %	1.05155	1619.26	0.9999	0.01
+ 0 %	0.95695	1619.27	1.0000	0.00
-10 %	0.86036	1619.27	1.0000	0.00
-20 %	0.76476	1619.28	1.0001	0.01
-30 %	0.66917	1619.28	1.0001	0.01
-40 %	0.57358	1619.28	1.0001	0.01
-50 %	0.47798	1619.29	1.0002	0.02

Table 3 $(t_3 = 20.93)$

k	Value	TVC	Ratio	Changes
+50 %	31.39	-4772.97	-3.95	6392.25
+40 %	29.30	-3174.97	-2.96	4794.24
+30 %	27.21	-1746.21	-2.08	3365.48
+20 %	25.12	-477.67	-1.29	2096.94
+10 %	23.02	644.70	0.61	974.570
+ 0 %	20.93	1619.27	0.00	0.00000
-10 %	18.84	2460.79	0.52	841.520
-20 %	16.74	3181.43	0.97	1562.16
-30 %	14.65	3783.43	1.37	2164.16
-40 %	12.56	4279.52	1.65	2660.25
-50 %	10.47	4678.75	1.89	3059.48

Table 4 $(T = 21)$

h	Value	TVC	Ratio	Changes
+50 %	31.5	2170.52	0.34	551.25
+40 %	29.4	2042.63	0.26	423.36
+30 %	27.3	1923.56	0.19	304.29
+20 %	25.2	1813.31	0.12	194.04
+10 %	23.1	1711.88	0.06	92.610
+ 0 %	21.0	1619.27	0.00	0.0000
-10 %	18.9	1535.48	-0.05	83.790
-20 %	16.8	1460.51	-0.09	158.76
-30 %	14.7	1394.36	-0.14	224.91
-40 %	12.6	1337.03	-0.18	282.24
-50 %	10.5	1288.52	-0.21	330.75

VI. CONCLUSION

An inventory replenishment policy is developed for deteriorating items with time-dependent demand. The rate of deterioration is time-proportional and the time to deterioration is followed by trapezoidal type distribution. In the present paper, we consider the model of quadratic demand and extended it to a time-dependent demand rate and shortage in inventory and also using trapezoidal type distribution. A numerical example is taken to illustrate the theory. The sensitivity of the optimal solution to change in the parameter value is examined. From the above analysis, it is seen α, β, k, h are the critical parameter in the sense that any error in the estimation of α, β, k, h. Again the above analysis shows that great care should be taken to estimate the value of the parameter α, β, k, h. The proposed models can be extended in several ways. The model can also be generalized to consider time-varying cost parameters and other practical aspects.

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