TWO DIMENSIONAL STEADY INCOMPRESSIBLE LAMINAR BOUNDARY LAYER ALONG A POROUS WALL IN TANI'S FLOW

[1]Dr. Krishnandan Prasad Singh and [2]Manish Kumar

[1]University Professor, Principal
R.D. College, Sheikhpura
T.M. Bhagalpur University, Bhagalpur
[2]Research Scholar
Department of Mathematics
T.M. Bhagalpur University, Bhagalpur.

Abstract

In the present paper an investigation has been made in to the laminar incompressible boundary layer with continuous suction along a porous wall in Tani's flow with potential flow velocity is given by $U(x)=U_0\left(l-\frac{x^2}{a^2}\right)$ and the momentum integral and the kinetic energy integral have been solved with the aid of eight degree.

Keyword : *Potential flow, pressure gradient stream function, kinetic energy, momentum integral.*

Introduction

A set of solution of boundary layer equations with average pressure gradient was found by Tani and L. Howarth. These equation relates to the potential flow given by $U(x)=Uo-ax^n,(n=1,2,3,...)$ where U_0 is the entrance velocity and 'a' is a reference length. Constitution a generalised of the flow along a flat plate and becomes identical with it when a = 0 with a>o. This defines a potential flow the pressure rise. Tani used the series expension for the stream function and found that separation occurred at $\frac{x}{a}=0.271$ in the case of the potential flow $U(x)=U_0\left(l-\frac{x^2}{a^2}\right)$ Following M.R. Head, the momentum integral and kinetic energy integral equation have been solved with the aid of eight degree polynomial velocity profile $\frac{u}{U}=f(\eta)=a_0+a_1\eta+a\eta+a_2\eta^2+...a_8\eta^8$

Point of separation have been obtained and compared with known result. The result obtained for solid wall are in excellent agreement with the known result.

Equations

For the two dimensional boundary layers along porous wall the momentum integral equation, the kinetic energy integral equation and the wall compatibility conditions are

$$\frac{dt^*}{dx} = \frac{2}{\bar{U}}\left[1-(H+2)\Lambda+\lambda\right] \quad (1)$$

$$\frac{dH_\varepsilon}{d\bar{x}} = \frac{1}{\bar{U}t^*}\left[2D - H_\varepsilon\left\{1-(H-1)\Lambda\right\}+\lambda\right] \quad (2)$$

and $m = -\Lambda + \lambda l$ (3)

where $\lambda^t = \frac{V_s\delta}{\upsilon}, \Lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx} = t^*\frac{d\bar{U}}{d\bar{x}}$

and $\lambda = \frac{\theta}{\delta}\lambda^t = \frac{U_s\theta}{\upsilon}$

Velocity Profile

The eight degree polynomial velocity profile

$$\frac{u}{U} = f(\eta) = 1-(1-\eta)^5\left[\begin{array}{l}1+\left(2.774+\frac{15+\lambda^t}{45}k_8\right)\eta \\ +\left(3.07-\frac{6+\lambda^t}{9}k_8\right)\eta^2+1.61\eta^3\end{array}\right] \quad (4)$$

Where k_8 in a parameter connected with the wall porosity $\lambda^t = \frac{V_s\delta}{\upsilon}$ with the pressure gradient of the external flow $U\left(\frac{dU}{dx}\right)$ and the wall compatibility condition

$$\lambda\left(\frac{\theta}{\delta}\right)^2\left(2.226+\frac{15+\lambda}{45}k_8\right)+\left(\frac{\theta}{\delta}\right)^2\times 2k_8-\Lambda = 0 \quad (5)$$

Where $\lambda = \frac{U_s\theta}{\upsilon}$

$\lambda < 0$ suction

$\lambda > 0$ injuction

For the family of velocity profiles the variations of the boundary layer characteristic θ/δ, H, H_E, *l* and D against the parameter K_8 for three different values of $\lambda^t = 0, -0.5$ and –1.0 respectively

Boundary layer along porous wall in tani's flows

It is proposed to investigate the boundary layer with Suction along porous wall in Tani's flow for which the potential flow velocity in given by $U(x) = U_0\left(1 - \frac{x^2}{a^2}\right)$. This type of flow occurs when a straight channel with parallel walls in succeeded by diverging channel with adverse pressure gradient. U_0 is the velocity at the entrance where the diverging flow start and 'a' in reference length. For thin flow

$$\frac{U(x)}{U_0} = 1 - \frac{x^2}{a^2}$$

i.e., $\bar{U} = 1 - \bar{x}^2$

and $\Lambda = \frac{\theta^2}{\upsilon}\frac{dU}{dx} = t^*\frac{d\bar{U}}{d\bar{x}} = -2\bar{x}t^*$

Solution with the aid of eight degree velocity pro file

The momentum integral education (1), the kinetic energy integral education (2) and the wall compatibility condition (5) becomes

$$\frac{dt^*}{dx} = f\left(\bar{x}, t^*, H_\varepsilon\right) \quad (6)$$

Where $f\left(\bar{x}, t^*, H_\varepsilon\right) = \frac{2}{1-\bar{x}^2}\left[1 + 2\bar{x}t^*(H+2) + \frac{\theta}{\delta}\lambda^1\right]$

$$\frac{dH_\varepsilon}{d\bar{x}} = g\left(\bar{x}, t^*, H_\varepsilon\right) \quad (7)$$

Where $g\left(\bar{x}, t^*, H_\varepsilon\right)$

$$= \frac{1}{t^*\left(1-\bar{x}^2\right)}\left[2D - H_\varepsilon\left\{1 + 2\bar{x}t^*(H-1) + \frac{\theta}{\delta}\lambda^1\right\} + \frac{\theta}{\delta}\lambda^1\right]$$

and $\left(\lambda^{12} + 15\lambda^1 + 90\right)k_8 = -90 \times t^*\left(\frac{\delta^2}{\theta^2}\right)$

$$= -100.17\lambda^1 \quad (8)$$

Values at the starting point

The entrance section $\left(\bar{x}=0\right)$ of the diverging flow is taken as the starting point for the numerical step by step calculation

At $\bar{x}=0, t^{*}=0$ and hence from equation (8) we have

$$k_8 = -\frac{100.17}{\lambda'^{2}+15\lambda^{1}+90} \tag{9}$$

The starting values for three different values of λ^1 are obtained

(i) $\lambda^1 = 0$

then $k_8 = 0$

and $\frac{\theta}{\delta} = 0.09997$

$H = 2.5900$

$l = 0.2225$

$H_\varepsilon = 1.5699$

$D = 0.1685$

(ii) $\lambda^1 = -0.5$

then $k_8 = 0.6052$

and $\frac{\theta}{\delta} = 0.09928$

$H = 02.5391$

$l = 0.2403$

$H_\varepsilon = 1.5750$

$D = 0.1718$

(iii) $\lambda^1 = -1.0$

then $k_8 = 1.318$

and $\frac{\theta}{\delta} = 0.0984$

$H = 2.4870$

$L = 0.2594$

$H_{\varepsilon} = 1.5811$

$D = 0.1756$

Solution of the momentum integral and the kinetic energy integral equation

With the values at the starting point numerical solutions of the momentum integral equation (6) and the kinetic energy integral equation (7) have been obtained for $\lambda^1 = 0, -0.5$ and -1.0.

Numerical integrations have been carried out for constants values of $\lambda^1 = 0, -0.5$ and -1.0 by Runge-kutta method and by Adam's method.

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