# "ON PARA COMPLEX MANIFOLD"

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**Summary:**

In this paper we have been introduced para complex manifold and contravariant almost para analytic vector. Some important results have been investigated.

## 1.1 INTRODUCTION:

Let $V_n$ be a $2n$ dimensional differentiable manifold. Let $p$ $E$ $V_n$ let us denote the tangent space of $V_n$ at $p$ by $T_p(V_n)$

In the following let $X, Y, Z ...... \in T_p(V_n)$ also referred as vector fields on $V_n$. Let us take an endomorphism $F : T_p(V_n) \longrightarrow T_p(V_n)$

If $F^2 = I_n$ i.e. $F^2 X = X$

where $F$ has differentiability of class $C^r$ then $F$ is called almost para complex structure on $V_n$, and $V_n$ is referred as almost para complex manifold with structure $F$.

**Example**: Let us consider $V_n$ on which $F$ is given by

$$((F)) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

then it is easy to verify that $F^2 = I_4$

**Theorem 1.2:**

Let us put from (2.1) we have

$$\mu F' \underline{\underline{def}}\ F\ \mu$$

$$(\mu F')(X) = F\ \mu(X)$$

$$(\mu F')(X) = F\ \mu(F' X)$$

$$\mu\, F'^2\ X = F^2 \mu\ X$$

$$\mu F'^2 X\ =\ \mu\ X$$

$$\boxed{F'^2\ x = I}$$

where μ is the singular tensor of type (1,1) then *F'* also give an almost para complex structure.

Almost para complex structure is not unique.

**2. Nijenhuis Tensor 2.2**

$$N(X,Y)\, \underline{\underline{def}}\, [\bar{X},\bar{Y}] + \overline{\overline{[X,Y]}} - \overline{[X,Y]} - \overline{[X,\bar{Y}]}$$

Change the relation between Nijenhuis tensor on almost para complex structure.

**Solution**:

$$N[X,Y] = (\bar{X},\bar{Y}) + \overline{\overline{[X,Y]}} - [\bar{X},\bar{Y}] - [\bar{X},\bar{Y}]$$

$$\left[\bar{X},\bar{Y}\right] = D_{\bar{x}}^{\bar{y}} - D_{\bar{y}}^{\bar{x}}$$

$$\left[\bar{X},\bar{Y}\right] = \left(D_{\bar{x}}F\right) + F\left(D_{\bar{x}}Y\right) - \left(D_{\bar{y}}F\right) - F\left(D_{\bar{y}}\right)$$

$$\overline{\overline{[X,Y]}} = D_x Y - D_y X$$

**Nijenhuis Tensor** on an almost para complex.

$$N\overline{\left[\bar{X},\bar{Y}\right]} = N\left[\bar{X},Y\right] = -N\left[\bar{X},Y\right] = N\left[X,\bar{Y}\right]$$

equivalent to

$$N\left[\bar{X},\bar{Y}\right] = N[X,Y] = N\left[\bar{X},Y\right] = N\overline{\left[X,\bar{Y}\right]}$$

**Proof**:

$$N(X,Y) = \left[\bar{X},Y\right] + \overline{\overline{[X,Y]}} - \overline{\left[\bar{X},Y\right]} - \overline{\left[X,\bar{Y}\right]}$$

$$N(X,Y) = \left[\bar{X},Y\right] + \overline{\overline{\left[\bar{X},\bar{Y}\right]}} - \overline{\left[\bar{\bar{X}},Y\right]} - \overline{\left[\bar{X},\bar{\bar{Y}}\right]}$$

$$= [X,Y] + \left[\bar{X},\bar{Y}\right] - \overline{\left[X,\bar{Y}\right]} - \overline{\left[\bar{X},\bar{Y}\right]}$$

$$N\overline{\left[\bar{X},\bar{Y}\right]} = \overline{\left[\bar{X},\bar{Y}\right]} + \overline{\left[\bar{X},\bar{Y}\right]} - \overline{\overline{\left[X,\bar{Y}\right]}} - \overline{\overline{\left[\bar{X},\bar{Y}\right]}}$$

$$= \overline{[X,Y]} + \overline{\left[\bar{X},\bar{Y}\right]} - \left[X,\bar{Y}\right] - \left[\bar{X},Y\right]$$

$$N\overline{\left[\bar{X},Y\right]} = \overline{\left[\bar{\bar{X}},\bar{Y}\right]} + \overline{\left[\bar{X},Y\right]} - \overline{\overline{\left[\bar{\bar{X}},Y\right]}} - \overline{\overline{\left[\bar{X},\bar{Y}\right]}}$$

$$=\overline{[X,Y]}+\overline{[\bar{X},Y]}-[X,Y]-[\bar{X},\bar{Y}]$$

$$N\overline{[X,Y]}=\overline{[\bar{X},\bar{Y}]}+\overline{[X,Y]}-\overline{\overline{[X,Y]}}-\overline{\overline{[X,\bar{Y}]}}$$

$$=\overline{[X,Y]}+\overline{[\bar{X},Y]}-[X,Y]-[X,\bar{Y}]$$

$$N\overline{[X,\bar{Y}]}=\overline{[\bar{X},\bar{Y}]}+\overline{[X,Y]}-\overline{\overline{[\bar{X},Y]}}-\overline{\overline{[X,\bar{Y}]}}$$

$$=\overline{[X,Y]}+\overline{[X,\bar{Y}]}-[\bar{X},\bar{Y}]-[X,Y]$$

$$\overline{[X,Y]}=\overline{D_{\bar{x}}Y}-\overline{D_y\bar{X}}$$

$$=\overline{D_{\bar{x}}Y}-\left[\left(D_yF\right)(X)+F\left(\overline{D_yX}\right)\right]$$

$$=\overline{D_{\bar{x}}Y}-\overline{\left(D_yF\right)}(X)-FD_yX$$

$$=\overline{D_{\bar{x}}Y}-\overline{\left(D_yF\right)}(X)-D_yX$$

$$\overline{[X,\bar{Y}]}=\overline{D_x\bar{Y}}-D_{\bar{y}}X$$

$$=\left(D_xF\right)(X)+F\overline{D_x(Y)}-D_{\bar{y}}X$$

$$=\overline{\left(D_xF\right)}(Y)+D_xY-D_{\bar{y}}X$$

Now Nijenhuis Tensor,

$$N[X,Y]=[\bar{X},\bar{Y}]+\overline{\overline{[X,Y]}}-\overline{[\bar{X},Y]}-\overline{[X,\bar{Y}]}$$

Putting the value

$$(D_{\bar{x}}F) + F(D_{\bar{x}}Y) - (D_{\bar{y}}F)(X) - (F D_{\bar{y}}X) + D_xY - D_yX$$

$$- \overline{D_xY} + \overline{(D_yF)}X + D_yX - (D_xF)(Y) -$$

$$D_xY + \overline{D_y(X)}$$

$$\boxed{N[X,Y] = (D_{\bar{x}}F)(Y) - (D_{\bar{y}}F)(X) + \overline{(D_yF)}(X) - (D_xF)(Y)}$$

Some theorem we have

rank $((F)) = n$

**Proof**:

$$\bar{X} = FX = 0 \Rightarrow \bar{X} = 0 \Rightarrow X = 0$$

hence $\bar{X} = 0$ has only trivinal solution on $x$=0 consequently rank $((F)) = n$. It can also be seen that $\|F\| \neq 0$ so rank $F = n$ in above example rank $F = X$ become $\|F\| = 1 \neq 0$ .

## REFERENCES:


1. M.M. Tripathi and S.S. Shukla : Semi-invariant submanifolds of nearly kerimotsu manifolds. Bull. Calcutta Math. Soc. 95 (2003).
2. Mishra, R.S. (1972) : Integrability conditions of almost contact manifolds tensor, N.S. 2, 211-216.
3. Mishra, R.S. (1989) : Almost contact metric manifolds, J. Nath. Acad. Math, Vol. 7.
4. Mishra, R.S. and H.B. Pandey : On almost complex spaces, Prog. Math. 9(2) (1975), 32-35.