A Study of edge labelling of a Bloom graph B(m,n) and its topological properties.

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Abstract :

In this paper , we are going tosee how theedges of the bloom graph are labelled with certain conditions and we study some of its topological properties including its Hamiltonian and Pancyclity Property .

Keywords :

Edge labelling, Bloom graph, Planarity, Interconnection network,Hamiltonicity, Pancyclicity.

1. Introduction

Graph Labellings, where the vertices and edges are assigned real values subject to certain conditions , have often been motivated by their utility to various applied fields and their intrinsic mathematical interest. Graph labellings were first introduced in the mid sixties in the intervening years. " A dynamic survey of graph labelling was done by Gallian,Joseph [1]. Some more works on labelling is attributed to Graham and Sloane in 1980 [2]. Now we are going to see a different kind of graph which is known as Bloom graph.
Planar and Regular graphs plays an important role in the field of graph applications. Some graphs are planar but not regular and may be regular graphs are not planar also. We introduce a new kind of graph named as bloom graph, which is both planar and regular.

Graphs can be used to model interconnection networks in which vertices correspond to processors of the network and the edges correspond to communication links. A new interconnection network topology called the bloom graph has introduced.

The performance of interconnection network is a critical issue in parallel computing. This has been a major driving force for the research of inter connection networks. The study of interconnection networks in parallel computing system includes the performance and cost issues.

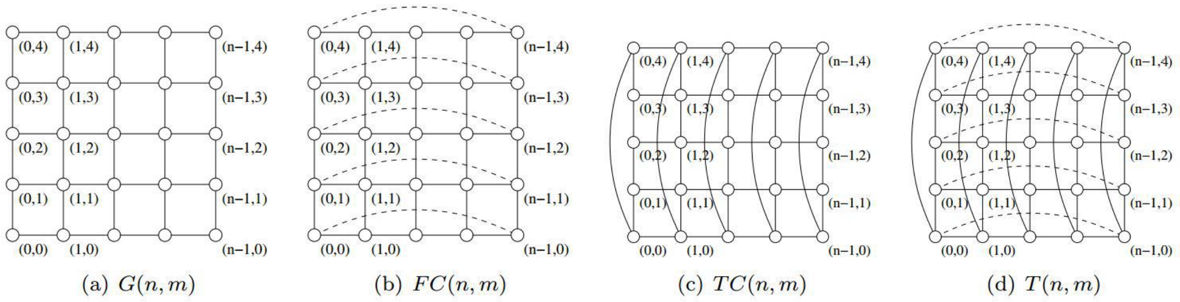


Fig 1. Grid, Cylinder and Torus
Grid and cylinder are planar but not regular.Whereas, torus is regular but not planar. In this paper, we introduce a new kind of graph whichis both planar and regular

2. Bloom Graph

Definition

The Bloom Graph B(m,n),m,n>1 is defined as follows: V[B(m,n)] ={(x,y) | 0 < x < m-1, 0 < y < n 1}, two distinct vertices (x1, y1) and (x2, y2) being adjacent if and only if

(i) x1 = x2 -1 and y1 = y2

(ii) x1 = x2 = 0 and y1+1 ≡ y2 mod n

(iii) x1 = x2 = m and y1+1≡ y2 mod n

(iv) x1 = x2 -1 and y1 +1≡ y2 mod n

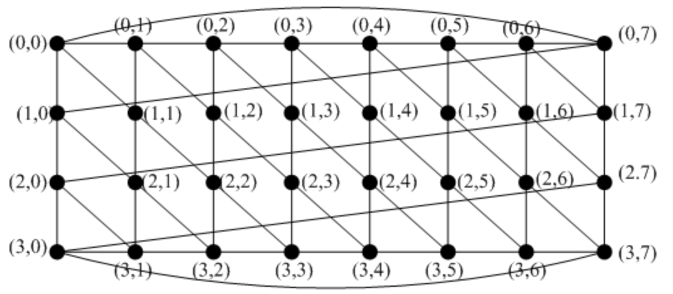


Fig 2.

The first condition describes the vertical edges, the second and third condition describes the horizontal edges in the top most and the lower most rows respectively. Condition four describes the slant edges.(See Figure 2)

3.Edge Labelling of a Bloom Graph

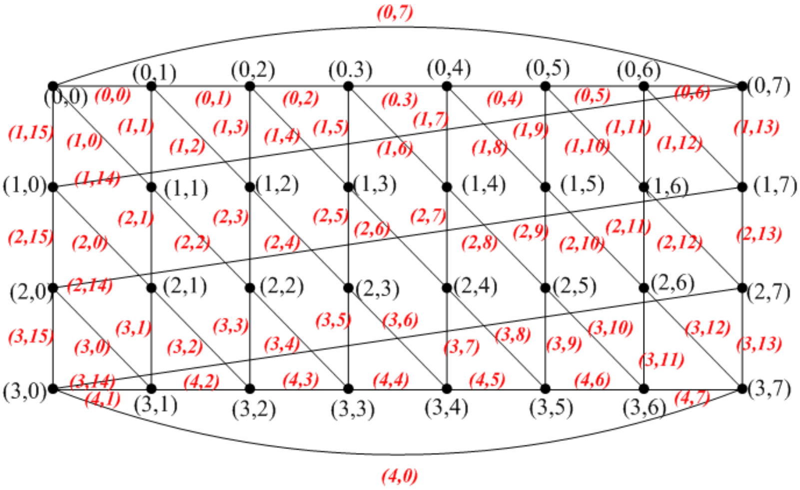
Edge labeling of bloom graph is as follows:-

1. If the vertices (0, y) and (0, y + 1(mod n)) are adjacent then label the edge as (0, y).

2. If the vertices (m-1, y) and (m-1, y +1(mod n)) are adjacent then label the edge as (m, y + 1).

3. If the vertices (x, y) and (x+1, y) are adjacent then label the edge as (x+1, 2y-1(mod 2n)).

4. If the vertices (x, y) and (x + 1, y +1(mod n)) are adjacent then label the edge as (x + 1, 2y).



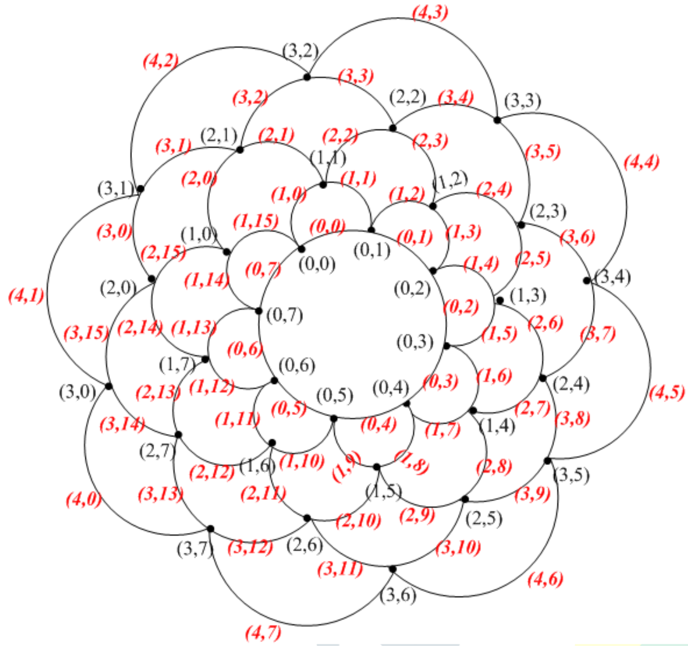


fig 3.

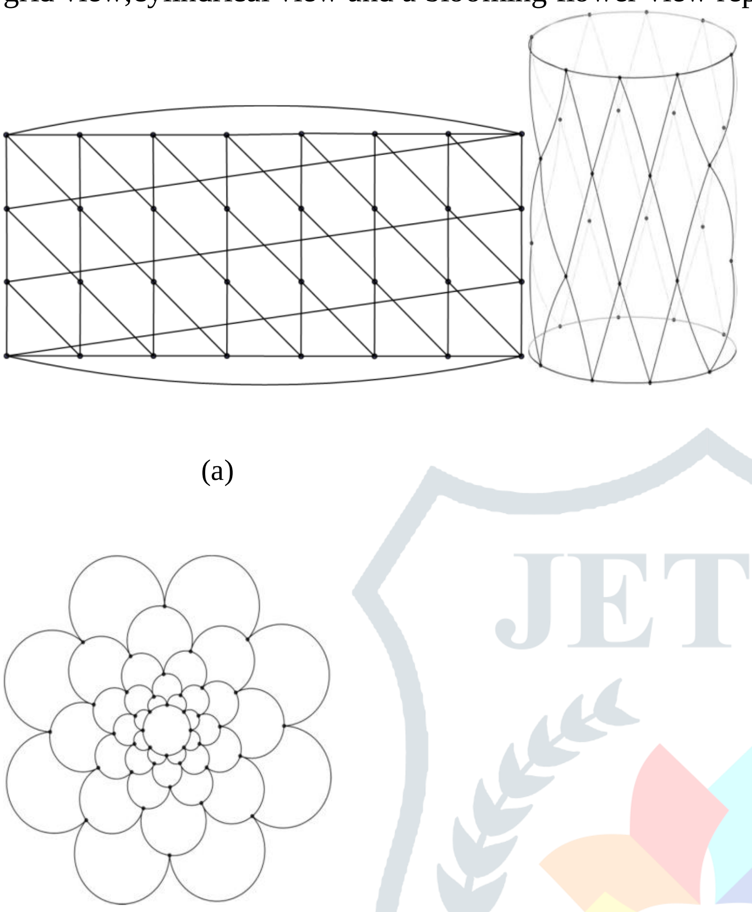
1.The first and the Second condition describes the horizontal edges in the top most and lower most rows,

2,The condition three describes the vertical edges

3,Condition fourdescribes the slant edges

4. Properties of a bloom graph

The Planarity of bloom graph can be understand .In order to understand the planarity of the bloom graph, it can redrawn as in fig 4 (b) and (c). The graphs in fig 3 (a) , (b) and (c) are isomporphic to each other giving a grid view,cylindrical view and a blooming flower view repectively.



(a) (b)

(c)

Fig 4. Isomorphism of B(m,n)

Properties of Bm,n

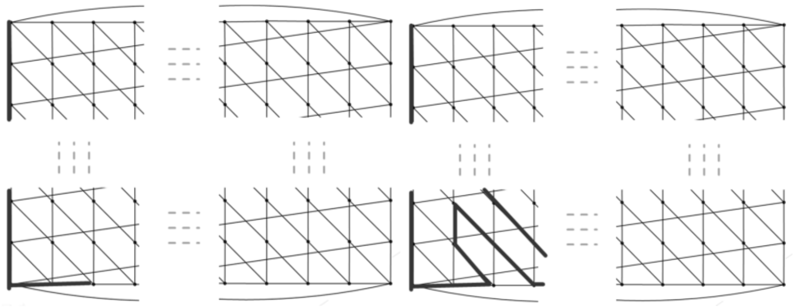
1. Number of vertices of bloom graph is mn.
2. Number of edges of bloom graph is 2mn.
3. Diameter of bloom graph is [m+n]/2
4. Bloom graph is planar.
5. Bloom graph is 4 regular.
6. Crossing number of bloom graph is zero.
7. Vertex connectivity of bloom graph is 4.
8. Edge connectivity of bloom graph is 4..

5. Hamiltonicity

Theorem : Bloom Graph B(m,n) is hamiltonian.

Proof: The Hamiltonicity of B(m,n) can be discussed in two cases on the parity of m and n.

Case 1: If m and n are of same parity. Then follow the Hamiltonian cycle as described in fig 5



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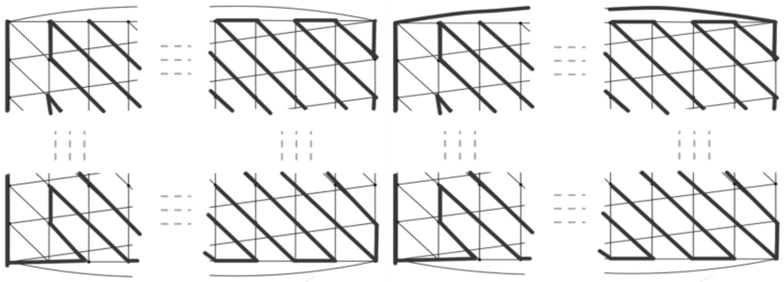
Step 1 Step 2



Step 3 Step 4

Fig 5.

Case ii: If m and n are of different parity. Then follow the Hamiltonian cycle as described in fig 6.

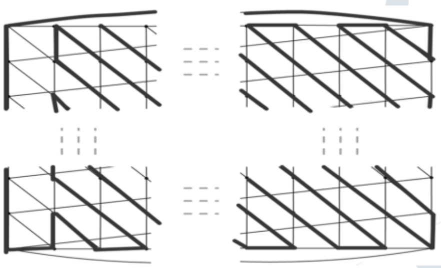


Fig 6.

6.Pancyclicity

Theorem ..B(m,n) is pancyclic.

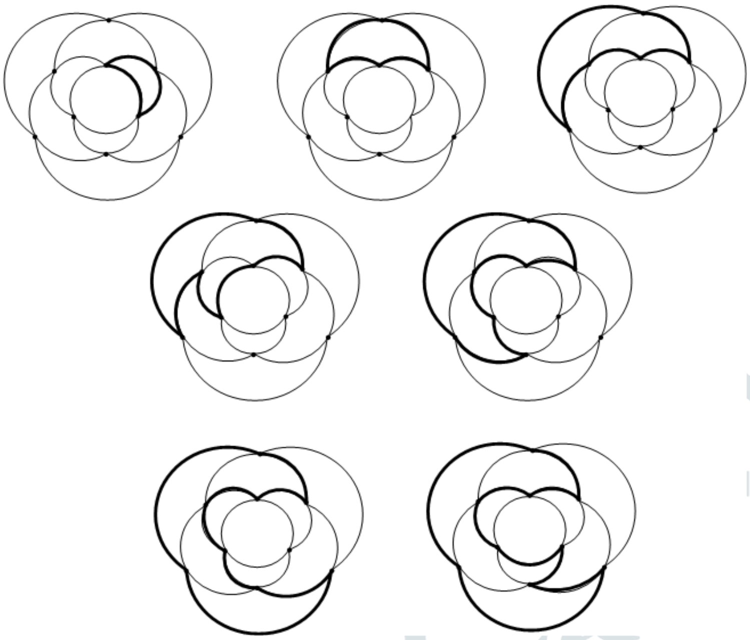


Fig 7.

Proof. Bloom graph B3,3 is pancyclic. i,e B3,3 contains cycles of length 3, 4, _ _ _ , 8, 9. (See Figure 7)
Assume that B3,k is pancyclic. According to theorem B3,k is Hamiltonian. The Hamiltonian cycle can be constructed as follows. (1, 1) → (2, 2) → (1, 2) → (2, 3) → _ _ _ → (1, k - 1) → (2, k) → (1, k) → (2, k) → (3, k) → (3, k -1) _ _ _ → (3, 1) → (2, 1) → (1, 1). In B3,k+1, cycle of length 3k+1 can be constructed as follows. (1, 1) →

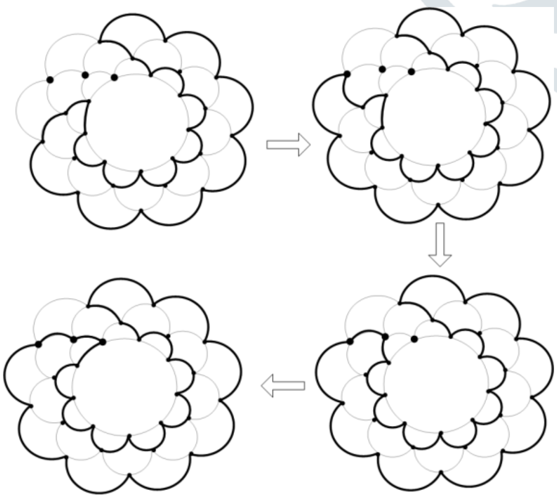


Fig 8.
Figure 8 Hamiltonian cycle in B3,8 of length 24 and the cycles of length 25, 26, 27 in B3,9
(2, 2) → (1, 2) → (2, 3) → _ _ _ → (1, k - 1) → (1, k) → (2, k) → (3, k + 1) → (3, k) → (3, k - 1) → _ _ _ → (3, 1) → (2, 1) → (1, 1).
(1, 1) → (2, 2) → (1, 2) → (2, 3) → _ _ _ → (1, k - 1) → (2, k) → (1, k) → (2, k + 1) → (3, k + 1) → (3, k) → (3, k -1) → (3, 1) → (2, 1) → (1, 1) and (1, 1) → (2, 2) → (1, 2) → (2, 3) → _ _ _ → (1, k - 1) → (1, k) → (1, k + 1) → (2, k + 1) → (3, k + 1) →

(3, k) → (3, k - 1) → (3, 1) → (2, 1) → (1, 1) is the constructing method to obtain cycles of length 3k+2 and 3k+3 respectively. Figure 8.shows the Hamiltonian cycle

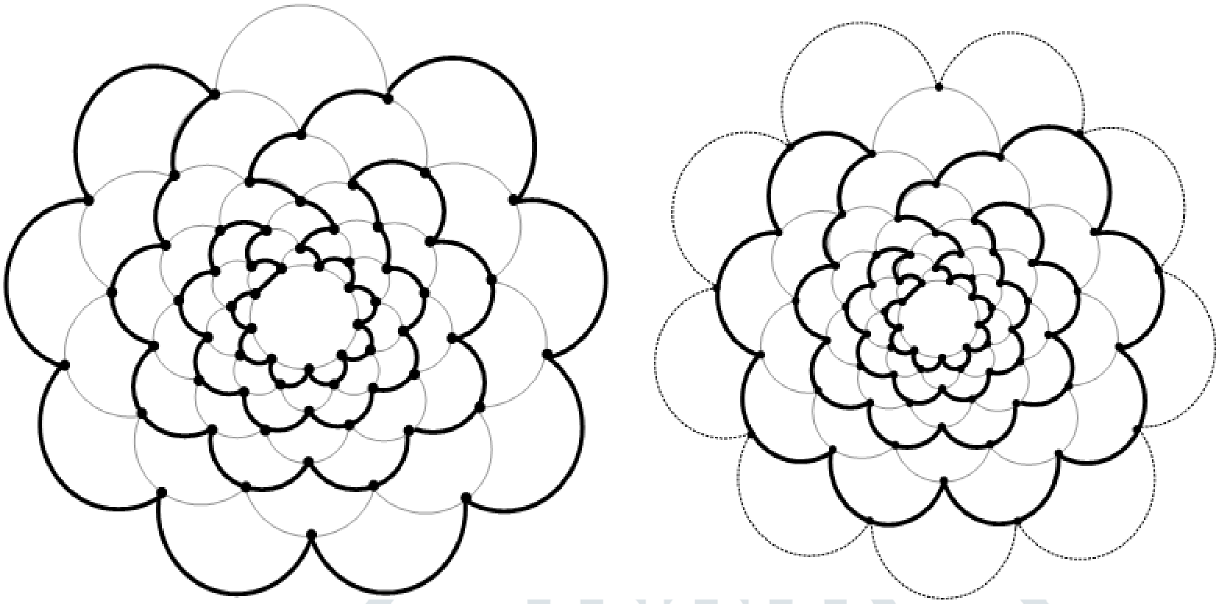


Fig 9.

Figure 9. Hamiltonian cycle in B7,9 and newly added layer with cycle of length 71.
in B3,8 and the cycles of length 25, 26, 27 in B3,9.
Assume that Bh,k is pancyclic for some positive integers h, k. According to Theorem there exist a Hamiltonian cycle of length hk.

Case (1) When h is odd.

The construction of Hamiltonian cycle is as follows. (1, 1) → (1, 2) → (1, 3) → _ _ _ → (1, k) → (2, k) → (3, k) → (4, k) → (3, k □ 1) → (4, k - 1) → _ _ _ → (3, 3) → (4, 3) → (5, 3) _ _ _ → (h -1, 4) → (h -2, 4) → _ _ _ → (h - 1, 2) → (h, 3) → (h, 2) → (h, 1) → (h, k) → _ _ _ → (h, 4) → (h - 1, 3) _ _ _ → (5, 2) → (4, 2) → (3, 2) → (2, 1) → (1, 1).
Figure 9 shows the Hamiltonian cycle when h = 7. Add one layer to Bh,k, the outer edges in Bh,k split by two and resulting a cycle of length k(h+1)-1. From this cycle of length k(h+1)-1, exclude one vertex and include one outer edge resulting a cycle of length k(h+1)-2. Repeating this process results cycles of length k(h+1)-(k-1)(See figure 10)

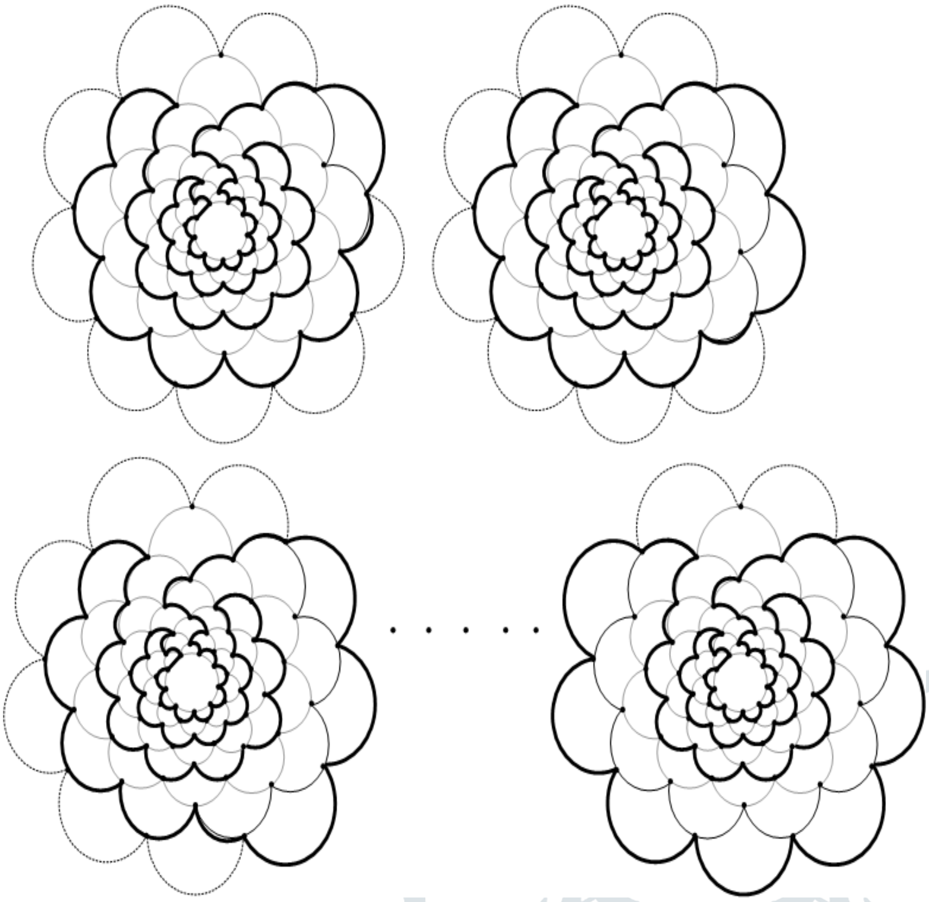


Figure 10. Construction of cycles of length 70 to 64 in $B_{7,9}$

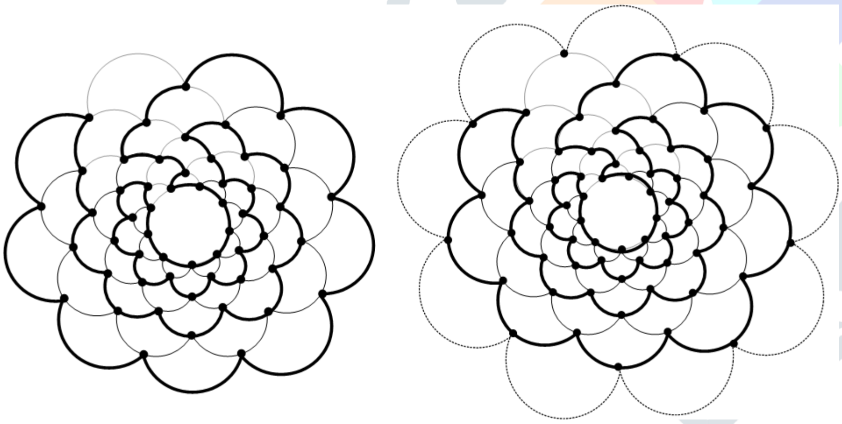


Figure 11, Hamiltonian cycle in B6,9 of length 54 and newly added layer in B6,9 with cycle of length 62

Case (2) When h is even.

The construction of Hamiltonian cycle is as follows. (1, 1) → (1, 2) → (2, 3) → _ _ _ → (1, k) → (2, k) → (3, k) → (2, k -1) → (3, k - 1) → (2, k - 2) → _ _ _ → (3, 3) → (2, 3) → (3, 2) → _ _ _ → (h - 2, 1) → (h - 1, 2) → (h, 2) → (h, 1) → (h, k) → _ _ _ → (h, 3) → (h -1, 3) _ _ _ → (4, 2) → (3, 1) → (2, 1) → (1,1). Figure 11 shows the Hamiltonian cycle when h = 6 and the newly added layer in B6,9 with cycle of length 62. Add one layer to Bh,k, the outer edges in Bh,k split by two and resulting a cycle of length k(h + 1) - 1. From this cycle of length k(h + 1) - 1, exclude one vertex and include one outer edge resulting a cycle of length k(h+1)-2. Repeating this process results cycles of length k(h + 1) - (k -1). (See Figure 12).

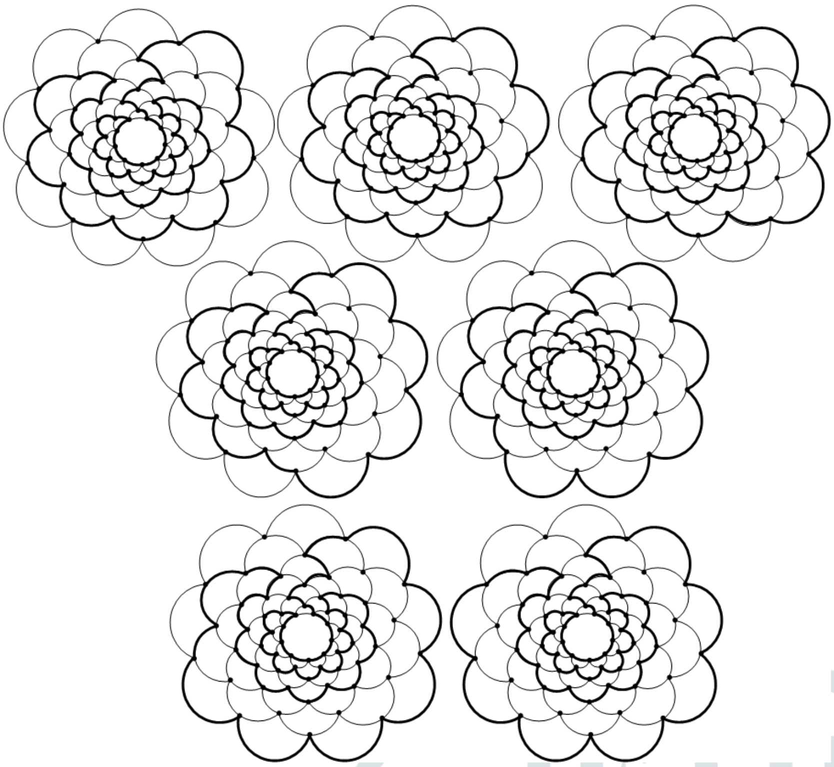


Figure 12. Construction of cycles in B6+1,9 of length 62 to 55.

7.Conclusion

In this paper we computed the edge labelling of the bloom graph B(m,n).The basic topological properties of Bloom graph have been discussed. The Hamiltonicity have also been proved. Thepancyclicity property have been discussed.

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