Recognizability Of Finite And Infinite Triangular Pictures By Online Tessellation Automata

K. Bharathivelan[1], V. Devi Rajaselvi[2] and S. Saravanan[3]

1^2 Department of Mathematics, Hindustan College of Arts and Science, Chennai, Tamilnadu.

3 Department of Mathematics, Dr. M.G.R. Government Arts and Science College for Women, Villupuram, Tamilnadu.

Abstract:

Pictures could be regarded as sentences in a picture language. This suggested the idea that notions and techniques of language theory could be extended to two dimensional environments. The recognizability of rectangular and hexagonal picture languages using tiling systems is already studied. Using coordinate systems in triangular grid we have defined the recognizability of triangular picture languages. We have investigated the notion of recognizability of iso - triangular picture languages by a new formalism called online tessellation automata. On the other hand infinite pictures are digitized images which occur in the grid of two dimensional planes. The study about the recognizability of infinite arrays is interesting ones. We extend this study to infinite triangular pictures. Also the notion of recognizability of infinite triangular pictures by a new formalism called online tessellation automata is studied.

Keywords: Online Tessellation Automata, Infinite Triangular Array Languages.

Introduction: The generalization of finite state automaton to two dimensional languages can be attributed to M Blum and C Hewitt [1] and they have introduced the notion of 4-way automaton. In 1991, A. Restivo and D. Giammarresi defined the family REC of recognizable picture languages [6]. An interesting model of 2D automaton to recognize REC class of picture languages is the two-dimensional online tessellation acceptor [4]. In [12], the authors introduced Wang automaton, a model of automaton for recognition of rectangular picture languages, which is based on Wang tiles. The Wang automaton combines features of both online tessellation acceptor and 4-way automaton but differ in having many scanning strategies to visit the input picture.

The recognizability of rectangular and hexagonal picture languages using tiling systems is studied in [2, 3, 5, 6, 11]. Using co-ordinate systems in triangular grid we have defined the recognizability of triangular picture languages. Differently the study of iso -triangular pictures [4] is a subject of great interest with remarkable applications.

On the other hand infinite pictures are the digitized images which occur in the grid of two dimensional plane. The study about the recognizability of infinite arrays is given in [2]. Also in [10], we learn about recognizable infinite array languages. In [13] we get idea about automata on infinite objects. We extend this study to infinite triangular array languages and we find the recognizability of infinite triangular pictures by online tessellation automata. In [8, 9] the authors introduced domino systems for infinite array languages. Here the notion of recognizability of infinite triangular pictures by a new formalism called online tessellation automata is introduced.

1. ISO TRIANGULAR PICTURES

To describe connected array of pictures in triangular grid, the coordinate system for the nodes of the triangular picture is defined in the following way.

We assign the co-ordinate values to the points inductively. If the co-ordinates of a point are (a_1, a_2, a_3) then the co-ordinates of its neighbour in the direction X are (a_1+1, a_2, a_3-1) and in the opposite direction (a_1-1, a_2, a_3+1). Similarly in the direction Y are (a_1-2, a_2+2, a_3) and in the opposite direction of Y are (a_1+2, a_2-2, a_3). According to the way the co-ordinates are introduced. Moving in the direction Z, the co-ordinates are (a_1, a_2-2, a_3+2) and in the opposite direction (a_1, a_2+2, a_3-2).

Definition 1.1: An iso -triangular picture p over the alphabet Σ is an isosceles triangular arrangement of symbols over Σ. The set of all iso -triangular pictures over the alphabet Σ is denoted by Σ_T^{**}. An iso-triangular picture language over Σ is a subset of Σ_T^{**}. Given an iso -triangular picture p the number of rows (counting from the bottom to top) denoted by r(p) is the size of an iso -triangular picture. The empty picture is denoted by Λ.

Iso-triangular pictures can be classified into four categories.

1. Upper iso-triangular picture 2. Lower iso-triangular picture 3.Right iso-triangular picture
4. Left iso-triangular picture

Definition 1.2: If $p \in \Sigma_T^{**}$, then $\hat{p}$ is the iso-triangular picture obtained by surrounding p with a special boundary symbol $\# \notin \Sigma$.

For example if $p = \begin{array}{ccc} & c & \\ c & c & c \end{array}$ then $\hat{p} = \begin{array}{ccccccc} & & & \# & & & \\ & & \# & c & \# & & \\ & \# & c & c & c & \# & \\ \# & \# & \# & \# & \# & \# & \# \end{array}$

Definition 1.3: Let $p \in \Sigma_T^{**}$ be an iso-triangular picture. Let Σ and Γ be two finite alphabets and $\pi : \Gamma \rightarrow \Sigma$ be a mapping which we call a projection. The projection by mapping π of an iso-triangular picture $p' \in \Gamma_T^{**}$ such that $\pi(p(i, j, k)) = p'(i, j, k)$.

1.1 RECOGNIZABILITY OF ISO - TRIANGULAR PICTURES USING ONLINE TESSELLATION AUTOMATA

In this paper we define a two direction iso-triangular online tessellation automata referred as 2DIOTA to accept languages of iso-triangular picture languages.

Definition 1.1.1: A non-deterministic (deterministic) two direction iso-triangular online tessellation automaton is defined by $A = (\Sigma, Q, q_0, F, \delta)$ where

Σ is the input alphabet, Q is the finite set of states,

$q_0 \in Q$ is the initial state, $F \subseteq Q$ is the set of final states,

$\delta : Q \times Q \times \Sigma \rightarrow 2^Q$ ($\delta : Q \times Q \times \Sigma \rightarrow Q$) is the transition function.

A 2DIOTA is composed of a two dimensional array of cells in a triangular grid indexed by N^3. Initially the symbol p(i, j, k) of the input picture p is stored on each cell c(i, j, k). A run on the picture p consists in computing on each cell c(i, j, k) a state $q_i \in Q$ according to the transition function δ and the symbol p(i, j, k). Here $\delta(q_j, q_i, a) = p$

A 2DIOTA accepts the input iso-triangular picture p if there exists a run on $\hat{p}$ such that the state associated to the position c(s, t, u) is a final state where c(s, t, u) is the last symbol of the last row. The set of all iso-triangular pictures recognized by $\mathcal{A}$ is denoted by L($\mathcal{A}$). Let L(2DIOTA) be the set of iso-triangular picture languages recognized by 2DIOTAs.

Example 1.1.1:

A 2 direction iso-triangular online tessellation automaton to recognize the local iso-triangular picture language L(Δ) shown below

Let Σ = {a, b} be a finite alphabet. Let

$$\Delta = \left\{ \begin{array}{ccc} & \# & \\ \# & \# & \# \end{array}, \begin{array}{ccc} & b & \\ \# & \# & \# \end{array}, \begin{array}{ccc} b & b & b \\ & \# & \end{array}, \begin{array}{ccc} \# & b & b \\ & \# & \end{array}, \begin{array}{ccc} b & b & \# \\ & \# & \end{array}, \right.$$

$$\begin{array}{ccc} & \# & \\ \# & b & b \end{array}, \begin{array}{ccc} & a & \\ b & b & b \end{array}, \begin{array}{ccc} & \# & \\ b & b & \# \end{array}, \begin{array}{ccc} \# & a & a \\ & b & \end{array}, \begin{array}{ccc} a & a & \# \\ & b & \end{array},$$

$$\begin{array}{ccc} & \# & \\ \# & a & a \end{array}, \begin{array}{ccc} & b & \\ a & a & a \end{array}, \begin{array}{ccc} & \# & \\ a & a & \# \end{array}, \begin{array}{ccc} \# & b & b \\ & a & \end{array}, \begin{array}{ccc} b & b & b \\ & a & \end{array},$$

$$\left. \begin{array}{ccc} b & b & \# \\ & a & \end{array}, \begin{array}{ccc} & \# & \\ \# & a & \# \end{array}, \begin{array}{ccc} \# & a & \# \\ & b & \end{array}, \begin{array}{ccc} a & a & a \\ & b & \end{array} \right\}$$

Then $L_1 = L(\Delta) = \left\{ \begin{array}{ccc} & a & \\ b & b & b \end{array}, \begin{array}{ccccc} & & a & & \\ & b & b & b & \\ a & a & a & a & a \end{array}, \begin{array}{ccccccc} & & & a & & & \\ & & b & b & b & & \\ & a & a & a & a & a & \\ b & b & b & b & b & b & b \end{array}, \ldots \right\}.$

The language L(Δ) is the set of all iso-triangular pictures with size $k \geq 2$ with a's appearing in the odd rows and b's appearing in the even rows. Clearly L(Δ) is local.

A 2 direction iso-triangular online tessellation automaton to recognize the above local iso-triangular picture language is given by $\mathcal{A} = (\Sigma, Q, q_0, F, \delta)$ where $\Sigma = \{a, b\}$ is the input alphabet $Q = \{q_0, q_1, q_2\}$, $F = \{q_1, q_2\}$

Let q_0 be the state associated with all border symbols # and the transition function δ is given by (1) $\delta(q_0,q_0, a) = q_1$ (2)$\delta(q_0,q_0, b) = q_2$ (3)$\delta(q_1,q_2, b) = q_2$ (4)$\delta(q_0,q_2, b) = q_2$ (5) $\delta(q_2,q_1, a) = q_1$ (6) $\delta(q_0,q_1, a) = q_1$

1.2 RECOGNIZABILITY INFINITE TRIANGULAR ARRAYS BY ONLINE TESSELLATION AUTOMATA

In this section we extend the concept of local triangular languages to ωω-triangular arrays. We define ωω-local triangular languages by requiring a set of windows of size 2 to occur in the infinite triangular arrays. This family coincides with the family of adherences of the local picture languages of infinite triangular arrays. We then consider an extended notion of ωω-local languages by requiring certain windows of size 2 to occur infinitely by often in the infinite triangular arrays. The resulting class strictly contains the ωω local language family. We define the notion of a ωω-recognizable language. This language is accepted by the two-direction online tessellation automaton reading infinite triangular arrays

Definition 1.2.1: If $p=(a_{ij})$,i=1,2,... and j=1,2,...and $p \in \Sigma_T^{\omega\omega}$.

For $p \in \Sigma_T^{**} \cup \Sigma_T^{\omega\omega}$,a prefix of p is triangular array p=(a_{ij}),a prefix of q is triangular array $q=(a_{ij})$, i=1,2......l, j=1,2,3...........r; if $1 \leq l$, r $\leq \infty$ and l, r $\leq$ m if p is of size m. We then write $q \leq p$ and if $q \neq p$ and we write $q<p$. For $p \in \Sigma_T^{**} \cup \Sigma_T^{\omega\omega}$, the set of all prefixes of p is denoted by FG(p).

Definition 1.2.2: For $L \subseteq \Sigma_T^{**}$,we define Lim(L)={ $p \in \sum_T^{\omega\omega}$ / FG(p)∩L is infinite}

Definition 1.2.3: For $L \subseteq \Sigma_T^{**}$,we define adh (L)={ $p \in \sum_T^{\omega\omega}$/FG(p)⊆FG(L) where FG(L)=$\bigcup_{p \varepsilon L} FG(p)$.

Definition 1.2.4: Given a triangular array p, finite or infinite , we denote $B_k(p)$, the set of all sub triangular arrays of p of size k such that if p is of size m then $k \leq m$.

Definition 1.2.5: For any triangular array $p \in \Sigma_T^{**}$,of size m we denote b(p),the triangular array of size m+2, obtained by surrounding p by a special boundary symbol $\# \notin \Sigma$.For $p \in \Sigma_T^{\omega\omega}$,b(p) is the infinite triangular array obtained by placing a row of #s below the first row of p and #s above the elements of first right slanting line of p. Let $p \in \Sigma_T^{\omega\omega}$ .Then inf $B_2(b(p))$ is the collection of all sub-triangular arrays of size 2 in $B_2(b(p))$ which occur infinitely many times in b(p).

Example 1.2.1: Consider the infinite triangular picture**,** where the entries in the (i,j) the position $1 \le i \le m$, $1 \le j \le n$ are a and other entries are b.

$$
b(p) = \begin{array}{r}
. \;\; . \;\; . \;\; . \;\; . \;\; . \\
\# \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \\
\# \;\; b \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \\
\# \;\; b \;\; b \;\; b \;\; . \;\; . \;\; . \;\; . \;\; . \\
\# \;\; a \;\; a \;\; a \;\; b \;\; b \;\; . \;\; . \;\; . \;\; . \\
\# \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \\
\# \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \\
\# \;\; a \;\; a \;\; a \;\; . \;\; . \;\; . \;\; . \;\; b \;\; b \;\; b \;\; . \;\; . \\
\# \;\; a \;\; a \;\; a \;\; . \;\; . \;\; . \;\; . \;\; b \;\; b \;\; b \;\; b \;\; b \;\; . \\
\# \quad \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \# \;\; \#
\end{array}
$$

Then $B_2(b(p))$ =

$$
\left\{
\begin{array}{c}\# \\ \# \; \# \; \#'\end{array}
\begin{array}{c}\# \\ \# \; \# \; a'\end{array}
\begin{array}{c}\# \\ \# \; a \; a'\end{array}
\begin{array}{c}a \\ \# \; \# \; \#'\end{array}
\begin{array}{c}b \\ \# \; \# \; \#'\end{array}
\begin{array}{c}a \\ a \; a \; a'\end{array}
\begin{array}{c}a \\ a \; a \; b'\end{array}
\begin{array}{c}a \\ a \; b \; b'\end{array}
\begin{array}{c}b \\ b \; b \; b'\end{array}
\begin{array}{c}b \\ a \; a \; a'\end{array}
\begin{array}{c}b \\ a \; a \; b'\end{array}
\begin{array}{c}b \\ a \; b \; b'\end{array}
\begin{array}{c}\# \\ \# \; b \; b\end{array}
\right\}
$$

$$
\text{Inf } B_2(b(p)) = \left\{
\begin{array}{c}b \\ \# \; \# \; b'\end{array}
\begin{array}{c}b \\ b \; b \; b'\end{array}
\begin{array}{c}\# \\ \# \; b \; b\end{array}
\right\}
$$

Here $\begin{array}{c}b \\ b \; b \; b\end{array} \in \text{inf } B_2(b(p))$ where as $\begin{array}{c}a \\ a \; a \; a\end{array} \notin \text{inf } B_2(b(p))$.

In this section the notion of acceptance of languages of infinite arrays by online tessellation automata is extended to the acceptance of infinite triangular arrays by online tessellation automata.

Definition 1.2.6: A nondeterministic two direction online tessellation automaton (2DOTA) is $A=(\Sigma,Q,q_0,F,\delta)$ where Σ is an input alphabet, Q is a finite set of states ,$\delta: Q \times Q \times \Sigma \rightarrow 2^Q$ is a transition function. A computation by a two-direction OTA on an infinite triangular array p where

$$b(p) = \begin{array}{ccccccccccc} & & & & & \therefore & & & & & \\ & & & & \# & a_{31} & a_{32} & & & & \\ & & & \# & a_{21} & a_{22} & a_{23} & a_{24} & & & \\ & & \# & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & & \\ \# & \# & \# & \# & \# & \# & \# & \# & \# & \# & \# \end{array}$$

with $a_{ij} \in \Sigma$ and # is a special symbol not in Σ is done as follows.

At time $t=0$, an initial state $q_0 \in Q_0$ is associated with all the positions of b(p) holding #. At time t=1,a state from $\delta(q_0, q_0, a_{11})$is associated with the position (1, 1) holding a_{11}.At time t =2,states are associated simultaneously with positions (1,2) and (2,1) respectively holding a_{12} and a_{21}.If s_{11} is the state associated with the position (1,1), then the state associated with the position (1,2) is an element of $\delta(q_0,q_0,a_{12})$. If s_{12} is the state associated with the position (1,2), then the state associated with the position (2,1) is an element of $\delta(a_{11},s_{12},a_{21})$.We then proceed to the next vertical.

(i.e) $\delta(q_0, q_0, a_{11}) = s_{11}$, $\delta(q_0, q_0,a_{12}) = s_{12}$, $\delta(s_{11}, s_{12},a_{21}) = s_{21}$, $\delta(q_0, q_0,a_{13}) = s_{13}$, $\delta(s_{12}, s_{13},a_{22}) = s_{22}$, $\delta(s_{21}, s_{22},a_{31}) = s_{31}$ ……………etc

The states associated with each position (i,j) by the transition function, depends on the states already associated with the position (i, j-1),(i, j+1) where the entry is a_{ij}. Let s_{ij} be the state associated with the position (i,j) where the entry is a_{ij}. A run (or a computation) of an infinite triangular array is an element of Q_T^{ω}. A run for an infinite triangular array is a sequence of states $s_{11}s_{12}s_{21}s_{13}s_{22}s_{31}$…and it is denoted by r(p).

If A is non deterministic, a run for an infinite triangular array is a set, but in the deterministic case it is a singleton set. If $p \in \Sigma_T^{\omega\omega}$, the set of runs of p is denoted by R(p).For $s \in Q_T^{\omega}$, we define inf (s) as the set of all states which repeat infinitely many times in s. The language of infinite triangular arrays recognized by the non deterministic two direction online tessellation automation A is $L_T^{\omega\omega}(A)=\{p \in \Sigma_T^{\omega\omega} : inf(r(p)) \cap F \neq \phi$, for some $r(p) \in R(p)\}$. Hence we note that if L(A) is the set of infinite triangular arrays generated by A, then $L(A) \subseteq FG(L_T^{\omega\omega}(A))$.

CONCLUSION: Recognizability of triangular and infinite triangular picture languages by online tessellation automaton has been defined. We will study the recognizability of infinite triangular pictures by P systems and pasting systems in future.

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