CONTRIBUTIONS TO HIGH FREQUENCY ENGINEERING

Dr. John Daniel, D.Sc. degree researcher, Mumbai University, Mumbai, India.

Abstract: Two types of antennas namely large single dish and array antennas are used for radio telescope construction. In part-I, by combining these two techniques, improved methods are suggested for the existing antennas of radio telescopes. In part-II, the relative convergence criteria are proved by many different methods. These proofs very clearly show that relative convergence does not occur due physical problems like edge diffraction and such convergence are purely due to mathematical approximation problems. Proofs are given for both waveguide wall discontinuity and the dielectric discontinuity. Since so far no proof was published in the literature, new relative convergence formulae are reported in the literature, even though actually they are not new problems. Such reported problems are proved to be same as that of original relative convergence. Reason for faster convergence of convergence of complex power (C.C.P.T.) method as compared with mode matching method is also explained. Power matching method using Perseval's theorem is shown to be more efficient than mode matching and C.C.P.T.

Key Words: **Radio Telescopes, Single dish antennas, Array antennas and improved design, Mode matching methods, Conservation of Complex Power Techniques, Relative Convergence**

I. Introduction

Radio telescopes demand antennas with very high directive gains. This requirement is achieved by using two classes of antennas. One is building very large single dish antenna and the other is by building antenna arrays. There were two types of arrays developed for radio telescope building. One is linear array and the other is interferometer array. Historically, some countries focused on developing low frequency telescopes and others focused on the development of high frequency antennas [1 2]. However, radio astronomers all over the world are focusing on building very large arrays and networking of radio telescopes like SKA (Square Kilometer Array) at low and high frequencies [1 2]. J. D. Kraus of Ohio state university developed aperture array type antennas and also parabolic reflector type antennas for radio telescope building purposes [3]. Many astronomers focused on developing interferometer array antennas. In part-I, by combining all the techniques developed in the past, several new types of radio astronomy antenna design techniques are enumerated.

Relative convergence problems were first reported by Raj Mittra in 1963 [9] and then by Lee, et al in Iris discontinuity problems in 1970 [10]. Relative convergence problems were reported by Mittra, et al [11] again in moment methods too. Relative convergence problems were proved to originate from edge condition problems. Leroy [12] has avoided RC by conditioning the linear system of equations and shown that RC does not arises from edge scattering problems. Later on in 1981 Safavi Naini, et al [13] reported relative convergence in CCPT. Then Shih, et al [14] analyzed relative convergence in step discontinuity waveguide scattering problems. Then, J El Hadad, et al [15] and J.A. Fleming [16] analyzed and reported complex relative convergence in Stripline discontinuities and computer magnetic storage device respectively. Antoni Barlabé, et al [17] analyzed the RC problems in a Finline structure. But so far, in the in the international literature, relative convergence formula was not derived from mode matching or power matching equations, even though John Daniel derived the formula and published in a local Pune, India conference in 2007 [18] .

Therefore, in part-II, the law of relative convergence is proved by many different methods. These proofs very clearly prove that relative convergence does not occur due physical problems like edge diffraction and such convergence are purely due to mathematical approximation problems. Proofs are given for both waveguide wall discontinuity and the dielectric discontinuity. Since so far no proof was published in the literature, new relative convergence formulae are reported in the literature [15 16 17], even though actually they are not new problems. Such reported problems are proved to be same as that of original relative convergence. Reason for faster convergence of convergence of complex power (C.C.P.T.) method [13] as compared with mode matching method is also explained.

II. PART-I: IMPROVED DESIGN AND DEVELOPMENT TECHNIQUES FOR RADIO TELESCOPE ANTENNAS

2.1. Improved horn aperture antenna-I [3 4]

Arno Penzias and Robert Wilson discovered cosmic microwave background radiations using a horn aperture antenna. Instead of using a single large aperture, if array of smaller apertures are used over the same single aperture area, the directive gain of the aperture could be improved. This improvement is made possible because of antenna array radiation pattern multiplication principle. As per this principle, array radiation pattern is equal to element pattern multiplied by array pattern obtained by replacing actual array elements by isotropic radiators of uniform radiation strength. Since single aperture pattern is multiplied by another large aperture pattern, the resulting pattern is a very narrow beam pattern. In this way, aperture gain of the horn antenna is improved for almost the same collecting area. By using interferometer array technique, much larger gain could be achieved with very few smaller aperture horn antennas since interferometer antenna gain depends on the distance between array elements and not on aperture area.

2.2. Improved horn aperture antenna-II [5]

A new type of high gain horn antenna could be built by using the technique of solar energy receiver. To collect large amount of solar energy, many smaller plane reflectors are placed on the ground in all directions from a solar energy collector tower. The plane reflectors receive the solar energy from the sun and reflect the received energy towards the collector plane placed on the top of the tower. In this way, all the plane reflectors reflect the solar power towards a small collector plate and the plate is heated to a very high temperature. Using this technique, the directive gain of a horn aperture antenna could be improved.

The solar energy collector tank placed on the tower could be replaced by four wide angle horn antennas placed in four directions of the tank and focused on the ground plane. The four horn antennas will receive signals from the smaller plane reflectors placed on the ground. All the ground plane reflectors are focused on the desired source of the sky. In this way, the power gain of the horn antenna could be improved. Using this technique, a very high gain interferometer could be constructed.

2.3. Improved aperture array antenna [3 4]

J.D. Kraus built a two dimensional linear array of helical antennas on a rotatable square plane made by steel. However, beyond certain dimensions gain can't be improved due to increased cable losses. But now, by recording the signals received by the elements at the same time without using any cable and then by computer based digital signal processing techniques, Kraus type two dimensional aperture array of any dimension could be made. Light weight fiber glass material could be used to make the plane of any size. In this way a rotatable very large aperture array could be built. Number of elements required could be reduced if interferometer technique is used. Smaller dish antennas could be used instead of helical antennas. Patch antenna elements also could be used in the place of helical antenna. In this way, low cost, light weight and high gain radio astronomy antennas could be built. If the distance between two consecutive elements $d = \lambda/3$, the gain of the array is maximized. Using this principle also, gain of the aperture array could be improved.

2.4. Improved Reflector Antenna [3]

J. D. Kraus built a parabolic reflector type antenna for building a wideband radio telescope. This antenna consists of three basic elements. One is a rotatable plane reflector which receives signal from the sky and reflect the received signal towards the parabolic reflector which is located at the opposite direction of the rotatable plane reflector at a distance. The fixed parabolic reflector focuses the received plane wave from the flat reflector. The focal point is on the ground and a movable receiver receives the signal at the focal point. By rotating the plane reflector, signals from the sky in different directions could be received at different focal points on the ground. Even though this antenna is a high gain and wideband antenna, it needs a vertical parabolic reflector focused on the ground in all directions of the ground plane to receive signals from all horizontal directions and the plane reflector has to be rotated in both angular directions of the spherical coordinate system. Therefore, construction of such antennas is highly expensive. Therefore, many improvement techniques for this type of antenna are suggested in the following lines.

Major drawback of the antenna is that it can't receive signal in the vertical direction from the sky since the plane reflector can't reflect the signal received from the sky in the vertical direction towards the parabolic reflector. This problem could be solved by constructing a temporary aperture array on the plane reflector to receive signals in the vertical direction. Instead of constructing a vertical parabolic reflector in all directions of the ground plane from the center of the plane reflector, the vertical parabolic reflector built to receive the signal in one ground plane direction could be rotated in the horizontal direction along with the movable receiver and the plane reflector, if all the three elements of the system are connected by steel rods in the horizontal direction. In this way the cost of the system could be reduced. Light weight fiber glass materials could be used to construct the reflectors.

A two element interferometer could be built by directly receiving the signal reflected by half of the plane reflector (First receiver) and from the parabolic reflector which focuses the signal on the ground plane (second receiver). The signal to the second receiver comes from the other half of the plane reflector. This interferometer system could be simplified by constructing two cylindrical parabolic reflectors and placed in the vertical plane one over the other. One cylinder could receive signals directly from the sky and the other cylinder could receive signal from the plane reflector. In this way, vertical size of the plane reflector and the need for a movable receiver are reduced. The need for a rotatable plane reflector also could be eliminated by using a movable plane reflector on the ground plane and rotating both the vertical cylindrical parabolic reflectors. One cylinder will receive signals directly from the sky and the other from the ground plane reflector.

2.5. Improved T shaped Antenna [1 2 4 7 8]

T shaped antenna was used for developing very low frequency radio telescopes in Ukraine. This type of antenna is basically a low frequency fat dipole antenna built using long wires with tapered edges. Such type of antenna has a bandwidth of 1:3. Therefore, Ukraine radio telescope operates in the frequency range of 10MHz to 30MHz bandwidth. Folded dipole antennas have a bandwidth equal to two times the bandwidth of half wave dipole antenna. Therefore, if the half wave dipole antennas in the T shaped wire mesh fat antennas are replaced by folded wire antennas with bi-cone feed, bandwidth of the T shaped antenna could be improved to 1:8 and low frequency radio telescopes could be built to operate in the frequency range of 10MHz to 80MHz. Similar performance could be obtained by folding a half wave cylindrical wire mesh and feeding the folded cylindrical antenna with bi conical feed.

2.6. Improved V shaped closed loop Antenna [1 2 4 6 7 8]

Closed V shaped antennas have a bandwidth of 1:8 and such type of antenna is used in modern low frequency radio telescope construction. A closed V shaped antenna is a magnetic dipole log periodic antenna. Therefore, as per the Rumsey's principle, such antennas have a wide bandwidth. Low frequency antennas used in modern low frequency radio telescope has a bandwidth of 1:8. Since fat antennas have a bandwidth of 1:3, if a fat closed V shaped wire antenna is constructed using V shaped wire antennas, bandwidth of 1:30 could be achieved. This means construction of a low frequency antenna with bandwidth in the range of 10MHz to 300MHz is possible. Similar performance could be obtained by folding a cylindrical long wire mesh into a fat V shaped closed loop antenna and feeding through bi conical feed. In the place of V shaped closed loop antennas, V shaped open ended, electric dipole log periodic antennas or Yagi - Uda antennas or helical dipole antennas or conical spiral dipole antennas could be used to build very wide band antennas.

III. PART-II: PROOF OF THE RELATIVE CONVERGENCE CRITERIA

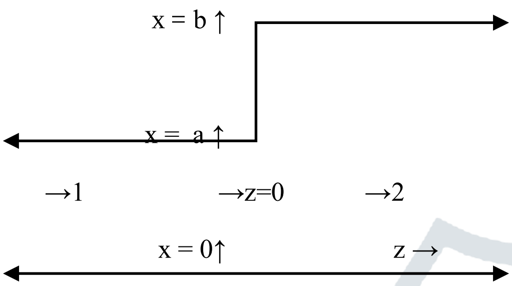

Figure - 1

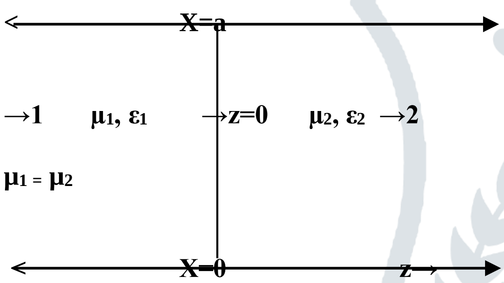

Figure - 2

Let us consider a parallel plate waveguide with a step discontinuity as soon in the figure-1. Dimensions of the waveguide are also specified in the diagram. Let us assume that a plane wave is propagating in the positive x direction.
The incident wave, reflected wave and transmitted wave at the discontinuity are matched to satisfy the boundary conditions at z = 0 and the equations for E and H fields are given by the following equations [19].

$\sum A_n \operatorname{Sin}(\pi nx/a) = \sum B_n \operatorname{Sin}(\pi nx/b)$, for n is a positive integer whose value varies from $-\infty$ to ∞ (1)

$\sum A_n.(\beta_n).\operatorname{Sin}(\pi nx/a) = \sum B_m.\beta_m.\operatorname{Sin}(\pi mx/b)$ (2)

Where β_n and β_m are propagation factors in the region one and two respectively.

3.1. METHOD-I

Let us assume that the modes are truncated to N and M in the region 1 and 2 respectively for determination of amplitude of the Fourier series in the equations (1) and (2). Therefore, in the region one number of modes (N) and the field E_a are expanded to M and E_b respectively. Therefore, N α a and M α b since waveguides in both sides are of the same type. Therefore, N/M = a/b which is the condition for relative convergence to converge to correct value. If this condition is not satisfied, error occurs, even if N and M are very large, while computing by using matrix algebra which can very well amplify the error while finding inverse of matrices.

If the discontinuity is dielectric discontinuity instead of waveguide wall discontinuity as shown in the figure-2, then the amplitudes of the modes depends on dielectric constants of the medium the speed of electromagnetic wave is $1/\sqrt{\mu\varepsilon}$ where μ and ε are permeability and the permittivity of the medium. As per the equations (1) and (2) $E/H = \omega\mu/\beta_n$ for each mode in the side one. Therefore, for N and M th modes this ratio E/H in each side must be equal. Therefore, $\beta_N = \beta_M$ where $\beta_N = \sqrt{(\omega^2\mu\varepsilon_1 - (N.\pi/a)^2)}$ and $\beta_M = \sqrt{(\omega^2\mu\varepsilon_2 - (M.\pi/a)^2)}$. Therefore, $N/M = \sqrt{\varepsilon_1/\varepsilon_2}$ where ε_1 and ε_2 are permittivity in the region one and two respectively.

3.2. METHOD-II

Method – I is based on matching modes. Method – II is based on matching power. As per Poynting theorem, power per unit area in a parallel plate waveguide is E^2/η where η is intrinsic impedance of the medium. This power on both sides of the waveguide at the metal discontinuity = $E_1^2/\eta_1 = E_2^2/\eta_2$. $E_1^2 = E_2^2$ since $\eta_1 = \eta_2$. Energy in the waveguide is α a in the region 1. Therefore, $(E_1/E_2)^2$ = a/b and N/M since energy is directly proportional to number of modes in the waveguide. For dielectric waveguide discontinuity, $(E_1/E_2)^2 = \eta_1/\eta_2 = \sqrt{\varepsilon_1/\varepsilon_2}$ and N/M since energy is directly proportional to number of modes in the waveguide. In the power matching methods, number of modes in the waveguide is equal to twice the number of modes in mode matching methods. Therefore, power matching methods like CCPT converge two times faster than the mode matching methods.

3.3. METHOD-III

In the equation (1) and (2), $A_N.\text{Sin}(N.\pi x/a) = B_M.\text{Sin}(M.\pi x/b)$ and $A_N.(\beta_N).\text{Sin}(\pi Nx/a) = B_M.\ \beta_M.\text{Sin}(\pi Mx/b)$. Therefore, N/M = a/b from the first equation and $\beta_N = \beta_M$ for a = b from the second equation since $A_N = B_M$. Therefore, $\sqrt{\varepsilon_1/\varepsilon_2}$ = N/M.

3.4. METHOD-IV

The equations (1) and (2) are just equating two Fourier series of two periodic functions of x. Periods of the functions of x are 2a and 2b with Fourier coefficients $A_N = \int F_a(x).dx$, where $F_a(x) = f_a(x)\ \text{Sin}(N\pi x/a)$ and integration is over x= 0 to x= a and $B_M = \int F_b(x).dx$, where $F_b(x) = f_b(x).\text{Sin}(M\pi x/b)$ and integration is over x=0 to x=b. The periods of $F_a(x)$ and $F_b(x)$ are also 2a and 2b respectively. Since the field in the second waveguide is radiated by the first waveguide, $\partial F_a(x)/\partial x = \partial F_b/\partial x$. From this equation, we get, N/M = a/b. Power in the left hand side of the discontinuity is equal to that of right hand side. So, N/M = $\sqrt{\varepsilon_1}/\sqrt{\varepsilon_2}$ for dielectric discontinuity. In this method also, we can see that the number of modes required in mode matching methods for convergence is two times that of power matching methods.

3.5. METHOD-V

From the equation (1) $A_N = B_M$ and from (2), $\beta_N A_N = \beta_M B_M$ where $\beta_N^2 = (\omega/c)^2 - (N\pi/a)^2$ and $\beta_M^2 = (\omega/c)^2 - (M\pi/a)^2$. Therefore, N/M = a/b for conducting wall discontinuity and N/M = $\sqrt{\varepsilon_1}/\sqrt{\varepsilon_2}$ for dielectric discontinuity with a=b.

3.6. METHOD-VI

As per the Poynting theorem, complex power on two sides of the discontinuity must be equal. From this principle, we get $A_N = B_M$ and a. $\beta_N^2\ A_N$ = b. $\beta_M^2\ B_M$. Therefore, N/M = a/b for conducting walls and N/M = $\sqrt{\varepsilon_1}/\sqrt{\varepsilon_2}$ for dielectric discontinuity with a=b. From this method also we can see that the number of modes required in power matching methods is only one half of that required in mode matching methods.

3.7. METHOD-VII

The equations (1) and (2) are just equating two Fourier series of two periodic functions of x. Periods of the functions of x are 2a and 2b with Fourier coefficients $A_N = \int F_a(x).dx$, where $F_a(x) = f_a(x)\ \text{Sin}(Nx\pi/a)$ and integration is over x= 0 to x= a and $B_M = \int F_b(x).dx$, where $F_b(x) = f_b(x).\text{Sin}(M\pi x/b)$ and integration is over x=0 to x=b. If n = N and m = M, $\pi N/a = \pi M/b \approx \pi l$ where l is an integer, since $A_N = B_M \approx 0$. Similarly, if a = b and if the dielectric medium of the waveguides are different, then $A_N^2 = B_M^2 = \eta_1.(\int F_a(x).dx)^2 = \eta_2.(\int F_b(x).dx)^2 \approx 0$. Therefore, $\sqrt{\eta_1}\ \int f_a(x)\ \text{Sin}(Nx\pi/a).dx = \sqrt{\eta_2}\ \int f_b(x).\text{Sin}(M\pi x/a)dx$ and therefore, frequencies of $F_a(x)$ and $F_b(x)$ are equal. Therefore, $\sqrt{\eta_1}.2.\ (N\pi/a) = \sqrt{\eta_2}\ .2.(M\pi/a)$ and N/M = $\sqrt{\varepsilon_1}/\sqrt{\varepsilon_2}$. Therefore, number of modes required for convergence in mode matching methods (2N, 2M) is two times the number of modes required in power matching methods for convergence (N, M).

3.8. METHOD-VIII

This method is to prove that the power matching methods converges two times faster than that of mode matching methods. The equations (1) and (2) can be expressed in complex Fourier series form. The Fourier coefficients are directly proportional to Fourier transformation of field in the waveguide in the x dimension. The fundamental frequency in the x dimension of complex Fourier spectrum in mode matching method is two times that of power matching methods. Therefore, number of modes required in mode matching methods for convergence is two times the number of modes required in power matching methods.

3.9. NEW RELATIVE CONVERGENCE PROBLEMS

J.L. Fleming [17] analyses the effect of truncation of a Fourier series on the solution of a Laplace's equation. Convergence of Fourier series is a very well known fact. Since there is only one Fourier series in the equation, relative convergence can't occur since mismatch problem does not arises. Therefore, Fleming's analysis is not required. Fleming analyzed the convergence problem due to the absence of proof in the literature to relative convergence problems.

CCPT is proved to be faster than the convergence rate of mode matching method. But in both methods, complex algebraic operations are required to be carried out by the computer. Since equations (1) and (2) are Fourier series equations, Parseval's theorem could be used to solve the waveguide discontinuity problems in which all operations are real. So, one half of the storage space is reduced and much less computing time is required for mode matching method and convergence rate is twice faster than that of mode matching method like CCPT.

J. El Hadad, et al [16] found that correct convergence occurs in the finline step discontinuity problem only if $R = M/Q \geq R_a = 1.5b/W_a$ where $W_a = (d_1 + d_2)/2$, d_1 and d_2 are aperture heights on the two sides of aperture discontinuity. As per the Hoffmann's publication [16], R = 1.5b/W where W is the height of the aperture for finline without any aperture discontinuity. As per the relative convergence proofs presented in this letter, convergence condition R = b/W for conductor discontinuity and $R = \sqrt{\varepsilon_1}/\sqrt{\varepsilon_2}$ for dielectric discontinuity. In this case, both discontinuities are present at the same location. Therefore, $R = (\sqrt{\varepsilon_1}/\sqrt{\varepsilon_2}).b/W = 1.5b/W$ since for air and dielectric interface in the finline $\sqrt{\varepsilon_1}/\sqrt{\varepsilon_2} = 1.5$. Therefore, for finline with aperture discontinuity, convergence condition is $R = 1.5b/(d_1.d_2) \geq R_a = 1.5b/W_a$ where $W_a = (d_1 + d_2)/2$.

IV. CONCLUSION

In part-I, Improved design and development techniques for horn aperture antenna, array antennas, reflector antennas, T-Shaped and V-shaped antennas are suggested. The author hopes to apply the suggested techniques in practice in future depending upon the availability of funds. In part-II, the relative convergence criteria are proved by many different methods. Proofs are given for both waveguide wall discontinuity and the dielectric discontinuity. New relative convergence problems reported in the literature are proved to be same as that of original relative convergence. Reason for faster convergence of convergence of complex power (C.C.P.T.) method as compared with mode matching method is also explained. Power matching method using Perseval's theorem is shown to be more efficient than mode matching methods and C.C.P.T.

V. ACKNOWLEDGEMENTS

The author is very much grateful to Prof. R. H. Macphie, University of Waterloo, Canada for introducing C.C.P.T. and relative convergence problems and University of Waterloo, Canada for a research assistantship during 1990.

REFERENCES

[1]. P.Zarka, M.Tagger, L.Denis and Nenufar-France collaboration. 2016. Nenufar, A giant low frequency radio telescope in nancy,,https://nenufar.obs-nancay.fr/img/pdf/ nenufar_sondra -20160527.pdf.

[2]. Anastasiya boiko, "ground based radio telescopes and their applications for space weather", kuleuven,leuven,belgium,https://perswww.kuleuven.be/~u0052182/weather/radiotelescopes.pdf.

[3]. J. D. Kraus.2004. Radio astronomy. Cygnus-Quasar books, 2nd edition, pp.6.1 – 6.118.

[4]. E.C. Jordan and K.G. Balmain.2007. Electromagnetic waves and radiating systems" Prentice Hall of India pvt. Ltd., New Delhi, pp.359-374, 380 – 396, 401-406, 589-599, 601-626, 488- 505.

[5]. Hans muller steinhagen.2004. Concentrating solar power- a review of the technology. Quarterly of the royal academy of engineering, Ingenia.

[6]. C.A. Balanis.1997.Antenna theory - analysis and design, John wiley and sons, Inc., pp. 441-570.

[7]. K.C.Gupta.1989. Microwaves. Wiley eastern ltd., New Delhi, pp.8-17.

[8]. Joseph A. Edminister.1986. Electromagnetics. Schaum's outline series, Mcgraw Hill book company, Singapore, pp.81-95, 139-158.

[9]. R.Mittra.1963. Relative convergence of the solution of a doubly infinite set of equations. Journal of Research of the National Bureau of Standards - D, Radio propagation, pp. 245-254.

[10]. Lee, S.W., W.R. Jones and J.J. Campbell.1970. Convergence of Numerical Solution of Iris-discontinuity Problems, Hughes Aircraft Co., Fullerton, CA, FR 70-14-594.

[11]. R.Mittra, T.Itoh and T.S.Li. 1972. Analytical and numerical studies of the relative convergence phenomenon arising in the solution of an integral equation by the moment method. IEEE trans. Microwave Theory Tech., Vol.MTT-20, pp.96-104.

[12]. Leroy, M. 1983. On the Convergence of Numerical Results in Modal Analysis. IEEE Trans. Antennas Propagation, 31, pp.655-659.

[13]. R Safavi Naini and R H, MacPhie. 1981. On solving waveguide Junction scattering problems by the conservation of complex power technique. IEEE Trans. Microwave Theory and Tech., vol. MT3-29, pp 337-343.

[14]. Shih Y.C. and K.G. Gray. 1983. Convergence of Numerical Solutions of Step-type Waveguide Discontinuity Problems by Modal Analysis. IEEE MTT-S Microwave Symp. Digest, Boston, MA, pp.233-235.

[15]. Antoni Barlabé, Adolfo Comerón, and Lluís Pradell. 2000. Generalized transverse resonance analysis of planar discontinuities considering the edge effect. IEEE transactions on guided wave letters, vol. 20, no. 12, pp.517-519.

[16]. J. El Hadad, A. Barlabé, A. Comerón. 2008. Minimization nof relative convergence phenomenon in the analysis of finline step discontinuities using the weighting functions that incorporate the edge effect in the GTR method. International journal of microwave and optical technology, vol. 3, no. 2,pp.106-109.

[17]. John L. Fleming. 2008. Convergence Analysis of a Fourier-Based Solution Method of the Laplace Equation for a Model of Magnetic Recording. Hindawi Publishing Corporation, Mathematical Problems in Engineering, Volume 2008, Article ID 154352, 11 pages.

[18]. John Daniel .2007. Proof of Relative Convergence. Proceedings of National Conference, SVCPC, Pune.

[19]. E.C.Jordan and K.G.Balmain. 1983. Electromagnetic waves and Radiating Systems. Prentice Hall of India Pvt. Ltd., New Delhi - 110001, pp.177-188.