A Note on Fuzzy Semi Volterra Spaces

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Abstract. In this paper, the relationship between fuzzy semi Volterra spaces and other fuzzy topological spaces such as fuzzy semi Baire spaces, fuzzy semi σ-Baire spaces, fuzzy semi strongly irresolvable spaces, fuzzy semi P-spaces, fuzzy semi D-Baire spaces, fuzzy semi hyperconnected spaces and fuzzy semi submaximal spaces are established. Also, the conditions under which a fuzzy semi Baire space becomes a fuzzy semi Volterra space are studied in this paper.

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1 Introduction

Till 1965, Mathematicians were concerned only about well-defined things and smartly avoided any other possibilities which are more realistic in nature. In 1965, L.A.Zadeh [13] introduced the concept of fuzzy set to accommodate real life situations by giving partial membership to each element of a situation under consideration. Fuzzy sets allow everyone us to represent vague concepts expressed in natural language. The representation depends not only on the concept but also on the context in which it is used. This inspired mathematicians to fuzzify mathematical structures. General topology is one of the important branches of mathematics in which fuzzy set theory has been applied systematically. Based on the concept of fuzzy sets invented by Zadeh, C.L.Chang [3] introduced the concept of fuzzy topological spaces in 1968 as a generalization of topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. In this paper, the relationship between fuzzy semi Volterra spaces and other fuzzy topological spaces such as fuzzy semi Baire spaces, fuzzy semi σ-Baire spaces, fuzzy semi strongly irresolvable spaces, fuzzy semi P-spaces, fuzzy semi D-Baire spaces, fuzzy semi hyperconnected spaces and fuzzy semi submaximal spaces are established. Also, the conditions under which a fuzzy semi Baire space becomes a fuzzy semi Volterra space are studied in this paper.

2 Preliminaries

In 1965, L.A.Zadeh [13] introduced the concept of fuzzy set λ on a base set X as a function from X into the unit interval $I = [0,1]$. This function is also called a membership function. A membership function is a generalization of a characteristic function.

Definition 2.1 *[3] Let λ and μ be fuzzy sets in X. Then for all $x \in X$,*

- $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
- $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
- $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = max\{\lambda(x), \mu(x)\}$,
- $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = min\{\lambda(x), \mu(x)\}$,
- $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in X, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by

$\psi(x) = sup_i\{\lambda_i(x), x \in X\}$, and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.2 *[3] A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:*

- $\Phi, X \in T$,
- If $A, B \in T$, then $A \cap B \in T$,
- If $A_i \in T$, for each $i \in I$, then $\cup_{i\in I} A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T) is a fuzzy topological space or fts in short. Every member of T is called a T-open fuzzy set. A fuzzy set is T-closed if and only if its complement is T-open. When no confusion is likely to arise, we shall call a T-open (T-closed) fuzzy set simply an open (closed) fuzzy set.

Lemma 2.1 *[1] Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define the fuzzy semi-closure and the fuzzy semi-interior of λ as follows:*

- $scl(\lambda) = \wedge \{\mu/\lambda \leq \mu, \mu$ is fuzzy semi-closed set of $X\}$
- $sint(\lambda) = \vee \{\mu/\mu \leq \lambda, \mu$ is fuzzy semi-open set of $X\}$.

Lemma 2.2 *[1] For a fuzzy set λ of a fuzzy space X,*

- $1 - scl(\lambda) = sint(1 - \lambda)$ and
- $1 - sint(\lambda) = scl(1 - \lambda)$.

Lemma 2.3 *[1] For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X. Then, $\vee \; cl \; \lambda_\alpha \leq cl(\vee \lambda_\alpha)$. In case $\mathcal{A}$ is a finite set, $\vee \; cl \; \lambda_\alpha = cl(\vee \lambda_\alpha)$. Also $\vee \, int \; \lambda_\alpha \leq int(\vee \lambda_\alpha)$.*

Definition 2.3 *[2] A fuzzy set λ in a fuzzy topological space X is called fuzzy semi-open if $\lambda \leq clint(\lambda)$ and fuzzy semi-closed if $intcl(\lambda) \leq \lambda$.*

Definition 2.4 *[11] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi G_δ-set in (X, T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi open sets in (X, T).*

Definition 2.5 *[11] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi closed sets in (X, T).*

Definition 2.6 *[10] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy semi dense if there exists no fuzzy semi closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $scl(\lambda) = 1$.*

Definition 2.7 *[9] Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy semi nowhere dense set if there exists no non-zero fuzzy semi-open set μ in (X, T) such that $\mu < scl(\lambda)$. That is, $sintscl(\lambda) = 0$.*

Definition 2.8 *[9] Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called fuzzy semi first category if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy semi second category.*

Definition 2.9 *[9] If λ is a fuzzy semi first category set in a fuzzy topological space (X, T), then $1 - \lambda$ is called a fuzzy semi residual set in (X, T).*

Theorem 2.1 *[9] If λ is a fuzzy semi nowhere dense set in a fuzzy topological space (X, T), then $1-\lambda$ is a fuzzy semi dense set in (X, T).*

Theorem 2.2 *[12] If λ is a fuzzy semi dense and fuzzy semi G_δ-set in a fuzzy topological space (X, T), then $1-\lambda$ is a fuzzy semi first category set in (X, T).*

Theorem 2.3 *[12] In a fuzzy topological space (X, T), a fuzzy set λ is a fuzzy semi σ-nowhere dense set in (X, T) if and only if $1-\lambda$ is a fuzzy semi dense and fuzzy semi G_δ-set in (X, T).*

3 Fuzzy semi Volterra spaces and fuzzy semi Baire spaces

Proposition 3.1 *Let (X, T) be a fuzzy topological space. If $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$, is a fuzzy nowhere dense set in (X, T), then (λ_i)'s are fuzzy nowhere dense sets in (X, T).*

Proof. Let $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(1-\lambda_i)$'s are fuzzy open sets in (X, T). Then $cl(\lambda) = cl(\vee_{i=1}^{\infty}(\lambda_i)) \geq \vee_{i=1}^{\infty} cl(\lambda_i)$. Since (λ_i)'s are fuzzy closed sets in (X, T), $cl(\lambda_i) = \lambda_i \ldots\ldots(1)$ and hence $cl(\lambda) \geq \vee_{i=1}^{\infty}(\lambda_i)$. Then, $intcl(\lambda) \geq int(\vee_{i=1}^{\infty}(\lambda_i)) \geq \vee_{i=1}^{\infty}(int(\lambda_i))$. Since λ is a fuzzy nowhere dense set in (X, T), $intcl(\lambda) = 0$ and hence $0 \geq \vee_{i=1}^{\infty}(int(\lambda_i))$. That is, $\vee_{i=1}^{\infty}(int(\lambda_i)) = 0$. This implies that $int(\lambda_i) = 0$ and hence from (1), $intcl(\lambda_i) = int(\lambda_i) = 0$, implies that (λ_i)'s are fuzzy nowhere dense sets in (X, T).

In view of the above proposition 3.1, one will have the following proposition.

Proposition 3.2 *If a fuzzy nowhere dense set λ in a fuzzy topological space (X, T) is a fuzzy F_σ-set, then λ is a fuzzy first category set in (X, T).*

Proof. Let λ be a fuzzy nowhere dense set such that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$. Since λ is a fuzzy nowhere dense set in (X, T), by proposition 3.1, (λ_i)'s are fuzzy nowhere dense sets in (X, T). Hence $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T) implies that λ is a fuzzy first category set in (X, T).

Proposition 3.3 *Let (X, T) be a fuzzy topological space. If each fuzzy nowhere dense set λ in (X, T) is a fuzzy F_σ-set, then (X, T) is a fuzzy semi Baire space.*

Proof. Let λ be a fuzzy nowhere dense set such that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$. Then, by proposition 3.2, λ is a fuzzy first category set in (X, T). Now $int(\lambda) \leq intcl(\lambda)$ and $intcl(\lambda) = 0$, implies that $int(\lambda)$=0, where λ is a fuzzy first category set in (X, T). Then, by theorem ??, (X, T) is a fuzzy semi Baire space.

Proposition 3.4 *Let (X, T) be a fuzzy topological space. If each fuzzy nowhere dense set in (X, T) is a fuzzy F_σ-set in (X, T), then (X, T) is a fuzzy semi Volterra space.*

Proof. Let each fuzzy nowhere dense set in (X, T) be a fuzzy F_σ-set in (X, T) Then, by proposition 3.3, (X, T) is a fuzzy semi Baire space. Then $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Now $1 - int(\vee_{i=1}^{\infty}(\lambda_i)) = 1$. This implies that $cl(\wedge_{i=1}^{\infty}(1-\lambda_i)) = 1$. Since λ_i is a fuzzy nowhere dense set in (X, T), by theorem ??, $1-\lambda_i$ is a fuzzy dense set in (X, T). But $cl(\wedge_{i=1}^{\infty}(1-\lambda_i)) \leq cl(\wedge_{i=1}^{N}(1-\lambda_i))$ and hence $1 \leq cl(\wedge_{i=1}^{N}(1-\lambda_i))$. That is, $cl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$. Since λ_i is a fuzzy F_σ-set in (X, T), $1-\lambda_i$ is a fuzzy G_δ-set in (X, T). Hence $cl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$, where $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X, T). Therefore (X, T) is a fuzzy semi Volterra space.

Remark. *Consider the example ??, define the fuzzy sets Ω and Ψ on X as follows:*

$\Omega: X \to [0,1]$ is defined as $\Omega(a) = 0.4$; $\Omega(b) = 0.5$; $\Omega(c) = 0.3$.
$\Psi: X \to [0,1]$ is defined as $\Psi(a) = 0.3$; $\Psi(b) = 0.4$; $\Psi(c) = 0.2$.

Now $\Omega = [1-(\lambda \wedge \mu)] \vee [1-(\lambda\nu)] \vee [1-(\mu \wedge \nu)]$ and $\Psi = \{1-[\mu \vee (\lambda \wedge \nu)]\} \vee \{1-[\nu \vee (\lambda \wedge \nu)]\} \vee \{1-(\mu \vee \nu)\}$ are fuzzy F_σ-sets in (X,T) and hence Ω and Ψ are fuzzy nowhere sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Remark. *If the fuzzy nowhere dense sets in (X,T) are* **not** *fuzzy F_σ-sets in (X,T), then (X,T)* **need not** *be a fuzzy semi Volterra space eventhough (X,T) is a fuzzy semi Baire space. For, consider the example ??.*

The fuzzy sets $1-\nu$, $1-(\lambda \vee \nu)$, $1-(\mu \vee \nu)$ are fuzzy nowhere dense sets in (X,T) and $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T) and hence (X,T) is a fuzzy semi Baire space and $1-(\lambda \vee \nu)$ is not a fuzzy F_σ-set in (X,T). Now $\alpha = \{\lambda \wedge (\lambda \vee \mu) \wedge (\lambda \wedge \nu)\}$, $\beta = \{\mu \wedge (\mu \vee \nu) \wedge (\lambda \wedge \mu) \wedge [\mu \vee (\lambda \wedge \nu)] \wedge [\lambda \wedge (\mu \vee \nu)]\}$ and $\delta = \{\nu \wedge (\lambda \wedge \nu) \wedge (\mu \wedge \nu) \wedge [\nu \wedge (\lambda \vee \mu)]\}$ are fuzzy G_δ-sets in (X,T), but not fuzzy dense sets in (X,T) and there is no fuzzy dense and fuzzy G_δ-set in (X,T) such that $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$ and hence (X,T) is **not** a fuzzy semi Volterra space.

Remark. *A fuzzy semi Volterra space* **need not** *be a fuzzy semi Baire space. For, consider the following example:*

Example 3.1 *Let $X = \{a,b,c\}$. The fuzzy sets λ, μ and ν are defined on X as follows :*

$\lambda: X \to [0,1]$ is defined as $\lambda(a) = 0.8$; $\lambda(b) = 0.5$; $\lambda(c) = 0.7$.
$\mu: X \to [0,1]$ is defined as $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.4$.
$\nu: X \to [0,1]$ is defined as $\nu(a) = 0.4$; $\nu(b) = 0.7$; $\nu(c) = 0.8$.

Clearly $T = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \vee (\mu \wedge \nu), \lambda \wedge (\mu \vee \nu), \mu \vee (\lambda \wedge \nu), \mu \wedge (\lambda \vee \nu), \nu \vee (\lambda \wedge \mu), \nu \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \nu, 1\}$ is a fuzzy topology on X. Now $\alpha = \{\lambda \wedge (\lambda \vee \mu) \wedge (\lambda \wedge \nu) \wedge [\lambda \vee (\mu \wedge \nu)] \wedge [\mu \vee (\lambda \wedge \nu)] \wedge [\lambda \wedge (\mu \vee \nu)] \wedge [\nu \wedge (\lambda \vee \mu)]\}$ and $\beta = \{\nu \wedge (\lambda \vee \nu) \wedge (\mu \vee \nu) \wedge [\nu \vee (\lambda \wedge \mu)] \wedge (\lambda \vee \mu \vee \nu)\}$ are fuzzy dense and fuzzy G_δ-sets in (X,T) and $cl(\alpha \wedge \beta) = 1$ and hence (X,T) is a fuzzy Volterra space. The fuzzy nowhere dense sets in (X,T) are $1-\lambda$, $1-\mu$, $1-\nu$, $1-(\lambda \vee \mu)$, $1-(\lambda \vee \nu)$, $1-(\mu \vee \nu)$, $1-(\lambda \wedge \mu)$, $1-(\lambda \wedge \nu)$, $1-(\mu \wedge \nu)$, $1-[\lambda \vee (\mu \wedge \nu)]$, $1-[\lambda \wedge (\mu \vee \nu)]$, $1-[\mu \vee (\lambda \wedge \nu)]$, $1-[\mu \wedge (\lambda \vee \nu)]$, $1-[\nu \vee (\lambda \wedge \mu)]$, $1-[\nu \wedge (\lambda \vee \mu)]$, $1-[\lambda \vee \mu \vee \nu]$ and $int(\vee_{i=1}^{\infty}(\lambda_i)) = \lambda \wedge \mu \neq 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Therefore (X,T) is **not** a fuzzy semi Baire space.

The following proposition gives a condition for a fuzzy semi Volterra space to be a fuzzy semi Baire space.

Proposition 3.5 *If each fuzzy first category set λ is formed from the fuzzy dense and fuzzy G_δ-sets (λ_i)'s $(i = 1$ to $N)$ in a fuzzy Volterra space (X,T), then (X,T) is a fuzzy Baire space.*

Proof. Let (λ_i)'s $(i = 1$ to $N)$ be fuzzy dense and fuzzy G_δ-sets in a fuzzy Volterra space (X,T). Since (X,T) is a fuzzy Volterra space, by proposition ??, $int(\vee_{i=1}^{N}(1-\lambda_i)) = 0$. Now, by lemma 2.3, $\vee_{i=1}^{N}[int(1-\lambda_i)] \leq int(\vee_{i=1}^{N}(1-\lambda_i))$. This implies that $\vee_{i=1}^{N} int(1-\lambda_i) = 0$. Then $int(1-\lambda_i) = 0$. Since (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T), by theorem 2.2, $(1-\lambda_i)$'s are fuzzy first category sets in (X,T). By theorem ??, (X,T) is a fuzzy semi Baire space. Hence, if each fuzzy first category set in (X,T) is formed from the fuzzy dense and fuzzy G_δ-sets (λ_i)'s $(i = 1$to $N)$ in a fuzzy semi Volterra space, then (X,T) is a fuzzy semi Baire space.

Proposition 3.6 *If each fuzzy first category set is a fuzzy closed set in a fuzzy semi Baire space (X,T), then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Then, by theorem 2.2, $(1-\lambda_i)$'s are fuzzy first category sets in X. Now, by hypothesis, $(1-\lambda_i)$'s are fuzzy closed sets in (X,T) and hence (λ_i)'s are fuzzy open sets in (X,T). Therefore (λ_i)'s are fuzzy dense and fuzzy open sets in (X,T). Then, by theorem ??, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi Baire space, $int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$, where (μ_α)'s are fuzzy nowhere dense sets in (X,T). Let the first $N(\mu_\alpha)$'s be $(1-\lambda_i)$ in (X,T). But $int[\vee_{i=1}^{N}(1-\lambda_i)] \leq int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)]$ and $int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$, implies that $int[\vee_{i=1}^{N}(1-\lambda_i)] = 0$ and hence $int[1-\wedge_{i=1}^{N}(\lambda_i)] = 0$. Then $1 - cl[\wedge_{i=1}^{N}(\lambda_i)] = 0$. Hence $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 3.7 *If each fuzzy residual set is a fuzzy open set in a fuzzy semi Baire space (X,T), then (X,T) is a fuzzy semi Volterra space.*

Proof. Let each fuzzy residual set μ be a fuzzy open set in a fuzzy semi Baire space (X,T). Then $1-\mu$ is fuzzy first category set and fuzzy closed set in (X,T). Then, by proposition 3.6, (X,T) is a fuzzy semi Volterra space.

Proposition 3.8 *If a fuzzy topological space (X,T) is a fuzzy semi Baire and fuzzy P-space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ-sets (λ_i)'s are fuzzy open sets in (X,T). Then, (λ_i)'s are fuzzy dense and fuzzy open sets (X,T). By theorem ??, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi Baire space, $int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$, where (μ_α)'s are fuzzy nowhere dense sets in (X,T). Let the first $N(\mu_\alpha)$'s be $(1-\lambda_i)$ in (X,T). But $int[\vee_{i=1}^{N}(1-\lambda_i)] \leq int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)]$ and $int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$, implies that $int[\vee_{i=1}^{N}(1-\lambda_i)] = 0$ and hence $int[1-\wedge_{i=1}^{N}(\lambda_i)] = 0$. Then $1 - cl[\wedge_{i=1}^{N}(\lambda_i)] = 0$. Hence $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 3.9 *If a fuzzy topological space (X,T) is a fuzzy semi Baire and fuzzy semi submaximal space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy semi submaximal space, the fuzzy dense sets (λ_i)'s are fuzzy open sets in (X,T). Then, (λ_i)'s are fuzzy dense and fuzzy open sets (X,T). By theorem ??, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi Baire space, $int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$, where (μ_α)'s are fuzzy nowhere dense sets in (X,T). Let the first $N(\mu_\alpha)$'s $(1-\lambda_i)$ in (X,T). But $int[\vee_{i=1}^{N}(1-\lambda_i)] \leq int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)]$ and $int[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] = 0$, implies that $int[\vee_{i=1}^{N}(1-\lambda_i)] = 0$ and hence $int[1-\wedge_{i=1}^{N}(\lambda_i)] = 0$. Then $1 - cl[\wedge_{i=1}^{N}(\lambda_i)] = 0$. Hence $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

4 Fuzzy semi Volterra spaces and fuzzy semi strongly irresolvable spaces

Proposition 4.1 *If $\lambda = \vee_{i=1}^{N}(\lambda_i)$ where (λ_i)'s are fuzzy σ-nowhere dense sets in a fuzzy semi Volterra and fuzzy semi strongly irresolvable space (X,T), then λ is a fuzzy nowhere dense set in (X,T).*

Proof. Let $\lambda = \vee_{i=1}^{N}(\lambda_i)$, where (λ_i)'s are fuzzy σ-nowhere dense sets in a fuzzy semi Volterra and fuzzy semi strongly irresolvable space (X,T). Since (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T), by theorem, ??, $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy semi Volterra space,

$cl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$. Since (X,T) is a fuzzy semi strongly irresolvable space, $clint(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$. Then, $clint(1-\vee_{i=1}^{N}(\lambda_i)) = 1$. This implies that $1-intcl(\vee_{i=1}^{N}(\lambda_i)) = 1$ and hence $intcl(\vee_{i=1}^{N}(\lambda_i)) = 0$. Therefore $intcl(\lambda) = 0$. Hence λ is a fuzzy nowhere dense set in (X,T).

Proposition 4.2 *If a fuzzy topological space (X,T) is a fuzzy semi strongly irresolvable and fuzzy semi Baire space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in a fuzzy semi strongly irresolvable and fuzzy semi Baire space (X,T). Since (X,T) is a fuzzy semi strongly irresolvable space, $cl(\lambda_i) = 1$, implies that $clint(\lambda_i) = 1$. Then $1-clint(\lambda_i) = 0$. This implies that $intcl(1-\lambda_i) = 0$. Hence $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Suppose that (μ_i)'s $(i = 1 \quad to \quad \infty)$ are fuzzy nowhere dense sets in (X,T) in which the first $N(\mu_i)$'s be $(1-\lambda_i)$. Now $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{i=1}^{\infty}(\mu_i)$. Then, $int(\vee_{i=1}^{N}(1-\lambda_i)) \leq int(\vee_{i=1}^{\infty}(\mu_i))$. Since (X,T) is a fuzzy semi Baire space, $int(\vee_{i=1}^{\infty}(\mu_i)) = 0$. This implies that $int(\vee_{i=1}^{N}(1-\lambda_i)) = 0$. Now $int(1-\wedge_{i=1}^{N}(\lambda_i)) = 0$, implies that $1-cl(\wedge_{i=1}^{N}(\lambda_i)) = 0$. Then $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 4.3 *If a fuzzy topological space (X,T) is a totally fuzzy second category, fuzzy regular and fuzzy semi strongly irresolvable space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a totally fuzzy second category, fuzzy regular space, by theorem ??, (X,T) is a fuzzy semi Baire space. Then (X,T) is a fuzzy semi strongly irresolvable and fuzzy semi Baire space. Then, by proposition 4.2, (X,T) is a fuzzy semi Volterra space.

5 Fuzzy semi Volterra spaces and other fuzzy spaces

Proposition 5.1 *If a fuzzy topological space (X,T) is a fuzzy semi σ-Baire space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \infty)$ be fuzzy dense and fuzzy G_δ-sets in a fuzzy semi σ-Baire space (X,T). Consider the fuzzy set $cl(\wedge_{i=1}^{N}(\lambda_i))$. Now, $1-cl(\wedge_{i=1}^{N}(\lambda_i)) = int(1-\wedge_{i=1}^{N}(\lambda_i)) = int(\vee_{i=1}^{N}(1-\lambda_i))$. But $int(\vee_{i=1}^{N}(1-\lambda_i)) \leq int(\vee_{i=1}^{\infty}(1-\lambda_i))\ldots\ldots(1)$. Since the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T), by Theorem ??, $(1-\lambda_i)$'s are fuzzy σ-nowhere dense sets in (X,T). Also, since (X,T) is a fuzzy semi σ-Baire space, $int(\vee_{i=1}^{\infty}(1-\lambda_i)) = 0\ldots..(2)$. Hence, from (1) and (2), $int(\vee_{i=1}^{N}(1-\lambda_i)) = 0$. Then $int(1-\wedge_{i=1}^{N}(\lambda_i)) = 0$, implies that $1-cl(\wedge_{i=1}^{N}(\lambda_i)) = 0$. Hence $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 5.2 *If the fuzzy topological space (X,T) is a fuzzy semi D-Baire space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Then, by theorem 2.2, $(1-\lambda_i)$'s are fuzzy first category sets in (X,T). Since (X,T) is a fuzzy semi D-Baire space, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Suppose that (μ_i)'s $(i = 1 \quad to \quad \infty)$ are fuzzy nowhere dense sets in (X,T) in which the first $N(\mu_i)$'s be $(1-\lambda_i)$. Hence $\vee_{i=1}^{\infty}(\mu_i)$ is a fuzzy first category set. Again, since (X,T) is a fuzzy semi D-Baire space, $\vee_{i=1}^{\infty}(\mu_i)$ is a fuzzy nowhere dense set in (X,T). Then, by theorem ??, $(1-\vee_{i=1}^{\infty}(\mu_i))$ is a fuzzy dense set in (X,T) and hence $\wedge_{i=1}^{\infty}(1-\mu_i)$ is a fuzzy dense set in (X,T). That is, $cl(\wedge_{i=1}^{\infty}(1-\mu_i)) = 1$. Now $\wedge_{i=1}^{\infty}(1-\mu_i) \leq \wedge_{i=1}^{N}(1-\mu_i) = \wedge_{i=1}^{N}(1-(1-\lambda_i)) = \wedge_{i=1}^{N}(\lambda_i)$. Hence $cl(\wedge_{i=1}^{\infty}(1-\mu_i)) \leq cl(\wedge_{i=1}^{N}(\lambda_i))$ and hence $1 \leq cl(\wedge_{i=1}^{N}(\lambda_i))$. That is, $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$. Hence $cl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi

Volterra space.

Proposition 5.3 *If the fuzzy topological space (X,T) is a fuzzy semi Volterra and fuzzy P-space and if $\lambda = \wedge_{i=1}^{N} (\lambda_i)$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in X, then $cl(\lambda) = 1$ but $int(\lambda) \neq 0$, in (X,T).*

Proof. Let $\lambda = \wedge_{i=1}^{N} (\lambda_i)$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is fuzzy semi Volterra space, $cl(\lambda) = cl(\wedge_{i=1}^{N} (\lambda_i)) = 1$ implies that $cl(\lambda) = 1$. Now it has to be proved that $int(\lambda) \neq 0$. Since (X,T) is fuzzy semi Volterra space, as in the proof of proposition ??, there exists a fuzzy F_σ-set μ in (X,T) such that $1-\lambda < \mu$. Since μ is a fuzzy F_σ-set, $1-\mu$ is a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ-set $(1-\mu)$ is a fuzzy open set in (X,T). Then μ is fuzzy closed set in (X,T). Therefore $cl(\mu) = \mu$. Now $cl(1-\lambda) < cl(\mu)$. Therefore $1 - int(\lambda) < cl(\mu) = \mu$. Then $1-\mu < int(\lambda)$. Therefore $int(\lambda) \neq 0$, in (X,T).

Proposition 5.4 *If the fuzzy P-space (X,T) is a fuzzy semi hyperconnected space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy G_δ-sets (λ_i)'s are fuzzy open sets in (X,T). Then $\wedge_{i=1}^{N} (\lambda_i) \in T$. Since (X,T) is a fuzzy semi hyperconnected space, $\wedge_{i=1}^{N} (\lambda_i) \in T$, implies that $cl(\wedge_{i=1}^{N} (\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 5.5 *If the fuzzy topological space (X,T) is a fuzzy semi submaximal and fuzzy semi hyperconnected space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy semi submaximal space, $cl(\lambda_i) = 1$, implies that $\lambda_i \in T$. Now $\lambda_i \in T$ implies that $\wedge_{i=1}^{N} (\lambda_i) \in T$. Also since (X,T) is a fuzzy semi hyperconnected space, $\wedge_{i=1}^{N} (\lambda_i) \in T$, implies that $cl(\wedge_{i=1}^{N} (\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

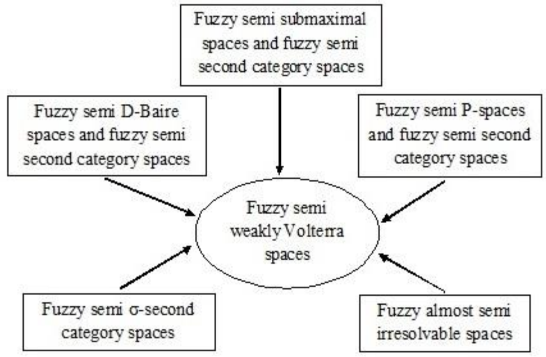
Proposition 5.6 *If a fuzzy topological space (X,T) is a totally fuzzy second category, fuzzy regular and fuzzy P-space, then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (X,T) be a totally fuzzy second category, fuzzy regular and fuzzy P-space. Let (λ_i)'s $(i = 1 to \quad \infty)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Since (X,T) is a P-space, (λ_i)'s are fuzzy G_δ-sets in (X,T) implies that (λ_i)'s are fuzzy open sets in (X,T). By theorem ??, $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X,T). Now the fuzzy set $\lambda = \vee_{i=1}^{\infty} (1-\lambda_i)$ is a fuzzy first category set in (X,T). Since (X,T) is a totally fuzzy second category, fuzzy regular space, by theorem ??, (X,T) is a fuzzy semi Baire space. Then by theorem ??, $int(\lambda) = 0$ in (X,T). This implies that $int(\vee_{i=1}^{\infty} (1-\lambda_i)) = 0$. Then $int(1 - \wedge_{i=1}^{\infty} (\lambda_i)) = 1 - cl(\wedge_{i=1}^{\infty} (\lambda_i))$ implies that $1 - cl(\wedge_{i=1}^{\infty} (\lambda_i)) = 0$. That is, $cl(\wedge_{i=1}^{\infty} (\lambda_i)) = 1$. Then, $cl(\wedge_{i=1}^{N} (\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T) [since $cl(\wedge_{i=1}^{\infty} (\lambda_i)) \leq cl(\wedge_{i=1}^{N} (\lambda_i))$]. Therefore (X,T) is a fuzzy semi Volterra space.

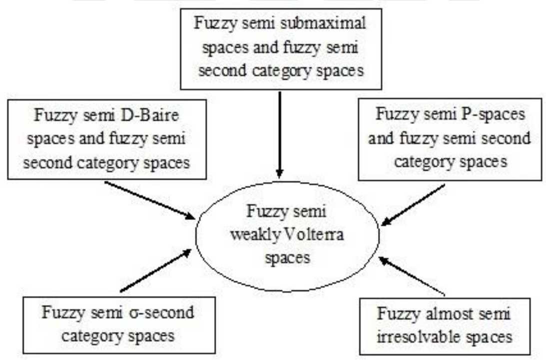
Proposition 5.7 *Let the fuzzy topological space (X,T) be a fuzzy maximally irresolvable and fuzzy semi hyperconnected space. Then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (X,T) be a fuzzy maximally irresolvable space. Then (X,T) is a fuzzy irresolvable space and every fuzzy dense set is a fuzzy open set in (X,T). Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy dense and fuzzy G_δ-sets in (X,T). Then by hypothsis, (λ_i)'s are fuzzy open sets in (X,T). Therefore $\wedge_{i=1}^{N} (\lambda_i)$ is fuzzy open in (X,T). Since (X,T) is a fuzzy semi hyperconnected space, $\wedge_{i=1}^{N} (\lambda_i)$ is a fuzzy dense set in (X,T). Hence $cl(\wedge_{i=1}^{N} (\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Remark. *The relationship among the classes of fuzzy semi Baire spaces, fuzzy semi Volterra spaces and fuzzy weakly Volterra spaces can be summarized in the following figure.*



Remark. *The relationship among the classes of fuzzy semi Baire spaces, fuzzy semi strongly irresolvable spaces, fuzzy semi submaximal spaces, fuzzy semi P-spaces, totally fuzzy second category spaces, fuzzy semi σ-Baire spaces, fuzzy semi D-Baire spaces, fuzzy regular spaces, fuzzy semi hyperconnected spaces and fuzzy semi Volterra spaces can be summarized in the following figure.*



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